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CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

LIKE FATHER, LIKE SON: SOCIAL NETWORKS, HUMAN CAPITAL INVESTMENT, AND SOCIAL MOBILITY

Antoni Calvó-Armengol
ICREA, Universitat Autònoma de Barcelona and Université de Toulouse.

Matthew O. Jackson
California Institute of Technology

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Antoni Calvó-Armengol   Matthew O. Jackson

Abstract

We build a model where an individual sees higher returns to investments in human capital when their neighbors in a social network have higher levels of human capital. We show that the correlation of human capital across generations of a given family is directly related to the sensitivity of individual investment decisions to the state of the social network. Increasing the sensitivity leads to increased intergenerational correlation, as well as more costly investment decisions on average in the society. We calibrate a simple threshold version of the model to data from a variety of EU nations. We also show how directly analyzing sensitivity of decisions to social circumstances can lead to information that is not captured by intergenerational correlation.

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Like Father, Like Son: Social Networks, Human Capital Investment, and Social Mobility

Antoni Calvó-Armengol*and Matthew O. Jackson†

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1 Introduction

Wages and social class are strongly correlated across parents and their children, and the strength of this intergenerational correlation is similar across countries, and persistent over time. Economists' evidence for this is based on parent-child correlation of (log) earnings and income. Recent estimates of the intergenerational correlation of long-run log earnings lie in the range $[.4, .6]$ for the U.S. and the U.K., and $[.2, .4]$ for Germany and Sweden.[1] Sociologists' analyses of intergenerational correlation focus on mobility tables. Given a hierarchy of occupational classes (or social classes), mobility tables relate the children-class destination to the parents-class origin. Odds ratios compute the relative likelihood of identical versus different parent-child class. Estimated odds ratios vary from 1 and 15 depending on the occupational class and for a large set of countries.[2]

It is important to note that there are strong similarities in mobility patterns despite drastic differences in countries in terms of their labor market regulations, level of development, education

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[1] See Björklund and Jännti (1999), Solon (2002), Piketty (2000) and references therein, in particular, Solon (1992), Zimmerman (1992), Bjö rklund and Jännti (1997), Dearden *et al.* (1997), and Mulligan (1997). Note that the data sets used for the different national studies differ in some important statistics (such as the average age of children and parents, etc.), which may introduce some national biases in the estimates. The income correlation is even higher and lies in the $[.7, .8]$ range for the U.S. (Mulligan 1997). Also, most of the data analyzed concerns only fathers and sons, which introduces some (gender) bias in the estimates. When daughters are included in the data set, the observed intergenerational correlation of income is usually higher (Mulligan 1997).

[2] See Ganzenboom and Treiman (1996) and Erikson and Goldthorpe (1992, 2002). Björklund and Jännti (1999) describe an alternative measure of social mobility based on the intergenerational correlation of an index of occupational status.

policies, and other characteristics.[3] Altogether, the observed dynastic correlation of earnings can hardly be explained by resorting to country-specific institutional and market variables.

In this paper, we propose a simple model based on social structure that exhibits strong family persistence in human capital investments and resulting earnings across generations.[4] In our model, skills are agent-specific. The intergenerational correlation in human capital investments and earnings is solely driven by the influence of the social setting on economic decisions. More precisely, we suppose that the marginal returns from higher human capital levels increase with the human capital composition of an agent's social network. This means that an agent's investment decisions are positively related to the state of the agent's social network. This, in turn, correlates with the parent's social decision to the extent that the social situations are related.

Economists and sociologists propose different channels for this social externality, including role-models, peer-pressure, and heredity of preferences or cultural traits.[5] Social networks, which are pervasively used in labor markets to disseminate job information, can also lead to substantial complementarities in human capital investment decisions.[6] We show that this social externality correlates human capital investments across generations, which further translates into correlation of parent-child earnings. We also establish a monotonicity of the intergenerational correlation as a function of the strength of the social externality. Higher sensitivity of investment decisions to social circumstance leads to high correlation across generations. Lastly, we show that higher sensitivity of investment decisions to social circumstance leads to less efficient investment decisions in a specific sense.

Of course, there are other theories of social immobility. Ours is complementary to those theories. The central economic theories of social mobility resort to the family transmission of economically relevant traits, capital market imperfections, or community-wide effects. For instance, wealth transfers in the form of bequests by altruistic parents are a clear source of dynastic correlation of income, while the genetic inheritance of productive abilities correlates human capital and labor earnings

[3]The time-series persistence also suggests some resilience of mobility patterns to school reforms and other labor market policies.

[4]Parent-offspring correlation in labor earnings explains much of the income correlation: "the main component (at least 70%) of the intergenerational correlation of welfare is due to the persistent inequality of labor earnings" Piketty (2000), p. 446.

[5]Role-model theories assert that children learn how to behave by observing the adults in their social network (Jencks and Mayer 1990). Exogenous norm-enforcing mechanisms also induce conformism among peers (Coleman 1990). A setting where preferences for conformity are endogenized appears in Calvó-Armengol and Jackson (2004,2005) for the case of "drop-out" decisions based on the state of one's network contacts. Heredity of preferences theory presumes that low-class children inherit preferences that discount future payoffs more than children from wealthier parents, and hence they invest less in human capital (Boudon 1973). Heredity of cultural traits theories claims that high-class children inherit better suited attitudes and aptitudes for culture and education, and then correlation follows from a parental earnings bias in social capital endowment for the offspring (Bourdieu and Passeron 1964).

[6]See Calvó-Armengol and Jackson (2004,2005) for details.

across generations (Becker and Tomes 1979). Also, imperfect credit markets impose borrowing constraints at the bottom of the earnings distribution, and so children of poor parents under-invest in human capital, and initial inequalities persist across generations (Loury 1981). Finally, segregation of individuals into homogeneous communities spurs positive spillover effects (e.g., local public goods) that homogenize economic outcomes across community members and generations (Bénabou 1993, 1996 and Durlauf 1996).

One caricature of the difference between our work and some of these others is as follows. Intergenerational interaction affects both the relative costs and benefits of investing in human capital, which in turn leads to correlation in decisions across generations. Things like wealth bequests, capital market frictions, and local public good provision effectively work on the cost side of the equation. Genetic inheritance can be thought of as working on either (or both) sides. Here we are focussing on the social interactions that operate effectively on the benefit side.[7]

While all of these factors are likely present, our model highlights the role of the social environment as a driver for social immobility, and shows how the sensitivity of investment decisions to social circumstances alone can drive intergenerational correlation. This is consistent with empirical evidence that identifies parental community traits and social background characteristics, instead of credit market imperfections or genetic inheritance, as the leading force in shaping the observed mobility patterns.[8] Thus, it is important to model social network influence on mobility and human capital investments.

We also note that with increased sensitivity to social circumstance comes increased overall investment costs, even for the same overall average level of investment. This can be seen as a type of inefficiency. Our conclusion that the dependence on a social channel leads to inefficient human capital investment decisions, echoes conclusions found by modeling immobility due to other sources. For instance, inefficiencies are also present in incomplete market models of social mobility (Loury 1981, and Banerjee and Newman 1991) and assortative matching models (Becker 1973, Cole, Mailath, and Postlewaite 1994).

Finally, we also show that there is information obtained by examining the parameters of our

[7] While we model it this way, it is not always exclusively so. For instance, access to well-placed friends may lower costs of capital or lower other barriers. We focus on the benefit side to isolate the importance of social interaction.

[8] The family inheritance of innate abilities is claimed to account for around 1/4 of the observed persistence in intergenerational earnings (Bowles and Gintis 2002). Borrowing constraints do not seem to explain more than 8% of the observed differences in returns to human capital investment, while non-cognitive skills related to the social background play a major causal role (Borjas 1992, Carneiro and Heckman 2002, Heckman and Krueger 2004). In France, higher social origin leads to better-paid first jobs for identical educational achievements, and the difference widens with seniority (Goux and Maurin, 1997). In the U.S., the convergence of the college enrollment rates per race, together with the persistence in the white-black wage differential, provides indirect evidence of the (long-standing) labor market value of the social background (Card and Krueger 1992, Kane 1994). More generally, the strength of the parents-children income relationship varies non-linearly with the parental neighborhood income levels, with a higher persistence at the two tails of the distribution – affluent and poor areas (Cooper *et al.* 1994).

model that is not exhibited by the use of correlation as a measure of immobility. To illustrate this point, we show how calibration of our model suggests that there are substantial differences in behavior and mobility across countries that are not being picked up by looking at intergenerational correlations.

Before proceeding, let us emphasize at the outset that although we focus on human capital investment and social mobility, the same model can be applied to a much wider variety of decisions that depend on social context. Effectively, many situations where individuals are making choices where an individual's choice of whether or not to undertake an action depends on neighbors' decisions can be viewed through this lens.

Section 2 presents the model. Section 3 describes an example with two dynasties. Section 4 analyzes a threshold investment model and estimates it for fourteen european countries using data from the European Community Household Panel. Section 5 presents results on the general model. Section 6 concludes. All proofs are relegated to an appendix.

2 The model

We focus on the behavior of a given community of families (dynasties), who interact in a symmetric way. This abstracts from more general networks of interactions, but provides a simple and clean analysis that already captures many of the critical effects.

2.1 Dynasties and (Random) Over-lapping Generations

There are n dynasties indexed by $i = 1, \ldots, n$. Time evolves in discrete periods indexed by $t = 1, 2, \ldots$. Each generation of a given dynasty consists of only one member. When there is no confusion, we identify current generation members by their dynasty index i.

At the beginning of each period, one dynasty is randomly chosen and its member replaced by his or her offspring. This happens with equal probability across dynasties.[9]

2.2 Human Capital Investment and the Social Setting

When they are born, offspring invest in human capital. This is a once-and-for-all decision.

We let human capital take two values: high and low. The analysis has an obvious extension to any finite set of values. Letting h_i^t denote the human capital of the agent in dynasty i at time t, we set $h_i^t = 1$ for the high level, and $h_i^t = 0$ for the low level.

Let $w_i(h)$ denote i's expected discounted stream of wages conditional on the vector of current human capital levels being h.[10]

[9] Therefore, the expected life span of any generation in dynasty i is $\sum_{t=1}^{+\infty} t\frac{1}{n}(1-\frac{1}{n})^t = n-1$. Thus, the length of a time period could be taken to be proportional to the expected lifetime of an agent divided by $n-1$.

[10] Given the Markov properties of the model, this is the same in any period, conditional on h.

We normalize $w_i(0, h_{-i}) = 0$ for all h_{-i}, so that if i does not invest in human capital then his or her wage is 0. We let $w_i(1, h_{-i}) > 0$ for all h_{-i} so that high human capital leads to higher wages than low human capital. More importantly, we assume that $w_i(1, h_{-i})$ is non-decreasing in h_j for all $j \neq i$. As we argue below, this monotonicity arises endogenously when social contacts convey job information, a fact largely documented in many labor markets.[11]

For simplicity in what follows we look at a symmetric setting. That is, we suppose that the functions w_i are the same across i, denoted simply by w, and that this function depends on h_{-i} in a symmetric manner. Thus, it depends only on the number of an agent's neighbors who have invested and not the specific identities. As will be clear, the model has straightforward extensions to cases where these simplifying assumptions are discarded.

We thus let $k^t = \sum_i h_i^t$, and for any i let $k_{-i}^t = \sum_{j \neq i} h_j^t$.

Agents are also born with a randomly drawn cost c of investing in the high human capital level, which is described by a cumulative distribution function $F(c)$. We assume that at birth each agent gets a draw from F. An agent's human capital level is 0 unless the agent decides to invest.

This cost encompasses innate skills and abilities, as well as direct costs of education. Note that we completely abstract from any correlation in costs of education across generations. *We do not do this because we believe that costs are independent across generations, but rather because we wish to isolate the social setting as a driver of social immobility.*[12]

Thus, an agent i's decision can be completely characterized by a vector $p = (p_0, \ldots, p_{n-1})$, where $p_k = F(w(1, k))$ is the probability that he or she invests in a high human capital level when the human capital levels of his or her neighbors are summarized by $k_{-i} = k$. The human capital level is 0 with complementary probability $1 - p_k$.

The non-decreasing nature of $w(1, \cdot)$ in k_{-i} implies that $p(k_{-i})$ is also non-decreasing in k_{-i}. The human capital composition of the social setting positively affects individual human capital investment decisions, as it increases the marginal returns from education.

As mentioned in the introduction, economists and sociologists have proposed and documented a variety of channels for this positive social externality, including role-model theory, peer-pressure for conformism, heredity of cultural traits or skills. Given the direct modeling of the social interaction through the variation of p_k as a function of k, our model encompasses all such channels. Our model is also compatible with the prevalent use of social networks in many labor markets, and its effect on the individual and aggregate dynamics of labor market outcomes, which we discuss shortly.

[11] See, for instance, the recent evidence reported in Santamaría-García (2003), as well as the discussion and references in Calvó-Armengol and Jackson (2004) and the recent survey of Ioannides and Loury (2004).

[12] Intergenerational correlation in costs of education has been studied as a driver of social immobility, both theoretically and empirically. See Bowles and Gintis (2002) and references therein.

2.3 A Markov Process

The random overlapping generations model, together with the human capital investment decision with idiosyncratic costs generates a Markov process. The state is the number of agents k^t at the high human capital level at the end of a period t, and transition probabilities can be derived from the vector $p = (p_0, \ldots, p_{n-1})$.

This is a finite-state irreducible and aperiodic Markov process. We characterize the long-run steady-state distribution of this process.[13] Given the symmetry in the model, we need only to keep track of how many agents are of the high type in any period. Thus, the steady-state of the Markov process can be described by $\mu = (\mu_0, \ldots, \mu_n)$, where μ_k is the probability that k agents are of the high type.

We examine the parent-offspring correlation in human capital levels under this steady-state distribution.

2.4 An Example: Labor Market Networks

The critical assumption for our results is that p_k is non-decreasing in k.

We now briefly show that one (among many) justifications for this assumption is having job information dispersed through social connections. Such a formal model is fully developed by Calvó-Armengol and Jackson (2004,2005), so we simply describe the results informally here, and refer the reader to those articles for the details.

Assume that individuals are connected by a social network, where network links represent direct communication channels between pairs of agents who know each other. The labor market is subject to turnover, with some agents randomly losing their jobs and others randomly hearing of available jobs in any give time period. When a currently employed worker cannot obtain a wage raise with one of these outside offers, he or she relays this information to his or her friends or acquaintances in the social network. When information about job opportunities is disseminated in this way, wages and employment are positively correlated across agents in both the short and long run.[14]

Let individual payoffs be the expected discounted stream of wages conditional on a given network. The correlation in wages implies that individual payoffs depend positively on the status of the whole of the individual's component of network. Therefore, the payoff to every agent in the

[13] For such a Markov process, it is well-known that the steady-state distribution has several nice features. First, it represents the relative frequencies spent in each state over long time horizons. Second, starting with a random draw from that distribution, the distribution over next period states is governed by the same distribution. Third, starting from any state, given a long enough horizon, the probability that one will end up in any give state is approximately given by the steady state distribution.

[14] See Propositions 1 and 2 in Calvó-Armengol and Jackson (2004) for the case of constant wages, a fixed network and uniform information transmission to direct connections, and Theorem 1 in Calvó-Armengol and Jackson (2005) for the general case with heterogeneous wages, general networks (random, weighted, directed, etc.) and general information diffusion (relayed information, preferential passing, etc.).

market is non-decreasing in the human capital level of every other agent. Formally, $(1, k_{-i})$ is non-decreasing in k_j for each $j \neq i$.

For the purpose of illustration, let us explore a very simple example where n agents are connected through a complete network (they all know each other directly). In each period, a currently employed worker loses his or her job with a probability $b = .015$, and all workers (both employed an unemployed) independently hear of a new job opportunity with probability $a = .100$ (which roughly calibrates reasonable employment rates). If a worker is currently at the highest wage level, the worker passes any job information on to an unemployed neighbor. If the worker is not currently at the highest wage level, then he or she keeps the information. A low human capital level $h = 0$ is normalized to pay $w = 0$. A high human capital level $h = 1$ pays either $w = 1$ when the agent has heard of one job offer since their last unemployment spell, and $w = 2$ when the agent has heard about more than one job opportunity since their last unemployment spell.[15]

Number of agents	$n = 1$	$n = 2$	$n = 4$	$n = 8$	$n = 16$	$n = 32$	$n \to \infty$
$E_t[w_i^t]$	0.10	0.19	0.38	0.73	1.26	1.77	1.97
$\Pr\{w = 2\}/\Pr\{w = 1\}$	0	.05	.17	.47	1.4	6.6	∞

This table simply indicates how a given agent's wage prospects improve in a given period as the number of neighbors is increased. This calculation presumes all other agents are employed at the high wage level, and that the given agent starts unemployed. One can also easily do steady-state calculations, and for various network configurations, as reported by Calvó-Armengol and Jackson (2004,2005).

3 Correlation in Parent-Child Human Capital: An Example with Dyads

Before moving to a full analysis, we start with an example with just two dynasties ($n = 2$). This makes the intuition very clear.

The following figure summarizes the transition probabilities. A darkened node represents an agent with high human capital, while agents with low human capital are represented by empty nodes. Arrows represent state transitions with their corresponding probabilities. The probability with which the state does not change is indicated by the expression near the dyad.

[15] The higher wage could represent an expected improved match, or improved bargaining power during the wage-setting negotiation.

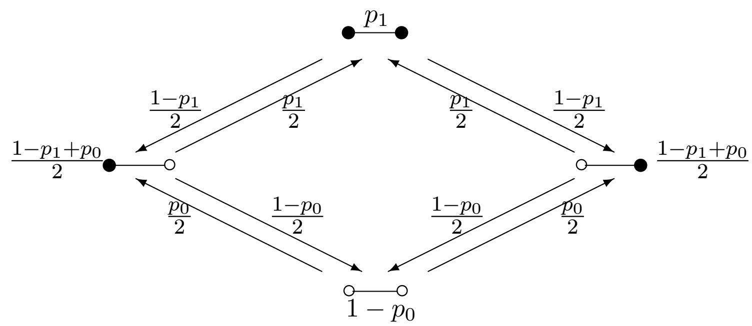


Figure 1: A Two Dynasty Markov Process

So, for instance, the p_1 at the top of the diagram is the probability that the state starts with both dynasties at high human capital and stays there.

The spread $p_1 - p_0$ reflects the sensitivity of human capital decisions to the social network. If p_0 is close to p_1, then one agent's wages and human capital investment decisions are largely independent of the status of the other agent. If the gap between these probabilities is large, then the decisions are very sensitive to the state of the other agent.

Let $\mu = (\mu_0, \mu_1, \mu_2)$ be the steady-state distribution, where μ_k is the probability that k agents have high human capital level. Direct calculations show that

$$\begin{bmatrix} \mu_2 \\ \mu_1 \\ \mu_0 \end{bmatrix} = \frac{1}{1+p_0-p_1} \begin{bmatrix} p_0 p_1 \\ 2p_0(1-p_1) \\ (1-p_0)(1-p_1) \end{bmatrix}.$$

We see that both dynasties are invested at the high human capital level with a long-run probability μ_2 that increases in both p_0 and p_1. In contrast, the probability of the joint low investment, μ_0, decreases in both p_0 and p_1. Finally, the probability that dyad members display different human capital levels μ_1 increases in p_0 but decreases in p_1.

We can easily compute the parent-offspring correlation in human capital levels under the steady-state distribution μ, denoted ρ. It follows that[16]

$$\rho = (p_1 - p_0)^2.$$

The intergenerational correlation increases with the square of the sensitivity of the human capital investment decision to the state of the other agent's status, $(p_1 - p_0)^2$. Note that the correlation is related to the difference between p_1 and p_0. If we simply have high levels of both p_1 and p_0, we would have low correlation. The spread between p_1 and p_0 captures how sensitive decisions are to the state of the network.

[16] The joint probability with which two consecutive generation members of the same dynasty have high human capital level is $p_1\mu_2 + \frac{1}{2}p_0\mu_1$. The expression follows from simple algebra. Note that the *intergenerational correlation* in human capital levels involves the joint probability of high human capital for *consecutive generations* of a same dynasty. This differs from the *correlation across dynasties*, which involves the joint probability of high (low) human capital for *contemporaneous generations*.

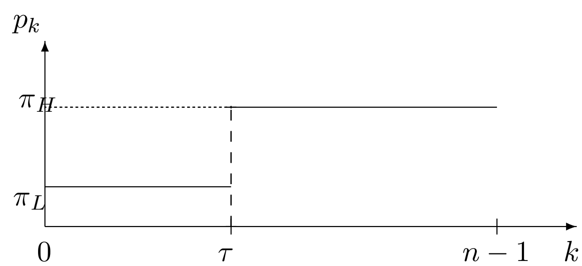
4 A Threshold Investment Model

We now turn to populations of size $n \geq 2$. We start with a specific, but interesting, model for human capital investment decisions. We call this a "threshold" model, since a given agent's decision on whether or not to invest in human capital depends on whether or not the number of high capital neighbors exceeds a given threshold.

We say that there are threshold investment decisions if there exists $1 \geq \pi_H \geq \pi_L \geq 0$ and $\tau \in \{1, \ldots, n-1\}$ such that $p_k = \pi_H$ if $k \geq \tau$ and $p_k = \pi_L$ is $k < \tau$. The difference between the high and the low success probability is $\pi_H - \pi_L$.

Holding τ fixed, when π_H increases and/or π_L decreases, individual investment decisions become more sensitive to the state of the society.

The transition probabilities are depicted below.



4.1 Human capital correlations under the threshold model

We now show that, holding τ fixed, an increase in π_H and a decrease in π_L that keep the average investment level $\overline{p}$ constant lead to an increase in intergenerational correlation.

If (π_H, π_L) and (π'_H, π'_L) are such that $\pi'_H > \pi_H$, $\pi'_L < \pi_L$ and $\overline{p}$ is the same under (n, τ, π_H, π_L) than under $(n, \tau, \pi'_H, \pi'_L)$, we say that (π'_H, π'_L) is a $\overline{p}$-preserving spread of (π_H, π_L).

Proposition 1 *An $\overline{p}$-preserving spread increases the (steady state) intergenerational correlation in human capital investments.*[17]

4.2 Fitting the Threshold Model to Data

Since the threshold investment model is fully described by (n, τ, π_H, π_L), it is easily calibrated. In order to minimize the degrees of freedom in the model, we fit a further simplified version of the threshold model. We restrict attention to the case where $\pi_L = 1 - \pi_H$ and $\pi_H \geq 1/2$.

The European Community Household Panel (ECHP) contains information on the level of human capital of representative households in various European member countries. These data allow us

[17] This refers to the correlation between parent and child investment states of any given dynasty at any time, when the initial distribution is the steady state distribution.

to infer the education transition matrices for combinations of parents and children.[18]

For this calibration, a human capital of 0 indicates has an individual pursued education to a point no further than high school graduation and 1 indicates education was pursued beyond high school.[19] Given the threshold model, any (n, τ, π) implies steady state probabilities of the four possible parent-child education combinations: 00, 01, 10, 11. So given the observed frequency distribution in the data, we find the parameters (n, τ, π) that lead to predictions that closely match those observed frequencies. Table 1 shows the calibrated values of π for various European countries and for all parent-offspring gender combinations when $n = 50$. An appendix contains the observed matrices, together with the estimated ones, the estimated τ's and an estimation error ε.[20] We also estimate the model for $n = 25$ and find almost no differences in the fits (see Tables 2 to 5 in the appendix).

	Father Daugh			*Father Son*			*Mother Daugh*			*Mother Son*		
Country	*Cor*	π	τ	*Cor*	π	τ	*Cor*	π	τ	*Cor*	π	τ
Austria	.123	.95	26	.057	.96	26	−.002	.97	30	.016	.96	29
Belgium	.027	.62	27	.174	.68	26	.014	.70	30	.100	.62	26
Denmark	.047	.73	35	.071	.77	34	.004	.75	35	.008	.74	21
Finland	−.128	.82	33	−.036	.85	32	−.061	.82	34	.014	.78	33
France	.119	.61	39	.221	.73	26	.093	.70	35	.176	.63	27
Germany	.107	.79	27	.111	.81	27	.067	.86	27	.045	.85	17
Greece	.003	.81	28	.070	.86	26	.011	.88	33	.001	.83	27
Ireland	.211	.82	26	.189	.84	26	.190	.85	26	.146	.83	26
Italy	.131	.94	26	.141	.95	26	.091	.96	26	.130	.95	26
Luxemburg	.289	.79	26	.083	.78	27	.145	.80	34	.070	.77	33
The Netherlands	.043	.89	31	.082	.87	31	−.021	.93	30	.005	.93	30
Portugal	.098	.94	26	.228	.96	26	.095	.96	26	.187	.94	16
Spain	.111	.81	26	.122	.84	26	.036	.86	32	.053	.82	32
United Kingdom	.045	.52	1	.070	.61	24	.023	.61	27	.095	.58	26

TABLE 1. ECHP intergenerational correlation in education vs. estimated investment probability

[18] We are grateful to Simona Comi for providing us with these education transition matrices corresponding to wave 5 (1998) of the ECHP. Comi (2003) contains an exhaustive analysis of European social mobility patterns with the ECHP database.

[19] Education beyond high school (tertiary education) is measured by Comi as still being in school at 20 years of age. It need not indicate that any higher degrees were earned.

[20] Given a value of n (here, $n = 25, 50$), our algorithm searches through a grid of admissible values for τ and π. The size of the grid is chosen in increments of .01 for π. The error ε is the Euclidean distance between the estimated and actual four-vectors.

in the threshold model.[21]

It is important to stress that the correlations and the implied parameters of the model give us different information. For instance, Denmark and the United Kingdom exhibit very similar father-daughter correlations (.045 and .047). Yet, the estimated values of π (and τ) differ drastically between both countries: a low .52 for the U.K. versus a higher .73 for Denmark.[22] Although similar in terms of correlation levels, the threshold model suggests, instead, that the influence of the social setting on education decisions (as measured through π) is much higher in Denmark than in the U.K. We can similarly analyze many other pairs of countries, such as Austria and France for father-daughter, Greece and the U.K. for father-son, France and Portugal for mother-daughter, etc.

Let us say a bit more about this. When seeing the low father-son correlation in Austria one would be tempted to conclude that this is a mobile society. However, when we see $\pi = .96$, we see a different picture. First, even with τ at 26, this leads to very little overall investment (.919 probability in the data that neither father nor son invested, and .920 from the fit of our model - see Table 3). Moreover, there is very high sensitivity to social setting, so that people whose majority of neighbors are not invested are extremely unlikely to invest, and people whose majority of neighbors are invested are extremely likely to invest. This is something that cannot be seen from the data, nor the correlation, and is inferable under the structure of the model. The fits imply that social circumstances are extremely important in determining individual decisions. In that sense, "mobility" has very little to do with one's own traits and everything to do with the social situation. Again, this contrasts with the data from the UK, where π is much closer to 1/2, where there is more overall investment, and where there is much less sensitivity to the social situation.

We also see some other interesting broader suggestions that come from the fits. With only one exception (in a case where the threshold does not really matter as π is nearly 1/2), the thresholds are all above $n/2$. This is consistent with the fact that in each country the average level of overall investment is less than 1/2. Second, and perhaps more important, most of the π's are fairly high - in a range of 3/4 or above. So, even though the intergenerational correlations lie mostly in a range from 0 to .2, we still see a substantial influence of the social situation on individual decisions.

[21] For each parent-offspring gender combination (father/daughter, father/son, mother/daughter, mother/son), the first column, *Cor*, reports the intergenerational correlation in education levels for the education transition matrices from wage 5 (1998) of the ECHP. We distinguish two education levels: (*i*) nor more than high school graduation, and (*ii*) past high school education. The second column, π, reports the estimated value for the investment probability in the threshold investment model when $n = 50$. The corresponding estimated values for τ and the error ε are reported in the appendix, together with estimations for π, τ, ε when $n = 25$. Data and estimations are reported for fourteen countries.

[22] The estimated value of τ for the U.K. is essentially irrelevant as a π close to 1 /2 implies that the investment is essentially independent of the state in any case. The comparison of father-son for the two countries leads to a situation where the τ is tied down for the *U.K.*.

This calibration exercise, although only illustrative, shows the parsimony of the threshold investments model and provides some across-country comparisons of the sources of social mobility. It also shows that this model can provide information that is not present in the correlations, and in particular information about the extent to which social situation influences individual decisions.

5 The General Case

We now turn to the analysis of the general model of investment decisions characterized by a vector of investment probabilities $p = (p_0, \ldots, p_{n-1})$.

5.1 A Simple Monotonicity Lemma

We begin with an obvious result that establishes monotonicity of the steady-state distribution of human capital decisions (in the sense of first-order stochastic dominance) with respect to the investment vector p.

We write $p' \geq p$ if $p'_k \geq p_k$, for all k, and $p' > p$ if $p'_k > p_k$, for all k. This could represent an increase in wages, a decrease in the cost distributions, or some other reason for increased investment as a function of the state of the network.

Lemma 1 *Let μ and μ' denote the steady-state distribution associated with p and p', respectively. If $p' \geq p$, then μ' first-order stochastically dominates μ. When $p' > p$, the first-order stochastic dominance is strict.*

5.2 Correlation in Parent-Child Human Capital

We now explore the relationship between the social sensitivity, as captured through p, and the intergenerational correlation in human capital decision, in the general model.

Intuitively, increasing the spread in p_k's across k makes the investment decision more sensitive to the social situation. That is, increasing p_k for high k and decreasing it for low k should lead the decision to invest to be more dependent on the social state. This in turn should increase the intergenerational correlation.

We capture this notion of increasing the spread in p_k's through the variance in the probability that a given agent invests. Let $Var(p)$ denote the variance in investment probabilities. So

$$Var(p) = \sum_k \mu_k^{-i} p_k^2 - (\sum_k \mu_k^{-i} p_k)^2 \tag{1}$$

where μ_k^{-i} is the steady-state probability that k agents other than some agent i are of the high type (under p).[23]

[23] We have $\mu_k^{-i} = \sum_{k=0}^{n-1} \frac{n-k}{n}\mu_k + \frac{k+1}{n}\mu_{k+1}$.

It turns out that keeping track of the variance in the p_k's alone is not enough to tie down intergenerational correlation, as it is this variance relative to the overall variation in an agent's state that is important. This normalization is necessary since it is only relative variation that matters in correlation comparisons rather than absolute levels.

The relevant normalizing variation is the variance of any given agent i's human capital state. In this simple 0-1 world that variance is simply

$$Var_p(h) = \overline{p}\,(1 - \overline{p})\,, \tag{2}$$

where $\overline{p}$ be the average steady-state probability that any given agent is of a high type given p.[24]

We can now provide a complete characterization of the relative orderings of correlations as dependent on properties of the p's.

Theorem 1 *The intergenerational correlation in human capital investments in any dynasty starting from the steady state is higher under p' than under p if and only if $\frac{Var(p')}{Var_{p'}(h)} > \frac{Var(p)}{Var_p(h)}$.*

An easy corollary of Theorem 1 is the following.

Corollary 1 *The intergenerational correlation in human capital investments in any dynasty starting from the steady state is higher under p' than under p if $Var(p') > Var(p)$ and $\overline{p} = \overline{p}'$.*

This follows from Theorem 1 by noting that if $\overline{p} = \overline{p}'$, then by (2), then $Var_{p'}(h) = Var_p(h)$.

The intuition behind the theorem and corollary is that as the correlation of the human capital investment with the state of the network increases, this leads to increased intergenerational correlation due to the fact that there is likely to be some overlap in the state of the network between the parent and the child.

So far, we have assumed that the offspring social background is the same as that of the parent when the parent dies. Suppose, instead, that this inheritance is only partial. The child's social universe is now mixed. He or she evolves in the same social universe as the parent with probability q, but belongs to a completely different social circle with probability $1-q$. More precisely, the child accesses the same network as his or her parent with probability q, and otherwise gets an entirely new network (with k_{-i}^t determined by a fresh draw from the steady state distribution). We model this process as if with probability q the child's investment decision is governed by p_k, and with probability $(1-q)$ it is given by $\overline{p}$. The influence of the social setting now depends on q, and we denote by p^q the resulting function.

Corollary 2 *The steady-state intergenerational correlation in human capital investments increases with q.*

[24] We have $\overline{p} = \sum_{k=0}^{n-1} \mu_k^{-i} p_k$.

In this case, it is easy to check that the conditions of Corollary 1 apply.

To the extent that the overlap of the social universe across generations is higher for the two-tails of the income distribution, this finding is consistent with the $U-$shaped intergenerational income correlation curve as a function of income levels documented by Cooper *et al.* (1994).[25]

5.3 Cost Efficiency

Beyond issues of equity and equality of opportunity, we should also care about social mobility for reasons of economic efficiency. We now examine relative mobility and efficiency in terms of overall investment costs. The idea is to track the distortions in investment that are present in a society. That is, suppose that one agent invests in human capital with a high cost due to the fact that his or her social network is in good shape, while another agent does not invest despite a much lower cost due to the fact that his or her network is in bad shape. This leads to more costly investment than would be present in a world where the agents were switched. In a perfect world, where we could exchange these two agents (and perhaps make some transfers) we would have an improvement.

A rough measure that keeps track of this distortion is the total expenditure on human capital investment of a society. That is, let us consider two societies described by p and p'. To keep a level playing field, let us compare societies with the same mean investment rate so that $\overline{p} = \overline{p}'$. We can then compare the average costs of investment in steady state.

To do this, let us derive an expression for the expected costs of investment in steady state. We do this under the supposition that any given agent's costs of investment are uniformly distributed on $[0, C]$. Recall, that the model is one where the benefits depend on the state of the system (k_{-i}) and then the costs are simply individual-specific. In particular, recall that $p_k = F(w(1, k))$. That is, the probability that i invests when k neighbors are invested is the probability that i's cost of investment is below $w(1, k)$, which is i's expected benefit from investing. Given that costs are uniformly distributed on $[0, C]$, $p_k = w(1, k)/C$. Conditional on investing when $k_{-i} = k$ we then can conclude that the expected costs are $w(1, k)/2$. Thus, these are $Cp_k/2$. Then, the average cost per capita of investing in high human capital can be written as

$$Cost(p) = \sum_{k=0}^{n-1} \frac{\mu_k^{-i} p_k}{\overline{p}} \frac{C}{2} p_k.$$

Here, $\mu_k^{-i} p_k / \overline{p}$ is the conditional probability that there were k other agents with high human capital at the time that i invested, conditional on i investing. We can then conclude the following.

[25] See Wright Mills (1945) for a seminal analysis of the social endogamy of the business elite in the U.S. and Wilson (1987) and Jencks and Mayer (1990) for an analysis of the social and economic consequences of living in the inner city in the U.S. Santamaría-García (2003) provides a model predicting a more extensive use of social contacts for job search among less-educated workers, consistent with empirical findings in many European countries and reported in the paper.

Theorem 2 *If $Var(p') > Var(p)$ and $\overline{p}' = \overline{p}$, then $Cost(p') > Cost(p)$.*

For two processes p and p' that have the same overall percentage of investment $\overline{p}$, the more sensitive one has a higher overall cost associated with it. It is thus less efficient in terms of the costs of investment for the same average level of investment.

6 Discussion

We have provided a parsimonious model that shows how individual decisions depend on the social state of a system, and how this leads to intergenerational correlation in decisions. Increased sensitivity of an individual's decision to the social state corresponds to increased intergenerational correlation. We also analyzed a special case of the model: the threshold investment model, which is a handy example and one for which the long-run steady-state distribution is easily characterized.

When n gets large, the steady-state distribution in the threshold model approximates a Poisson process. The steady-state distribution (under the restriction that $\mu_L = 1 - \mu_H$) is

$$\mu_k = \begin{cases} \mu_0 \binom{n}{k} \left(\frac{\pi_L}{1-\pi_L}\right)^k, \text{ for } k < \tau \\ \mu_0 \left[\frac{\pi_L(1-\pi_H)}{\pi_H(1-\pi_L)}\right]^\tau \binom{n}{k} \left(\frac{\pi_H}{1-\pi_H}\right)^k, \text{ for } k \geq \tau \end{cases},$$

where μ_0 normalizes the sum to one. When n gets large, we obtain two Poisson distributions to the right and to the left of τ, and appropriately renormalized.

In closing let us re-emphasize that our model encompasses other processes beyond the human capital investment and mobility application that we have analyzed. Consider, for instance, the following set up. Each period, a randomly selected newborn takes an 0-1 decision. With small probability ε, this decision is independent of the environment and decided by a coin toss. With complementary probability $1-\varepsilon$, the investment is context dependent. In that situation the newborn chooses 1 only if the first agent met has chosen 1, where the newborn is equally likely to meet any other agent in the society. The resulting probability of choosing 1 conditional on k other people in the society having chosen 1 is:

$$p_k = \frac{1}{2}\varepsilon + (1-\varepsilon)\frac{k}{n-1}.$$

This model is analyzed in Kirman (1993), in the context of ant behavior, as a proxy for some sorts of investment behavior. He shows that the continuous population limit of the steady-state distribution for this process is a Beta distribution.

We can apply Theorem 1 (and corollaries) to this process to draw some conclusions. For instance, they relate the value of ε to the intergenerational correlation levels in decisions. Decreasing ε amounts to a $\overline{p}$-preserving spread, and the correlation increases. Coupling this with Theorem 2, we can deduce overall cost effects of changing the likelihood of an independent decision (here, efficiency increases with ε). Our results thus provide a step towards measuring the efficiency effects of time series persistence.

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Appendix

The following lemma is useful.

Lemma **2** *For all* $0 \leq k \leq n-1$, $\mu_{k+1} = a_k \mu_k$, *where*

$$a_k = \frac{n-k}{k+1}\left(\frac{p_k}{1-p_k}\right),$$

and thus $\mu_k = a_j a_{j+1} \cdots a_{k-1} \mu_j$ *for all* $k > j$.

Proof of Lemma 2: Consider a state where exactly k agents are of high type. At steady-state, the inflow to and the outflow from this state exactly balance each other. This is written as

$$\begin{aligned}
\mu_0 p_0 &= \mu_1 \frac{1}{n}(1-p_0), \text{ for } k=0 \\
\mu_{k-1}\frac{n-(k-1)}{n}p_{k-1} + \mu_{k+1}\frac{k+1}{n}(1-p_k) &= \mu_k \frac{k}{n}(1-p_{k-1}) + \mu_k \frac{n-k}{n}p_k, \text{ for } 1 \leq k \leq n-1 \\
\mu_{n-1}\frac{1}{n}p_{n-1} &= \mu_n(1-p_{n-1}), \text{ for } k=n,
\end{aligned}$$

and the result follows. ■

Proof of Lemma 1: By Lemma 2, whenever $i > j$ we can write

$$\mu_i/\mu_j = a_j a_{j+1} \cdots a_{i-1}.$$

Noting that $p'_k \geq p_k$ implies $a'_k \geq a_k$ (with corresponding strict inequalities), it follows that

$$\mu'_i/\mu'_j \geq \mu_i/\mu_j$$

for all $i > j$, with strict inequality for some pairs when $p' \neq p$. Given that $\sum_{k=0}^{n} \mu'_k = 1 = \sum_{k=0}^{n} \mu_k$, the result follows directly. ■

Proof of Theorem 1: Let us consider any dynasty i. Consider any point in time (having started at time 0 from the steady state distribution) where a given newborn in role i is faced with the choice to invest. Let X be the number of other agents who have value 1 at that point in time. We know that

$$\Pr[X = k] = \frac{n-k}{n}\mu_k + \frac{k+1}{n}\mu_{k+1}. \tag{3}$$

Let Z be i's parent's value and Y be i's value. We can write the covariance of the parent and child's values as

$$Cov = \Pr[Z = Y = 1] - \Pr[Z = 1]\Pr[Y = 1].$$

We write this as

$$Cov = \left(\sum_{k=0}^{n-1} \Pr[Z = 1|X = k]\Pr[X = k|Y = 1]\Pr[Y = 1]\right) - \Pr[Z = 1]\Pr[Y = 1].$$

By definition, $\Pr[Y = 1] = \overline{p} > 0$. It then follows that

$$Cov = \overline{p}\left(\sum_{k=0}^{n-1} \Pr[Z = 1|X = k]\Pr[X = k|Y = 1]\right) - \overline{p}\Pr[Z = 1]$$

or

$$Cov = \overline{p}\sum_{k=0}^{n-1} \Pr[Z = 1|X = k]\left(\Pr[X = k|Y = 1] - \Pr[X = k]\right). \tag{4}$$

Note that $\Pr[Z = 1, X = k]$ (the probability that the parent is high human capital and there are k others of high human capital) is equal to $\mu_{k+1}(k+1)/n$. Note also that

$$\Pr[X = k|Y = 1] = \Pr[Y = 1|X = k]\frac{\Pr[X = k]}{\Pr[Y = 1]} = p_k\frac{\Pr[X = k]}{\overline{p}}.$$

Then, using (3), we rewrite (4) as

$$Cov = \overline{p}\sum_{k=0}^{n-1} \frac{\mu_{k+1}\frac{k+1}{n}}{\mu_k\frac{n-k}{n} + \mu_{k+1}\frac{k+1}{n}}\left(\Pr[X = k]\frac{p_k}{\overline{p}} - \Pr[X = k]\right). \tag{5}$$

Using the fact that $\mu_{k+1} = a_k \mu_k$ established in Lemma 2, (4) as

$$Cov = \overline{p} \sum_{k=0}^{n-1} \frac{a_k \frac{k+1}{n}}{\frac{n-k}{n} + a_k \frac{k+1}{n}} \left(\Pr[X = k] \frac{p_k}{\overline{p}} - \Pr[X = k] \right),$$

which, using the expression for a_k in Lemma 2 gives:

$$Cov = \sum_{k=0}^{n-1} p_k \Pr[X = k] \left(p_k - \overline{p} \right). \tag{6}$$

This implies that

$$Corr = \sum_{k=0}^{n-1} \frac{p_k}{\overline{p}} \Pr[X = k] \left(\frac{p_k - \overline{p}}{1 - \overline{p}} \right). \tag{7}$$

We rewrite this as

$$Corr = \frac{\sum_{k=0}^{n-1} p_k \mu_k^{-i} \left(p_k - \overline{p} \right)}{\overline{p}(1 - \overline{p})}.$$

Thus,

$$Corr = \frac{Var(p)}{Var_p(h)},$$

and the Theorem follows directly. ■

Proof of Proposition 1: From the proof of Theorem 1, we can write

$$\overline{p}(1 - \overline{p}) Corr = \sum_{k=0}^{n-1} \Pr[X = k] p_k \left(p_k - \overline{p} \right).$$

Given the threshold model, we can rewrite this as

$$\overline{p}(1 - \overline{p}) Corr = \pi_L \left(\pi_L - \overline{p} \right) \sum_{k=0}^{\tau-1} \Pr[X = k] + \pi_H \left(\pi_H - \overline{p} \right) \sum_{k=\tau}^{n-1} \Pr[X = k]. \tag{8}$$

Note that

$$\overline{p} = \pi_L \sum_{k=0}^{\tau-1} \Pr[X = k] + \pi_H \sum_{k=\tau}^{n-1} \Pr[X = k]. \tag{9}$$

Then, using (9) we rewrite (8) as

$$\overline{p}(1 - \overline{p}) Corr = \pi_L^2 \sum_{k=0}^{\tau-1} \Pr[X = k] + \pi_H^2 \sum_{k=\tau}^{n-1} \Pr[X = k] - \overline{p}^2 \tag{10}$$

Consider a differential change $(d\pi_H, d\pi_L)$ in (π_H, π_L) such that $d\pi_H > 0$, $d\pi_L < 0$, and $(\pi_H + d\pi_H, \pi_L + d\pi_L)$ is a $\overline{p}$-preserving spread of (π_H, π_L), that is, $d\overline{p} = 0$. In what follows, we use the following notation:

$$\sigma_\alpha^\beta = \sum_{k=\alpha}^{\beta} \Pr[X = k], \text{ for } \alpha \leq \beta.$$

Taking a differential of (10) under the constraint $d\overline{p} = 0$ gives

$$\overline{p}\,(1-\overline{p})\,dCorr = 2\pi_L \sigma_0^{\tau-1} d\pi_L + 2\pi_H \sigma_\tau^{n-1} d\pi_H + \pi_L^2 d\sigma_0^{\tau-1} + \pi_H^2 d\sigma_\tau^{n-1}. \tag{11}$$

Using (9), the condition $d\overline{p} = 0$ becomes

$$\sigma_0^{\tau-1} d\pi_L + \sigma_\tau^{n-1} d\pi_H + \pi_L d\sigma_0^{\tau-1} + \pi_H d\sigma_\tau^{n-1} = 0. \tag{12}$$

Multiplying (12) by $\pi_H + \pi_L$ and substracting it from (11) gives

$$\overline{p}\,(1-\overline{p})\,dCorr = (\pi_H - \pi_L)\left[\sigma_\tau^{n-1} d\pi_H - \sigma_0^{\tau-1} d\pi_L\right] - \pi_L \pi_H \left[d\sigma_0^{\tau-1} + d\sigma_\tau^{n-1}\right]. \tag{13}$$

Note that $\sigma_0^{\tau-1} + \sigma_\tau^{n-1} = 1$ (this is a sum of probabilities), and thus $d\sigma_0^{\tau-1} + d\sigma_\tau^{n-1} = 0$. We thus conclude that

$$\overline{p}\,(1-\overline{p})\,dCorr = (\pi_H - \pi_L)\left[\sigma_\tau^{n-1} d\pi_H - \sigma_0^{\tau-1} d\pi_L\right] > 0, \text{ when } \pi_H > \pi_L.$$

■

Proof of Theorem 2: We write

$$Cost(p) = \frac{C}{2\overline{p}} \sum_{k=0}^{n-1} \mu_k^{-i} p_k p_k, \tag{14}$$

which, by (7) we rewrite as

$$Cost(p) = \frac{C}{2}\left((1-\overline{p})Corr + \overline{p}\right).$$

The result then follows from Theorem 1 and the fact that $\overline{p}' = \overline{p}$. ■

Table 2. Father/Daughter.

data			estimation			threshold model		
AU	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.903	.033	0	.902	.048	0	.900	.048
1	.054	.008	1	.048	.003	1	.048	.005
			π=.95,τ=14,ε=.017			π=96,τ=26,ε=.017		
BE	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.357	.295	0	.257	.237	0	.356	.237
1	.181	.167	1	.237	.168	1	.237	.170
			π=.63,τ=14,ε=.080			π=.62,τ=27,ε=080		
DK	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.534	.061	0	.533	.197	0	.533	.197
1	.350	.054	1	.197	.073	1	.197	.073
			π=.73,τ=21,ε=.205			π=.73,τ=35,ε=.205		
FI	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.672	.053	0	.664	.151	0	.664	.151
1	.274	.002	1	.151	.034	1	.151	.034
			π=.82,τ=20,ε=.161			π=.82,τ=33,ε=.161		
FR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.366	.416	0	.368	.239	0	.366	.239
1	.071	.148	1	.239	.155	1	.239	.156
			π=.61,τ=16,ε=.244			π=61,τ=39,ε=.244		
GE	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.624	.063	0	.622	.163	0	.621	.166
1	.261	.052	1	.163	.052	1	.166	.047
			π=.80,τ=14,ε=.140			π=.79,τ=27,ε=.141		
GR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.646	.192	0	.648	.157	0	.648	.157
1	.124	.038	1	.157	.038	1	.157	.038
			π=.81,τ=15,ε=.048			π=.81,τ=28,ε=.048		

The first column reports the education transition matrices for father/daughter from wage 5 (1998) of the ECHP, from Comi. 0 stands for nor more than high school graduation; 1 stands for past high school education. Data is reported for seven countries: AU=Austria, BE=Belgium, DK=Denmark, FI=Finland, FR=France, GE=Germany, GR=Greece. Columns 2 and 3 report the estimated values for the education transition matrices for $n = 25, 50$. The corresponding parameters π, τ for the estimated threshold investment model are reported below the matrices. The error ε is equal to the Euclidean distance between the estimated and the actual distributions.

Table 2 (Father/Daughter) (Contd.)

data			estimation			threshold model		
IR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.645	.220	0	.648	.154	0	.643	.148
1	.063	.072	1	.154	.044	1	.148	.062
			π=.81,τ=14,ε=.116			π=.82,τ=26,ε=.112		
IT	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.876	.056	0	.874	.061	0	.880	.056
1	.055	.013	1	.061	.005	1	.056	.007
			π=.94,τ=14,ε.012			π=94,τ=26,ε=.007		
LX	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.580	.268	0	.577	.141	0	.576	.169
1	.045	.107	1	.141	.141	1	.169	.086
			π=.83,τ=13,ε=.163			π=.79,τ=26,ε=.160		
NL	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.791	.015	0	.792	.098	0	.792	.098
1	.188	.007	1	.098	.012	1	.098	.012
			π=.89,τ=18,ε=.122			π=.89,τ=31,ε=.122		
PO	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.865	.081	0	.865	.065	0	.870	.061
1	.043	.011	1	.065	.005	1	.061	.008
			π=.93,τ=14,ε=.028			π=.94,τ=26,ε=.028		
SP	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.623	.216	0	.622	.163	0	.624	.154
1	.098	.063	1	.163	.052	1	.154	.068
			π=.80,τ=14,ε=.085			π=.81,τ=26,ε=.084		
UK	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.246	.223	0	.230	.250	0	.230	.250
1	.254	.278	1	.250	.270	1	.250	.270
			π=.52,τ=25,ε=.032			π=.52,τ=1,ε=.032		

ECHP data and estimations are reported for the following seven countries: IR=Ireland, IT=Italy, LX=Luxemburg, NL=The Netherlands, PO=Portugal, SP=Spain, UK=The United Kingdom.

Table 3 (Father/Son)

data			estimation			threshold model		
AU	0	1	$n=25$	0	1	$n=50$	0	1
0	.919	.018	0	.922	.038	0	.920	.038
1	.060	.003	1	.038	.002	1	.038	.003
			π=.96,τ=14,ε=.030			π=.96,τ=26,ε=.030		
BE	0	1	$n=25$	0	1	$n=50$	0	1
0	.405	.227	0	.407	.200	0	.397	.218
1	.170	.198	1	.200	.194	1	.218	.167
			π=.73,τ=13,ε=.040			π=.68,τ=26,ε=.058		
DK	0	1	$n=25$	0	1	$n=50$	0	1
0	.587	.027	0	.585	.180	0	.585	.180
1	.356	.030	1	.180	.055	1	.180	.055
			π=.77,τ=21,ε=.234			π=.77,τ=34,ε=.234		
FI	0	1	$n=25$	0	1	$n=50$	0	1
0	.724	.032	0	.723	.128	0	.723	.128
1	.237	.006	1	.128	.023	1	.128	.023
			π=.85,τ=19,ε=.146			π=.85,τ=32,ε=.146		
FR	0	1	$n=25$	0	1	$n=50$	0	1
0	.481	.332	0	.477	.207	0	.478	.197
1	.058	.129	1	.207	.108	1	.197	.128
			π=.71,τ=14,ε=.196			π=.73,τ=26,ε=.194		
GE	0	1	$n=25$	0	1	$n=50$	0	1
0	.653	.053	0	.657	.151	0	.654	.154
1	.251	.043	1	.151	.041	1	.154	.038
			π=.82,τ=14,ε=.140			π=.81,τ=27,ε=.140		
GR	0	1	$n=25$	0	1	$n=50$	0	1
0	.726	.131	0	.728	.124	0	.721	.120
1	.111	.033	1	.124	.024	1	.120	.038
			π=.86,τ=14,ε=.017			π=.86,τ=26,ε=.016		

The first column reports the education transition matrices for father/son from wage 5 (1998) of the ECHP, from Comi. Data and estimations are reported for seven countries: AU=Austria, BE=Belgium, DK=Denmark, FI=Finland, FR=France, GE=Germany, GR=Greece.

TABLE 3 (FATHER/SON) (Contd.)

data			estimation			threshold model		
IR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.686	.196	0	.692	.138	0	.682	.134
1	.062	.056	1	.138	.032	1	.134	.049
			π=.84,τ=14,ε=.099			π=.84,τ=26,ε=.96		
IT	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.886	.047	0	.893	.052	0	.890	.052
1	.055	.012	1	.052	.003	1	.052	.006
			π=.95,τ=14,ε=.013			π=.95,τ=26,ε=.009		
LX	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.601	.266	0	.605	.169	0	.605	.172
1	.077	.056	1	.169	.057	1	.172	.052
			π=.79,τ=14,ε=.134			π=.78,τ=27,ε=.134		
NL	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.757	.011	0	.757	.113	0	.757	.113
1	.222	.010	1	.113	.017	1	.113	.017
			π=.87,τ=18,ε=.149			π=.87,τ=31,ε=.149		
PO	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.919	.038	0	.922	.038	0	.920	.038
1	.031	.012	1	.038	.002	1	.038	.003
			π=.96,τ=14,ε=.013			π=.96,τ=26,ε=.012		
SP	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.683	.156	0	.683	.141	0	.682	.134
1	.109	.051	1	.141	.035	1	.134	.049
			π=.83,τ=14,ε=.039			π=.84,τ=26,ε=.034		
UK	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.226	.237	0	.217	.235	0	.210	.241
1	.224	.312	1	.235	.314	1	.241	.309
			π=.65,τ=12,ε=.014			π=.61,τ=24,ε=.023		

ECHP data and estimations are reported for the following seven countries: IR=Ireland, IT=Italy, LX=Luxemburg, NL=The Netherlands, PO=Portugal, SP=Spain, UK=The United Kingdom.

Table 4 (Mother/Daughter)

data			estimation			threshold model		
AU	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.992	.037	0	.992	.038	0	.992	.038
1	.039	.001	1	.038	.002	1	.038	.002
			π=.96,τ=17,ε=.001			π=.96,τ=30,ε=.001		
BE	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.374	.314	0	.374	.238	0	.374	.238
1	.165	.147	1	.238	.151	1	.238	.151
			π=.62,τ=16,ε=.106			π=.62,τ=30,ε=.106		
DK	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.538	.068	0	.540	.195	0	.540	.195
1	.350	.045	1	.195	.070	1	.195	.070
			π=.74,τ=20,ε=.202			π=.74,τ=35,ε=.202		
FI	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.606	.043	0	.600	.175	0	.600	.175
1	.338	.013	1	.175	.050	1	.175	.050
			π=.78,τ=21,ε=.213			π=.78,τ=34,ε=.213		
FR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.381	.455	0	.391	.235	0	.390	.234
1	.055	.110	1	.235	.141	1	.234	.141
			π=.63,τ=17,ε=.287			π=.63,τ=35,ε=.287		
GE	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.714	.079	0	.710	.131	0	.713	.131
1	.176	.031	1	.131	.028	1	.131	.025
			π=.85,τ=14,ε=.069			π=.85,τ=27,ε=.069		
GR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.693	.202	0	.689	.141	0	.689	.141
1	.080	.025	1	.141	.029	1	.141	.029
			π=.83,τ=20,ε=.087			π=.83,τ=33,ε=.087		

The first column reports the education transition matrices for mother/daughter from wage 5 (1998) of the ECHP, from Comi. Data and estimations are reported for seven countries: AU=Austria, BE=Belgium, DK=Denmark, FI=Finland, FR=France, GE=Germany, GR=Greece.

Table 4 (Mother/Daughter) (Contd.)

data			estimation			threshold model		
IR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.663	.235	0	.666	.148	0	.662	.141
1	.046	.056	1	.148	.039	1	.141	.055
			π=.82,τ=14,ε=.135			π=.83,τ=26,ε=.134		
IT	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.891	.061	0	.893	.052	0	.890	.052
1	.040	.008	1	.052	.003	1	.052	.006
			π=.95,τ=14,ε=.016			π=.95,τ=26,ε=.015		
LX	0	1	$n = 25$	0	1	$n = 25$	0	1
0	.596	.330	0	.593	.177	0	.593	.177
1	.028	.046	1	.177	.053	1	.177	.053
			π=.77,τ=20,ε=.214			π=.77,τ=34,ε=.214		
NL	0	1	$n = 25$	0	1	$n = 25$	0	1
0	.856	.021	0	.856	.069	0	.856	.069
1	.126	.002	1	.069	.006	1	.069	.006
			π=.93,τ=18,ε=.072			π=.93,τ=30,ε=.072		
PO	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.866	.082	0	.865	.065	0	.870	.061
1	.041	.011	1	.065	.005	1	.061	.008
			π=.93,τ=14,ε=.030			π=.94,τ=26,ε=.030		
SP	0	1	$n = 25$	0	1	$n = 25$	0	1
0	.666	.254	0	.664	.151	0	.664	.151
1	.056	.028	1	.151	.034	1	.151	.034
			π=.82,τ=19,ε=.140			π=.82,τ=32,ε=.140		
UK	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.302	.302	0	.301	.248	0	.302	.247
1	.189	.208	1	.248	.203	1	.247	.205
			π=.55,τ=17,ε=.080			π=.58,τ=27,ε=.080		

ECHP data and estimations are reported for the following seven countries: IR=Ireland, IT=Italy, LX=Luxemburg, NL=The Netherlands, PO=Portugal, SP=Spain, UK=The United Kingdom.

Table 5 (Mother/Son)

data			estimation			threshold model		
AU	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.944	.019	0	.940	.029	0	.940	.029
1	.036	.001	1	.029	.001	1	.029	.001
			π=.97,τ=19,ε=.013			π=.97,τ=29,ε=.013		
BE	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.427	.282	0	.422	.222	0	.428	.210
1	.143	.147	1	.222	.135	1	.210	.152
			π=.68,τ=14,ε=.100			π=.70,τ=26,ε=.099		
DK	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.560	.035	0	.563	.188	0	.563	.188
1	.379	.026	1	.188	.063	1	.188	.063
			π=.75,τ=21,ε=.248			π=.75,τ=21,ε=.248		
FI	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.677	.024	0	.672	.148	0	.672	.148
1	.287	.012	1	.148	.032	1	.148	.032
			π=.82,τ=19,ε=.188			π=.82,τ=33,ε=.188		
FR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.474	.356	0	.477	.207	0	.469	.212
1	.057	.112	1	.207	.108	1	.212	.107
			π=.71,τ=14,ε=.212			π=.70,τ=27,ε=.212		
GE	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.732	.073	0	.731	.124	0	.730	.124
1	.171	.024	1	.124	.021	1	.124	.022
			π=.86,τ=15,ε=.070			π=.86,τ=17,ε=.070		
GR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.771	.151	0	.772	.106	0	.774	.106
1	.065	.013	1	.106	.016	1	.106	.015
			π=.88,τ=14,ε=.061			π=.88,τ=27,ε=.061		

The first column reports the education transition matrices for mother/son from wage 5 (1998) of the ECHP, from Comi. Data and estimations are reported for seven countries: AU=Austria, BE=Belgium, DK=Denmark, FI=Finland, FR=France, GE=Germany, GR=Greece.

Table 5 (Mother/Son) (Contd.)

data			estimation			threshold model		
IR	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.697	.214	0	.701	.134	0	.692	.131
1	.049	.041	1	.134	.030	1	.131	.046
			π=.84,τ=14,ε=.117			π=.85,τ=26,ε=.117		
IT	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.906	.051	0	.912	.043	0	.910	.043
1	.034	.009	1	.043	.002	1	.043	.004
			π=.96,τ=14,ε=.015			π=.096,τ=26,ε=.013		
LX	0	1	$n = 25$	0	1	$n = 25$	0	1
0	.642	.292	0	.640	.160	0	.640	.160
1	.036	.029	1	.160	.040	1	.160	.040
			π=.80,τ=20,ε=.181			π=.80,τ=33,ε=.181		
NL	0	1	$n = 25$	0	1	$n = 25$	0	1
0	.858	.019	0	.856	.069	0	.856	.069
1	.120	.003	1	.069	.006	1	.069	.006
			π=.93,τ=17,ε=.072			π=.93,τ=30,ε=.072		
PO	0	1	$n = 25$	0	1	$n = 25$	0	1
0	.912	.038	0	.912	.043	0	.912	.043
1	.039	.011	1	.043	.002	1	.043	.004
			π=.96,τ=14,ε=.011			π=.96,τ=16,ε=.010		
SP	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.734	.188	0	.737	.120	0	.731	.124
1	.056	.022	1	.120	.023	1	.124	.021
			π=.86,τ=14,ε=.094			π=.86,τ=32,ε=.094		
UK	0	1	$n = 25$	0	1	$n = 50$	0	1
0	.305	.328	0	.306	.237	0	.304	.242
1	.142	.226	1	.237	.219	1	.242	.213
			π=.64,τ=13,ε=.133			π=.61,τ=26,ε=.134		

ECHP data and estimations are reported for the following seven countries: IR=Ireland, IT=Italy, LX=Luxemburg, NL=The Netherlands, PO=Portugal, SP=Spain, UK=The United Kingdom.