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MEMBERSHIP IN CITIZEN GROUPS

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Abstract

We address the coordination problem of individuals deciding to join an association that provides a public good and selective benefits to its members, when ability of the association to fulfill its purposes depends on membership size. In a global game formulation, we show that a unique equilibrium with non-trivial membership exists, and we perform meaningful comparative statics. A unique equilibrium also obtains when agents are heterogeneous, and we show that heterogeneity decreases membership size. In a two-period setting, where seniority of membership entails additional benefits, we provide conditions for uniqueness of equilibrium, and show that the presence of seniority benefits increases membership in both periods.

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1 Introduction

We study the coordination problem of agents deciding to join a group that provides a public good and incentives excludable to non-members. The literature on private provision of public goods focuses mainly on the free rider problem that arises from strategic substitutability of one agent's contribution with other agents' contributions.[1] In "The Logic of Collective Action", Olson [16] suggests that membership in large associations can be explained, despite the extreme free rider problem, if the group is able to provide incentives excludable to non-members. These selective incentives can give a direct utility, like a magazine, or acquire value through social interaction, as in the case of reputation or peer pressure. In both cases, it is reasonable to assume that incentives display strategic complementarities: the more people join the group, the higher the value of being a member. This assumption generates a multiplicity of equilibria: nobody joins, everybody joins, and some intermediate case that is generically unstable. They all are unsuitable for comparative statics analysis.

Carlsson and Van Damme [2] and Morris and Shin [13] show how the introduction of noisy signals in games with strategic complementarities can generate a unique equilibrium.[2] Introducing a similar payoff uncertainty into our membership game, we show that

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[1]One example among many is Mailath and Postlewaite [11].

[2]"Multiplicity of equilibria arise largely as the unintended consequence of two modelling assumptions - the fundamentals are assumed to be common knowledge, and economic agents know others' actions in equilibrium. Both are questionable." Morris and Shin [13] .

in the unique equilibrium a positive measure of people joins and we show existence of a fee that maximizes the expected size of the group. We analyze the effects on membership decisions of increasing uncertainty about the group's ability to provide the public good, and of threats to the status quo provision level of the public good. Our results are consistent with empirical observations on large citizen groups that have attracted considerable attention in the political science literature.

We explore the effect of heterogeneity among prospective members on the equilibrium group size. We show existence and uniqueness of equilibrium when agents differ in their willingness to pay for selective incentives, and we show that, under certain conditions, increasing heterogeneity while keeping the mean willingness to pay constant decreases the equilibrium size of the group.

Finally, we analyze a dynamic version of the model. A membership decision is not once and for all: typically agents are invited to join or renew their memberships every so often. Moreover, it is reasonable to assume that after periods of learning and adjustment new opportunities (office positions, familiarity with the internal mechanism) are available within the group to returning members. This endogenously generates heterogeneity between "senior" and "junior" members. We provide sufficient conditions for existence and uniqueness of equilibrium and show that the existence of seniority benefits increases membership not only in the second period, but also in the first. Seniority benefits directly increase the value of joining immediately, but they also increase the payoff of waiting and joining in the future because of strategic complementarities. Hence, showing uniqueness of equilibrium is an interesting problem also from a theoretical perspective.

The remainder of the paper is organized as follows. Section 2 presents some examples of successful collective action, describes the typical membership contract and reviews the literature. The basic structure of the static model, the characterization of the equilibrium and the comparative statics results are presented in Section 3. Section 4 relaxes the assumption of homogeneity of agents. Section 5 contains the dynamic setting and section 6 concludes. Most of the proofs are in the Appendix.

2 Large Citizen Associations

The National Association Study shows that the mean and median membership size of US associations are, respectively, 27,575 and 750.[3] These numbers suggest that, although not many, very large associations exist. Examples we have in mind are environmentalist groups like NWF (4 million of members), WWF (1,000,000), Sierra Club (700,000) or professional groups like AEA (23,000).[4] Another example is represented by trade unions. All these groups provide some benefits for their members in exchange for a fee, and have been successful in producing a public good.

The political science literature has extensively analyzed the composition of selective incentives that citizen associations offer. The benefits provided by groups have been divided into material, solidary and purposive. The typical purposive benefit is the satisfaction derived from the actual contribution to a cause deemed worthy by the member. A solidary benefit derives from the opportunity of social interaction among members provided by the group. Material benefits include publications, discounts, T-shirts, etc.

The existing literature on warm-glow has already analyzed the role of purposive incentives in contribution games.[5] We focus on a material incentive like a publication for two reasons. First, a survey in Walker [18] on interests group in America shows that virtually every group in his sample of 206 citizen associations offers some kind of publication which is considered one of the most important benefit by members. Second, the role of solidary benefits seems modest, since, as reported in Putnam [17], 87 percent of Sierra Club members never attended a single group meeting.

One common criticism to the argument that successful collective action is due only to private benefits is that a competing firm not burdened by the cost of producing the public good can offer just the private benefit at a lower price. We believe that in the case of publications establishing a brand name through success in providing the public good gives the association some monopoly power over the private good. Moreover the production technology may display economies of scale. Discounts on postal rates, preferred tax

[3] As reported in Knoke [10].

[4] Data from the individual websites of each association as of 2003.

[5] See Andreoni [1].

treatments or free interns are other instances in which the provision of a public good gives a cost advantage in the production of the selective benefits.

We can think also to alternative reasons why the utility of membership may be increasing with the success of the group in providing the public good. For example, if an environmentalist group is expected to be successful in influencing the laws on car emissions, a reader would find the point of view expressed in the group newspaper to be a valuable piece of information when buying a new car. Success in accomplishing some environmental protection project may well enhance the quality of hiking and animal watching activities of members.[6]

We model the selective incentives provided by associations as a particular form of club good: excludable to non-members, non-rival and with negligible costs of congestion. This assumption captures a fundamental characteristic of the incentives packages we see in reality and uncovers the coordination problem agents face, since their payoff of joining displays strategic complementarities.

Two strands of literature are related to our work. The first deals with impure public goods, and the second with dynamic global games.

Cornes and Sandler [3, 4, 5] are the first to introduce an impure public good model in which the purchase of any quantity of an intermediate good makes available, through a joint production function, fixed proportions of public good and private characteristic. With sufficiently strong complementarities between the private characteristic and the public good, individual demand for the intermediate good may be increasing in the quantity demanded by others, thus alleviating the free rider problem. They do not address directly the issue of coordination among agents. Morgan [12] applies the impure public good approach to model public good provision and shows the existence of a unique equilibrium when the public good is funded through a lottery.

As for dynamic applications of global games, the two closest papers are Dasgupta [6] and Heidhues and Melissas [8]. Heidhues and Melissas [8] focus on cohort effects, while Dasgupta [6] focuses on social learning. In both papers, contrary to our paper, the

[6] See King and Walker [9].

decision to contribute is once and for all, and the interaction between learning by doing effect and strategic complementarities is absent.

3 The Model

There is a continuum of homogeneous agents of size 1. They decide independently and simultaneously whether or not to join a group. Let $k > 0$ be the cost of membership and $e \in [0,1]$ be the proportion of agents joining the group. The group's total revenues ke are used as an input in a binary production function. This function generates a pure public good and a non-rival club good that agents enjoy only if they are members. We use this assumption as a convenient way to model the observation that selective benefits are often tied to the success in providing the public good.

The production function is:

$$y(e) = \begin{cases} 0 & \text{if } ke < \theta \\ \bar{y} & \text{if } ke \geqslant \theta \end{cases},$$

where $\bar{y} > 0$ and θ is a stochastic threshold.[7]

The common utility function is:

$$\begin{cases} U\left(\lambda y\left(e\right), \left(1-\lambda\right) y\left(e\right)\right) - k & \text{if the agent joins the group} \\ U\left(0, \left(1-\lambda\right) y(e)\right) & \text{if she does not join the group,} \end{cases}$$

where the exogenous parameter $\lambda \in (0,1)$ represents the extent by which $y(e)$ can be excluded and jointly consumed only by members. $U(\cdot,\cdot)$ is increasing in both arguments and $U(0,0) = 0$. To simplify notation let the difference in the utility of joining and not joining, in case of success of the group be:

$$n = U\left(\lambda \bar{y}, \left(1-\lambda\right) \bar{y}\right) - U\left(0, \left(1-\lambda\right) \bar{y}\right)$$

The net utility from joining is:

$$\Pr\left\{ke \geq \theta\right\} \cdot n - k.$$

[7] An example of a similar stochastic binary production function can be found in Nitzan and Romano [15].

To rule out the uninteresting case in which joining is dominated we assume that:

$$n > k.$$

Consider first the perfect information case. Let $\theta = \overline{\theta} > 0$ be known and such that $k\overline{e} = \overline{\theta}$ for some $\overline{e} \in [0, 1]$, so that it is possible to produce the public good if enough agents join. The simultaneous move game always has two pure strategy equilibria: one in which nobody joins and one with everybody joins.

Assume now that the true level of the threshold is a random draw from a prior distribution that is normal with mean μ and variance σ^2, with $\mu \in (0, k)$. The equilibrium outcomes are $e = \{0, 1\}$, and

$$e = \frac{\Phi^{-1}\left(\frac{k}{n}\right)\sqrt{\sigma} + \mu}{k},$$

where Φ is the standard cumulative normal distribution.[8] This additional outcome derives from a mixed strategy symmetric equilibrium that is not stable. Note that all previous equilibria are not responsive to fundamentals, or unstable.

We now introduce asymmetric information. Each agent receives a signal θ_i of the true state θ. θ is normally distributed with mean μ and variance σ and

$$\theta_i = \theta + \sqrt{\tau}\varepsilon_i,$$

where $\varepsilon_i \sim N(0, 1)$ i.i.d. across agents and $\tau > 0$.[9]

Define the interim expected net payoff of membership for an agent with signal θ_i for given proportion of members e:

$$\pi(\theta_i, e) = \Pr\{ke \geq \theta|\theta_i\} n - k.$$

Note that there exist $\underline{\theta}$ and $\overline{\theta}$ such that if $\theta_i < \underline{\theta}$, then $\pi(\theta_i, e) > 0$ for all e; and if $\theta_i > \overline{\theta}$, then $\pi(\theta_i, e) < 0$ for all e. This means that there exist regions of the space of signals where agents have a dominant strategy. Moreover, agent i, conditional on her signal, puts strictly positive probability on the events $\theta_j < \underline{\theta}$ and $\theta_j > \overline{\theta}$ for each j.

[8] For $e = 1$ to be an equilibrium outcome we need $\frac{k}{n} \in \left(\Phi\left(\frac{-\mu}{\sqrt{\sigma}}\right), \Phi\left(\frac{k-\mu}{\sqrt{\sigma}}\right)\right)$.

[9] The normality assumption of θ allows us to derive closed form solutions. The uniqueness result of Theorem 1 would go through also assuming any prior probability distribution that satisfies the conditions in Morris and Shin [14].

Theorem 1 *There exists $\bar{\tau} > 0$ such that, for $\tau \in (0, \bar{\tau})$, there is a unique equilibrium in which players follow a switching strategy around θ_n i.e. they join the group if $\theta_i < \theta_n$ and stay out otherwise.*

Proof: see Appendix.

The result stated in Theorem 1 follows Morris and Shin [13, 14]. The proof has two parts. First we prove that there exist a unique switching strategy equilibrium. Then we show that a switching strategy is the only that survives iterated deletion of strictly dominated strategies. $\theta_i < \underline{\theta}$ and $\theta_i > \overline{\theta}$ define upper and lower dominance regions where each agent has a strictly dominant action. Starting from below (above), we can iteratively delete a sequence of strictly dominated strategies between the two regions since the expected payoff function is strictly increasing in the signal and displays strategic complementarities. When τ is sufficiently small the iterated deletion process converges to a unique point. In the limit:

$$\lim_{\tau \to 0} \theta_n = k \left(1 - \frac{k}{n}\right).$$

Note that, since the expected size of the group is

$$S = \Phi \left(\frac{\theta_n - \mu}{\sqrt{\sigma + \tau}} \right),$$

S is maximized at $k = \frac{n}{2}$.

3.1 Comparative Statics

"... people create and join organization in response to disturbances in the social environment," Hansen [7].

"... when persons face a threat to their livelihood or to rights they already enjoy, they are more likely to engage in collective action to protect these gains despite the problems posed by the public goods dilemma," Walker [19].

Walker [18] also underlines the attempts to frustrate antagonist associations by politicians through different means like challenges to their not-for-profit status or raising postal rates.

One possible way of analyzing the effect of disturbances in the social environment is to increase uncertainty in the ability of the association to provide the public good. By looking at the effect of the precision of the public component of the signal on θ_n we obtain:

Proposition 1 *There exists $\hat{\tau} > 0$ such that, for $\tau \in (0, \hat{\tau})$, the effect of an increase in σ increases the equilibrium size of the group if and only if $\mu > k\left(1 - \frac{k}{n}\right)$.*

Proof. The comparative statics of the size of the group S defined above with respect to σ is:

$$\frac{dS}{d\sigma} = \phi\left(\frac{\theta_n - \mu}{\sqrt{\sigma + \tau}}\right) \frac{\frac{d\theta_n}{d\sigma}\sqrt{\sigma + \tau} + \frac{1}{2\sqrt{\sigma+\tau}}(\mu - \theta_n)}{\sigma + \tau},$$

where $\phi(\cdot)$ is the standard normal density function and

$$\begin{aligned} \frac{d\theta_n}{d\sigma} &= \frac{d\theta^*}{d\sigma}\left(1 + \sqrt{\tau}\frac{1}{k\phi\left(\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)}\right) \\ \frac{d\theta^*}{d\sigma} &= \frac{\Phi^{-1}\left(\frac{\theta^*}{k}\right) + \frac{1}{2\sqrt{\sigma(\sigma+\tau)}}(2\sigma + \tau)\Phi^{-1}\left(\frac{k}{n}\right)}{\sqrt{\tau} - \sigma\frac{1}{k\phi\left(\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)}}. \end{aligned}$$

Note that

$$\lim_{\tau \to 0} \theta^* = k\left(1 - \frac{k}{n}\right)$$

which implies that

$$\lim_{\tau \to 0} \frac{d\theta_n}{d\sigma} = \lim_{\tau \to 0} \frac{d\theta^*}{d\sigma} = 0.$$

Since

$$\phi\left(\frac{\theta_n - \mu}{\sqrt{\sigma + \tau}}\right) > 0,$$

when τ is small, the sign of $\frac{dS}{d\sigma}$ is the same of $\mu - k\left(1 - \frac{k}{n}\right)$. ■

Intuitively, an increase in the variance of θ shifts probability mass away from the mean. When $\mu > k\left(1 - \frac{k}{n}\right)$, people choose not to join if they receive a signal close to the mean. Therefore increasing the dispersion of the signal increases the probability of receiving a signal lower than the equilibrium cut-off.

A raise in postal rates can be thought as an additional expense for the group and has the same effect of a tax on collected membership fees. If the tax is $\alpha \in (0, 1)$, the equilibrium is defined by the system

$$\theta_n = \theta^* + \sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{(1-\alpha)\,k}\right)$$

$$\Phi\left(\frac{\sqrt{\tau}\,(\theta^* - \mu)}{\sqrt{\sigma\,(\sigma+\tau)}} - \frac{\sqrt{\sigma}\Phi^{-1}\left(\frac{\theta^*}{(1-\alpha)k}\right)}{\sqrt{\sigma+\tau}}\right) = \frac{k}{n}.$$

The limit cutoff value is

$$\lim_{\tau\to 0}\theta_n = (1-\alpha)\,k\left(1-\frac{k}{n}\right),$$

and with similar calculations as above

$$\lim_{\tau\to 0}\frac{d\theta_n}{d\alpha} = -k\left(1-\frac{k}{n}\right) < 0.$$

Hence, an increase in postal rate decreases membership.

To analyze the effect of threats to the public good, let $x > 0$ be the status quo level of provision of the public good without collective action. In this case:

$$n = U\left(\lambda\bar{y}, x + (1-\lambda)\,\bar{y}\right) - U\left(0, x + (1-\lambda)\,\bar{y}\right).$$

Note that $\frac{dn}{dx} < 0$ if the cross derivative $U_{12}(\cdot,\cdot)$ is negative, and the size of the group increases with a reduction of the status quo level of public good provision. The assumption of $U_{12}(\cdot,\cdot) < 0$ means that the utility of the selective benefit is decreasing in the status quo level of public good provision. It does not conflict with the existence of strategic complementarities, and can be justified if we consider selective benefits like advocacy or representation before government.

4 Heterogeneous Agents

In this section we explore the consequences of heterogeneity in the population. Let an exogenously given fraction p of the population have utility function $U^n(\cdot,\cdot)$, and the remaining $(1-p)$ have utility function $U^s(\cdot,\cdot)$. Let:

$$\begin{cases} U^n\left(\lambda\bar{y},(1-\lambda)\,\bar{y}\right) - U^n\left(0,(1-\lambda)\,\bar{y}\right) = n \\ U^s\left(\lambda\bar{y},(1-\lambda)\,\bar{y}\right) - U^s\left(0,(1-\lambda)\,\bar{y}\right) = s \end{cases}$$

and we assume $s > n$. Our simple form of heterogeneity describes a situation in which there exists an exogenous proportion of agents who receive more value from the selective benefit given the same level of public good.[10] A sufficient condition on the utility functions is $U_1^n(\cdot,\cdot) < U_1^s(\cdot,\cdot)$. The information structure is the same as in section 3. In addition, for simplicity, we set $\sigma = 1$.

We first prove existence and uniqueness of an equilibrium in switching strategies along the lines of Theorem 1. This will allow us to explore the effect of increasing heterogeneity in the population on the equilibrium probability of providing the public good, and hence on the equilibrium size of the group.

Theorem 2 *There exist a $\bar{\tau} > 0$ such that, for $\tau \in (0, \bar{\tau})$, there is a unique equilibrium in which players of type $t = \{n, s\}$ follow a switching strategy around θ_t.*

The proof, along the lines of Theorem 1, is in the Appendix.

We now investigate the equilibrium effect of increasing heterogeneity among agents. We increase the net payoff of $1 - p$ agents by Δ, decreasing it for the remaining p agents by $\frac{1-p}{p}\Delta$. This spread holds constant the population mean net payoff.

Proposition 2 *There exist a $\bar{\tau}' > 0$ such that, for $\tau \in (0, \bar{\tau}')$, increased heterogeneity in the form of a mean preserving spread in net payoffs decreases the equilibrium size of the group.*

We leave the proof to the Appendix and outline the argument here. The probability of success required by the indifferent type with benefit n is $\frac{k}{n}$. When n decreases this probability must increase of a factor $\frac{k}{n^2}$. The probability of success required by the indifferent agent with benefit s decreases only by $\frac{k}{s^2}$. Since the probability of success is a weighted average of the proportion of agents joining, the decrease of type n cutoff more than compensates the increase in type s cutoff. The overall probability of success will

[10]An alternative way to introduce heterogeneity is consider a different membership fee. Given our production function this assumption has an ambiguos effect on the expected payoff of joining since some agents pay less but the positive externality on others is reduced. If the heterogeneity is in the utility cost of the fee then the result is unchanged.

decrease and the equilibrium size of the group will decrease as well. If we allow the group to charge different fees to different types of agents in order to maximize the probability of providing the public good, then heterogeneity in the form of a mean preserving spread is of no consequence. More generally, note that the limiting size of the group, as τ converges to zero, is an increasing and concave function of the distribution of valuations. Hence, if two distributions of benefits F and G are such that F second-order stochastically dominates G, then the equilibrium size of the group under F will be larger. Having the same mean and smaller variance is not sufficient: indeed, if F and G have the same mean, F has smaller variance than G, but G second-order stochastically dominates F, the equilibrium size of the group will be larger under G.[11]

5 The Dynamic Model

In this section we analyze a two-period extension of the model to study the effect of seniority benefits. After a period of learning and adjustment new opportunities (office positions, familiarity with the internal mechanism, etc.) are available within the group. In the first period agents consider not only the one shot gains from joining but also the net expected difference in second period utility between joining as new members or in a "senior" status.

If the signal of the first period contains valuable information about the second period state, the proportion of agents that joined in the first period is an additional piece of information upon which drawing inference on the new state. This information contagion has been investigated in a number of different papers[12]; here we are more interested in the "learning by doing" effect, and for simplicity we consider the new state independently drawn.

In the first period agents are homogeneous: they all get the same payoff from joining when the group is successful. In the second period a new state γ is drawn, agents receive a noisy signal of the true state but now those agents that joined in the first period have

[11] An example is available upon request.

[12] See for example Dasgupta [6].

higher utility if they decide to pay the membership fee and maintain their status for an additional period.

The introduction of seniority benefits increases directly the continuation value of joining in the first period but increases also the payoff of staying out because of strategic complementarities. In addition monotonicity of payoffs in the signal is not apparent, hence, showing uniqueness of an equilibrium is an interesting problem also from a theoretical perspective. The main result of this section is Proposition 3, which provides conditions for existence and uniqueness of an equilibrium of the dynamic game.

Let $\theta_i = \theta + \sqrt{\tau}\varepsilon_i$, $\gamma_i = \gamma + \sqrt{\tau}\eta_i$ be the signals in the first and second period respectively, where

$$\begin{aligned} \theta &\sim N(\mu, 1) \\ \gamma &\sim N(\mu, 1) \\ \varepsilon_i &\sim N(0, 1) \\ \eta_i &\sim N(0, 1), \end{aligned}$$

all independent, and $\mu \in (0, k)$.

An agent decides in the first period whether to join or not after having seen her signal and considering the continuation equilibrium of the second period. Different first period signals will determine the expected proportion of senior agents in the second period given the realized first period true state and the equilibrium strategy.

Using well-known properties of the normal distribution we have that:

$$\begin{gathered} \theta_j|\theta \sim N(\theta, \tau) \\ \theta|\theta_i \sim N\left(\frac{\mu\tau + \theta_i}{1+\tau}, \frac{\tau}{1+\tau}\right) \\ \gamma_j|\gamma \sim N(\gamma, \tau) \\ \gamma|\gamma_i \sim N\left(\frac{\mu\tau + \gamma_i}{1+\tau}, \frac{\tau}{1+\tau}\right). \end{gathered}$$

We first solve for the equilibrium in the second period for a given proportion $1-p$ of agents joining in the first period.

Denote by n the utility of an agent who joins the group for the first time when the group is successful. In the second period, senior members get $s > n$, while new members still receive n.

Conditional on the true state γ, if $1-p$ (p) agents follow a switching strategy around γ_s (γ_n), the proportion of agents joining in the second period is:

$$p\Phi\left(\frac{\gamma_n-\gamma}{\sqrt{\tau}}\right)+(1-p)\,\Phi\left(\frac{\gamma_s-\gamma}{\sqrt{\tau}}\right).$$

The critical value γ^* below which the group is successful is implicitly defined by:

$$k\left(p\Phi\left(\frac{\gamma_n-\gamma^*}{\sqrt{\tau}}\right)+(1-p)\,\Phi\left(\frac{\gamma_s-\gamma^*}{\sqrt{\tau}}\right)\right)=\gamma^*.$$

Conditional on γ_i, the expected payoff of type $1-p$ and p agents is respectively:

$$\begin{aligned}\Pr(\gamma\leq\gamma^*|\gamma_i)\cdot s-k &= \Phi\left(\frac{\gamma^*-\gamma_i+\tau(\gamma^*-\mu)}{\sqrt{\tau(1+\tau)}}\right)s-k\\ \Pr(\gamma\leq\gamma^*|\gamma_i)\cdot n-k &= \Phi\left(\frac{\gamma^*-\gamma_i+\tau(\gamma^*-\mu)}{\sqrt{\tau(1+\tau)}}\right)n-k.\end{aligned}$$

When $\gamma_i=\gamma_s$ (γ_n), $1-p$ (p) agents should be indifferent between joining or not. In equilibrium the following system has to be satisfied:

$$\left\{\begin{array}{r} p\Phi\left(\frac{\gamma_n-\gamma^*}{\sqrt{\tau}}\right)+(1-p)\,\Phi\left(\frac{\gamma_s-\gamma^*}{\sqrt{\tau}}\right)=\frac{\gamma^*}{k}\\ \Phi\left(\frac{\gamma^*-\gamma_s+\tau(\gamma^*-\mu)}{\sqrt{\tau(1+\tau)}}\right)=\frac{k}{s}\\ \Phi\left(\frac{\gamma^*-\gamma_n+\tau(\gamma^*-\mu)}{\sqrt{\tau(1+\tau)}}\right)=\frac{k}{n}.\end{array}\right. \tag{1}$$

We know from Section 4 that this system, for any $p\in(0,1)$, admits a unique solution $(\gamma_s,\gamma_n,\gamma^*)$ for τ small, with $\gamma_s>\gamma_n$.

Let $\Phi_{x|y}$ be the cumulative normal distribution of x given y with corresponding density $\phi_{x|y}$ and define

$$\left\{\begin{array}{l} g_n(\gamma_i,p)=\Phi_{\gamma|\gamma_i}\left\{k\left[(1-p)\,\Phi_{\gamma_j|\gamma}(\gamma_s)+p\Phi_{\gamma_j|\gamma}(\gamma_n)\right]\right\}n-k\\ g_s(\gamma_i,p)=\Phi_{\gamma|\gamma_i}\left\{k\left[(1-p)\,\Phi_{\gamma_j|\gamma}(\gamma_s)+p\Phi_{\gamma_j|\gamma}(\gamma_n)\right]\right\}s-k\end{array}\right. \tag{2}$$

to be the expected gains from joining in the second period for new (n) and senior (s) members that receive signal γ_i. These expressions are function of the second period signals, given proportion p of agents who did not join in the first period and second period equilibrium cutoffs (γ_n, γ_s).

If everybody follows a switching strategy around θ_n in the first period, the proportion of agents that joins is $(1-p) = \Phi_{\theta_j|\theta}(\theta_n)$, and we can redefine

$$\begin{aligned} g_n(\gamma_i, p) &= g_n(\gamma_i, \theta, \theta_n) \\ g_s(\gamma_i, p) &= g_s(\gamma_i, \theta, \theta_n). \end{aligned}$$

After some algebra, the net expected utility of membership conditional on θ_i when everybody else follows a switching strategy around θ_n is:

$$\begin{gathered} \pi(\theta_i, \theta_n) = \Phi_{\theta|\theta_i}\left[k\Phi_{\theta_j|\theta}(\theta_n)\right] n - k + \\ + \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{\gamma_s} g_s(\gamma_i, \theta, \theta_n)\, d\Phi_{\gamma_i}(\gamma_i) - \int_{-\infty}^{\gamma_n} g_n(\gamma_i, \theta, \theta_n)\, d\Phi_{\gamma_i}(\gamma_i) \right) d\Phi_{\theta|\theta_i}(\theta). \end{gathered} \tag{3}$$

To show that a cutoff strategy can be an equilibrium and to use iterated deletion to prove uniqueness of an equilibrium we need $\pi(\theta_i, \theta_n)$ to be monotone in θ_i.

First note that

$$\Phi_{\theta|\theta_i}\left[k\Phi_{\theta_j|\theta}(\theta_n)\right] n - k$$

is decreasing in θ_i. To show that $\pi(\theta_i, \theta_n)$ is decreasing in θ_i we prove in the Appendix that:

Claim 1 *For τ small enough*

$$\int_{-\infty}^{+\infty} \left(\int_{-\infty}^{\gamma_s} g_s(\gamma_i, \theta, \theta_n)\, d\Phi_{\gamma_i}(\gamma_i) - \int_{-\infty}^{\gamma_n} g_n(\gamma_i, \theta, \theta_n)\, d\Phi_{\gamma_i}(\gamma_i) \right) d\Phi_{\theta|\theta_i}(\theta)$$

is decreasing in θ_i.

A higher signal in the first period reduces the expected probability of providing the good, hence it reduces membership in the first period. In the second period there will be less senior members, who have the greatest incentive to join, and the probability of providing the public good will drop also in the second period. This reduces the payoff of membership for both senior and new members. We show in the appendix that for small τ, the effect on senior members dominates the one on junior members.

We can similarly show that $\pi(\theta_i, \theta_n)$ is increasing in θ_n. Now we are ready to state the main result of this section.

Proposition 3 *There exist a $\tilde{\tau} > 0$ and $b > 0$ such that, for $\tau \in (0, \tilde{\tau})$ and $(s - n) < \min\{b, k\}$, there is a unique equilibrium in which players follow a switching strategy around θ_n in the first period, and, in the second period, around γ_s if they joined in the first period, and around γ_n otherwise.*

From the previous discussion, we have shown that, for any strategy in the first period, there exists a unique continuation equilibrium in the second period if $\tau \in (0, \tilde{\tau})$. This equilibrium is a switching strategy around $\gamma_s(p)$ for players that joined in the first period, and around $\gamma_n(p)$ for players that did not. We have also shown that a switching strategy in the first period can be an equilibrium. What is left to check is the existence of dominance regions that allow us to start the iterated deletion process, and existence and uniqueness of θ^* that solves:

$$\pi(\theta^*, \theta^*) = 0.$$

We start the argument with existence and uniqueness of a switching strategy equilibrium. We can rewrite the last equation in two steps. Conditional on the true state θ, if everybody follows a switching strategy around θ_n, the proportion of agents joining in the first period is:

$$\Phi_{\theta_j|\theta}(\theta_n) = \Phi\left(\frac{\theta_n - \theta}{\sqrt{\tau}}\right).$$

The critical value θ^* below which the group is successful is implicitly defined by:

$$k\Phi\left(\frac{\theta_n - \theta^*}{\sqrt{\tau}}\right) = \theta^*.$$

An agent should be indifferent between joining or not in the first period when she receives a signal $\theta_i = \theta_n$. Using the definitions of $g_s(\gamma_i, \theta, \theta_n)$ and $g_n(\gamma_i, \theta, \theta_n)$ we obtain:

$$\left\{\begin{array}{c} \theta_n = \theta^* + \sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right) \\ \Phi\left(\frac{\sqrt{\tau}(\theta^* - \mu)}{\sqrt{1+\tau}} - \frac{\Phi^{-1}\left(\frac{\theta^*}{k}\right)}{\sqrt{1+\tau}}\right) = \frac{k}{n + Q(\theta^*)}, \end{array}\right. \tag{4}$$

where $Q(\theta^*)$ is

$$\int_{-\infty}^{+\infty} \left(\begin{array}{c} \int_{-\infty}^{\gamma_s} g_s\left(\gamma_i, \theta, \theta^* + \sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right) d\Phi_{\gamma_i}(\gamma_i) - \\ + \int_{-\infty}^{\gamma_n} g_n\left(\gamma_i, \theta, \theta^* + \sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right) d\Phi_{\gamma_i}(\gamma_i) \end{array} \right) d\Phi_{\theta|\theta_i = \theta^* + \sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)}(\theta). \tag{5}$$

From (4) it is clear that given θ^*, there exists a uniquely determined $\theta_n(\theta^*)$, and from the discussion above, there exist unique $\gamma^*(\theta^*)$, $\gamma_s(\theta^*)$, $\gamma_n(\theta^*)$. Showing existence and uniqueness of a switching strategy equilibrium is equivalent to showing that

$$\Phi\left(\frac{\sqrt{\tau}(\theta^* - \mu)}{\sqrt{1+\tau}} - \frac{\Phi^{-1}\left(\frac{\theta^*}{k}\right)}{\sqrt{1+\tau}}\right) = \frac{k}{n + Q(\theta^*)} \tag{6}$$

has a unique solution. In the Appendix we prove that this is the case if the difference between s and n is not too large, when τ is sufficiently small.

To establish existence of dominance regions, note that for very low first period signals joining is always a strictly dominant strategy. If the signal is sufficiently high, meaning that there is very small probability of getting the good, even if everybody else joins, stay out is dominant if the payoff difference between seniors and new members is not too high: $s - n < k$. Since $\pi(\theta_i, \theta_n)$ is decreasing in the first argument and increasing in the second, the same reasoning in Theorem 1 establishes that the equilibrium in cutoff strategy that we found is the unique equilibrium of this game.

A direct inspection of (6) shows that, since $Q(\theta^*) > 0$ and the LHS is decreasing in θ^* when τ is small, seniority benefits increase membership also in the first period.

6 Conclusion

Our model analyzes a membership game in which agents decide to join a group that provides a public good and selective incentives. When selective benefits are excludable, non-rival, and tied to the provision of public good a unique equilibrium exists. Our equilibrium analysis yields comparative statics predictions that are consistent with the main findings of the political science literature on large citizen associations. In the heterogeneous agent case we show in what sense heterogeneity can be detrimental when perfect screening is not feasible. We further analyze a dynamic version of the model, where heterogeneity emerges endogenously. This is one of the few examples to our knowledge of a dynamic version of global games with payoff complementarities between different periods. A unique equilibrium that exhibits persistence in membership exists, given restrictions on admissible continuation payoffs. This suggests that a multiple period version of the model will require even more stringent conditions and eventually might leave open the possibility of non uniquely determined equilibrium beliefs, leading back to multiple equilibria.

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Appendix

Proof of Theorem 1.

Let $\Phi_{x|y}$ be the cumulative normal distribution of x given y with corresponding density $\phi_{x|y}$. Using well-known properties of the normal distribution we have that:

$$\theta_j|\theta \sim N(\theta,\tau)$$
$$\theta|\theta_i \sim N\left(\frac{\mu\tau+\sigma\theta_i}{\sigma+\tau},\frac{\tau\sigma}{\sigma+\tau}\right).$$

Conditional on the true state θ, if everybody follows a switching strategy around θ_n, the proportion of agents joining is:

$$\Phi_{\theta_j|\theta}(\theta_n)=\Phi\left(\frac{\theta_n-\theta}{\sqrt{\tau}}\right).$$

The critical value θ^* below which the group is successful is implicitly defined by:

$$k\Phi\left(\frac{\theta_n-\theta^*}{\sqrt{\tau}}\right)=\theta^*.$$

Note that for any θ_n, there exists a unique θ^*that solves the above equation, and $\theta^* \in (0,k)$. Conditional on θ_i, the expected payoff of an agent is:

$$\Pr(\theta\le\theta^*|\theta_i)\,n-k=\Phi\left(\frac{\sigma(\theta^*-\theta_i)+\tau(\theta^*-\mu)}{\sqrt{\tau\sigma(\sigma+\tau)}}\right)n-k.$$

When $\theta_i=\theta_n$ the agent should be indifferent between joining or not. Therefore we obtain a system of two equation in two unknowns:

$$\begin{cases}\Phi\left(\frac{\theta_n-\theta^*}{\sqrt{\tau}}\right)=\frac{\theta^*}{k}\\ \Phi\left(\frac{\sigma(\theta^*-\theta_n)+\tau(\theta^*-\mu)}{\sqrt{\tau\sigma(\sigma+\tau)}}\right)=\frac{k}{n},\end{cases}$$

that is equivalent to:

$$\begin{cases}\theta_n=\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\\ \Phi\left(\frac{\sqrt{\tau}(\theta^*-\mu)}{\sqrt{\sigma(\sigma+\tau)}}-\frac{\sqrt{\sigma}\Phi^{-1}\left(\frac{\theta^*}{k}\right)}{\sqrt{\sigma+\tau}}\right)=\frac{k}{n}.\end{cases}$$

Let

$$W(\theta^*)\equiv\frac{\sqrt{\tau}(\theta^*-\mu)}{\sqrt{\sigma(\sigma+\tau)}}-\frac{\sqrt{\sigma}\Phi^{-1}\left(\frac{\theta^*}{k}\right)}{\sqrt{\sigma+\tau}}-\Phi^{-1}\left(\frac{k}{n}\right),$$

and note that:

$$\lim_{\theta^* \to 0} W(\theta^*) = \infty$$

$$\lim_{\theta^* \to k} W(\theta^*) = -\infty$$

$$\frac{dW}{d\theta^*} = \frac{1}{\sqrt{\sigma+\tau}} \left(\frac{\sqrt{\tau}}{\sqrt{\sigma}} - \frac{\sqrt{\sigma}}{k\phi\left(\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)} \right) <$$

$$< \frac{1}{\sqrt{\sigma+\tau}} \left(\frac{\sqrt{\tau}}{\sqrt{\sigma}} - \frac{\sqrt{2\pi\sigma}}{k} \right).$$

Since the last expression is negative when:

$$\sqrt{\tau} < \frac{\sigma\sqrt{2\pi}}{k},$$

the solution is unique for small τ.

In the limit:

$$\lim_{\tau \to 0} \theta_n = \lim_{\tau \to 0} \theta^* = k\left(1 - \frac{k}{n}\right).$$

The last step of the argument is to show that the above equilibrium in switching strategies is indeed the only one surviving iterated deletion of strictly dominated strategies. This part of the proof follows closely the argument in Morris and Shin [13]. Define:

$$\pi(\theta_i, \theta_n) = \Phi_{\theta|\theta_i}[k\Phi_{\theta_j|\theta}(\theta_n)]n - k$$

$\pi(\theta_i, \theta_n)$ is the expected utility of membership conditional on θ_i when everybody else follows a switching strategy around θ_n. If $\theta_i < \underline{\theta}$ join is a strictly dominant action so after the first stage of iterated deletion $\theta_n^1 \geq \underline{\theta}$. Now θ_{n2} is defined as the solution to:

$$\pi\left(\theta_n^2, \theta_n^1\right) = 0.$$

Note that by strategic complementarities joining below θ_n^2 strictly dominates any other strategy that prescribes joining below θ_n^1. By further iteration we can construct a sequence $\left\{\theta_n^k\right\}_{k=1}^{k=+\infty}$ that is increasing since $\pi(\cdot,\cdot)$ is decreasing in its first argument and increasing in the second. The smallest solution $\underline{\theta}^*$ to $\pi(\theta, \theta) = 0$ is the limit of the sequence. The same argument can be done starting from $\overline{\theta}$ to get a decreasing sequence with limit $\overline{\theta}^*$ that is the largest solution to $\pi(\theta, \theta) = 0$. But since we proved above that there is a unique solution θ^* to $\pi(\theta, \theta) = 0$ then $\underline{\theta}^* = \overline{\theta}^* = \theta^*$.

Proof of Theorem 2.

Let θ^* be the critical value below which the group is successful and θ_n, θ_s be the cutoffs. In equilibrium the following system has to be satisfied:

$$\begin{cases} p\Phi\left(\frac{\theta_n-\theta^*}{\sqrt{\tau}}\right)+(1-p)\,\Phi\left(\frac{\theta_s-\theta^*}{\sqrt{\tau}}\right)=\frac{\theta^*}{k} \\ \Phi\left(\frac{\theta^*-\theta_n+\tau(\theta^*-\mu)}{\sqrt{\tau(1+\tau)}}\right)=\frac{k}{n} \\ \Phi\left(\frac{\theta^*-\theta_s+\tau(\theta^*-\mu)}{\sqrt{\tau(1+\tau)}}\right)=\frac{k}{s}, \end{cases}$$

which is equivalent to:

$$\begin{cases} p\Phi\left(\sqrt{\tau}\,(\theta^*-\mu)-\sqrt{1+\tau}\Phi^{-1}\left(\frac{k}{n}\right)\right)+ \\ +(1-p)\,\Phi\left(\sqrt{\tau}\,(\theta^*-\mu)-\sqrt{1+\tau}\Phi^{-1}\left(\frac{k}{s}\right)\right)=\frac{\theta^*}{k} \\ \frac{(\theta_n-\theta^*)}{\sqrt{\tau}}=\sqrt{\tau}\,(\theta^*-\mu)-\sqrt{1+\tau}\Phi^{-1}\left(\frac{k}{n}\right) \\ \frac{(\theta_s-\theta^*)}{\sqrt{\tau}}=\sqrt{\tau}\,(\theta^*-\mu)-\sqrt{1+\tau}\Phi^{-1}\left(\frac{k}{s}\right). \end{cases}$$

Let

$$T\left(\theta^*\right)\equiv p\Phi\left(\sqrt{\tau}\,(\theta^*-\mu)-\sqrt{1+\tau}\Phi^{-1}\left(\frac{k}{n}\right)\right)+$$
$$+(1-p)\,\Phi\left(\sqrt{\tau}\,(\theta^*-\mu)-\sqrt{1+\tau}\Phi^{-1}\left(\frac{k}{s}\right)\right)-\frac{\theta^*}{k},$$

and note that:

$$\lim_{\theta^*\to 0} T\left(\theta^*\right)>0$$
$$\lim_{\theta^*\to k} T\left(\theta^*\right)<0$$
$$\frac{dT}{d\theta^*}\leq\sqrt{\frac{\tau}{2\pi}}-\frac{1}{k}.$$

Since the last expression is negative when

$$\sqrt{\tau}<\frac{\sqrt{2\pi}}{k},$$

the solution is unique for small τ.

To show the uniqueness of equilibrium define:

$$\begin{aligned} \pi_n\left(\theta_i,\theta_n,\theta_s\right) &= \Phi_{\theta|\theta_i}\left\{k\left[p\Phi_{\theta_j|\theta}(\theta_n)+(1-p)\,\Phi_{\theta_j|\theta}(\theta_s)\right]\right\}n-k \\ \pi_s\left(\theta_i,\theta_n,\theta_s\right) &= \Phi_{\theta|\theta_i}\left\{k\left[p\Phi_{\theta_j|\theta}(\theta_n)+(1-p)\,\Phi_{\theta_j|\theta}(\theta_s)\right]\right\}s-k \end{aligned}$$

as the expected utility of membership conditional on θ_i when everybody else follows a switching strategy around θ_n and θ_s.

Note that there exist $\bar{\theta}_n$, and $\bar{\theta}_s$ such that:

$$\begin{cases} \pi_n(\theta_i, \theta_n, \theta_s) \leq \Phi_{\theta|\theta_i}(k)\, n - k < 0 & \text{if } \theta_i > \bar{\theta}_n \\ \pi_s(\theta_i, \theta_n, \theta_s) \leq \Phi_{\theta|\theta_i}(k)\, s - k < 0 & \text{if } \theta_i > \bar{\theta}_s. \end{cases}$$

This means that if $\theta_i > \bar{\theta}_n$ stay out is a strictly dominant action for n types and, after the first stage of iterated deletion, $\theta_n^1 \leq \bar{\theta}_n$. Similarly we obtain $\theta_s^1 \leq \bar{\theta}_s$. Now θ_n^2, θ_s^2 are defined as the solution to:

$$\begin{aligned} \pi_n\left(\theta_n^2, \theta_n^1, \theta_s^1\right) &= 0 \\ \pi_s\left(\theta_s^2, \theta_n^1, \theta_s^1\right) &= 0. \end{aligned}$$

By further iteration we end up with a two dimensional sequence $\left\{\theta_n^k, \theta_s^k\right\}$ that is decreasing since $\pi_n(\cdot,\cdot,\cdot)$ and $\pi_s(\cdot,\cdot,\cdot)$ are decreasing in their first argument and increasing in the second and the third. The largest solution $\left\{\bar{\bar{\theta}}_n, \bar{\bar{\theta}}_s\right\}$ to the system

$$\begin{aligned} \pi_n(\theta_n, \theta_n, \theta_s) &= 0 \\ \pi_s(\theta_s, \theta_n, \theta_s) &= 0 \end{aligned}$$

is the limit of the sequence. The same argument can be done starting from above to get a decreasing sequence with limit $\left\{\underline{\underline{\theta}}_n, \underline{\underline{\theta}}_s\right\}$ that is the smallest solution to the above system. But since we proved that there is a unique solution to the system we are done.

Proof of Proposition 2.

The expected size of the group is:

$$S = p\Phi\left(\frac{\theta_n - \mu}{\sqrt{1+\tau}}\right) + (1-p)\,\Phi\left(\frac{\theta_s - \mu}{\sqrt{1+\tau}}\right).$$

Taking derivatives we get:

$$\Delta S = \frac{1}{\sqrt{1+\tau}}\left(p\phi\left(\frac{\theta_n - \mu}{\sqrt{1+\tau}}\right)\Delta\theta_n + (1-p)\,\phi\left(\frac{\theta_s - \mu}{\sqrt{1+\tau}}\right)\Delta\theta_s\right).$$

Applying the implicit function theorem we obtain expressions for $\Delta\theta_n$ and $\Delta\theta_s$. When τ goes to 0 we get:

$$\begin{aligned}
\lim_{\tau\to 0}\Delta\theta_s &= \lim_{\tau\to 0}\Delta\theta_n = -\left(\frac{k}{ns}\right)^2 (1-p)\left(s^2-n^2\right) < 0 \\
\lim_{\tau\to 0}\theta_s &= \lim_{\tau\to 0}\theta_n = \lim_{\tau\to 0}\theta^* = k\left(1-\frac{k}{ns}\left(ps+(1-p)\,n\right)\right).
\end{aligned}$$

Hence,

$$\lim_{\tau\to 0}\Delta S = -\left(\frac{k}{ns}\right)^2 (1-p)\left(s^2-n^2\right)\phi\left(k\left(1-\frac{k}{ns}\left(ps+(1-p)\,n\right)\right)-\mu\right),$$

and when $s=n$, $\lim_{\tau\to 0}\Delta S = 0$, otherwise $\lim_{\tau\to 0}\Delta S < 0$.

Proof of Claim 1.

We want to show that

$$\int_{-\infty}^{+\infty}\left(\int_{-\infty}^{\gamma_s} g_s\left(\gamma_i,\theta,\theta_n\right)d\Phi_{\gamma_i}\left(\gamma_i\right) - \int_{-\infty}^{\gamma_n} g_n\left(\gamma_i,\theta,\theta_n\right)d\Phi_{\gamma_i}\left(\gamma_i\right)\right)d\Phi_{\theta|\theta_i}\left(\theta\right)$$

is decreasing in θ_i.

From the exogenous heterogeneity case we know that:

$$g_n\left(\gamma_n,\theta,\theta_n\right) = g_s\left(\gamma_s,\theta,\theta_n\right) = 0. \tag{7}$$

$$\begin{aligned}
g_n\left(\gamma_i,\theta,\theta_n\right) &= \Phi_{\gamma|\gamma_i}\left\{k\left[\Phi_{\theta_j|\theta}\left(\theta_n\right)\Phi_{\gamma_j|\gamma}\left(\gamma_s\right) + \left(1-\Phi_{\theta_j|\theta}\left(\theta_n\right)\right)\Phi_{\gamma_j|\gamma}\left(\gamma_n\right)\right]\right\} n - k \\
g_s\left(\gamma_i,\theta,\theta_n\right) &= \Phi_{\gamma|\gamma_i}\left\{k\left[\Phi_{\theta_j|\theta}\left(\theta_n\right)\Phi_{\gamma_j|\gamma}\left(\gamma_s\right) + \left(1-\Phi_{\theta_j|\theta}\left(\theta_n\right)\right)\Phi_{\gamma_j|\gamma}\left(\gamma_n\right)\right]\right\} s - k
\end{aligned}$$

Note also that:

$$g_s\left(\gamma_i,\theta,\theta_n\right) = g_n\left(\gamma_i,\theta,\theta_n\right)\frac{s}{n} + k\frac{s-n}{n} \text{ for } \gamma_i \le \gamma_n,$$

and, using results derived above:

$$\frac{d\gamma_s}{dp} = \frac{d\gamma_n}{dp} = \frac{\frac{1}{\sqrt{\tau}}\left(\Phi\left(\frac{\gamma_s-\gamma^*}{\sqrt{\tau}}\right) - \Phi\left(\frac{\gamma_n-\gamma^*}{\sqrt{\tau}}\right)\right)}{\frac{1}{(1+\tau)}\left(p\phi\left(\frac{\gamma_n-\gamma^*}{\sqrt{\tau}}\right) + (1-p)\,\phi\left(\frac{\gamma_s-\gamma^*}{\sqrt{\tau}}\right) - \frac{1}{k}\frac{1}{\sqrt{\tau}}\right)} < 0, \tag{8}$$

since $\gamma_s > \gamma_n$, and the denominator is negative for small τ. This implies also that:

$$\begin{cases} \frac{d}{d\theta} g_n(\gamma_i, \theta, \theta_n) < 0 \\ \frac{d}{d\theta} g_s(\gamma_i, \theta, \theta_n) < 0. \end{cases} \tag{9}$$

An increase in θ, increases the proportion $p = 1 - \Phi_{\theta_j|\theta}(\theta_n)$ of non-joiners in the first period. Keeping γ_s and γ_n fixed, this reduces g_n and g_s, since $\gamma_s > \gamma_n$. Moreover, g_n and g_s are monotonically increasing in γ_s and γ_n.

Combining the previous results we obtain:

$$\frac{d}{d\theta_i}\left\{ \begin{array}{c} \int_{-\infty}^{+\infty}\int_{-\infty}^{\gamma_s} g_s(\gamma_i,\theta,\theta_n)\, d\Phi_{\gamma_i}(\gamma_i)\, d\Phi_{\theta|\theta_i}(\theta) + \\ -\int_{-\infty}^{+\infty}\int_{-\infty}^{\gamma_n} g_n(\gamma_i,\theta,\theta_n)\, d\Phi_{\gamma_i}(\gamma_i)\, d\Phi_{\theta|\theta_i}(\theta) \end{array} \right\} =$$

$$= \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{\gamma_n}\left[g_n(\gamma_i,\theta,\theta_n)+k\right]\frac{s-n}{n} d\Phi_{\gamma_i}(\gamma_i) + \int_{\gamma_n}^{\gamma_s} g_s(\gamma_i,\theta,\theta_n)\, d\Phi_{\gamma_i}(\gamma_i)\right]\frac{d\phi_{\theta|\theta_i}(\theta)}{d\theta_i} d\theta =$$

$$= \int_{-\infty}^{+\infty} J(\theta)\, H(\theta)\, d\theta,$$

where

$$\begin{aligned} J(\theta) &= \int_{-\infty}^{\gamma_n}\left[g_n(\gamma_i,\theta,\theta_n)+k\right]\frac{s-n}{n} d\Phi_{\gamma_i}(\gamma_i) + \int_{\gamma_n}^{\gamma_s} g_s(\gamma_i,\theta,\theta_n)\, d\Phi_{\gamma_i}(\gamma_i) \\ H(\theta) &= \frac{d\phi\left(\frac{\theta-\theta_i+\tau(\theta-\mu)}{\sqrt{\tau(1+\tau)}}\right)}{d\theta_i} = \frac{\theta-\theta_i+\tau(\theta-\mu)}{\tau(1+\tau)}\phi\left(\frac{\theta-\theta_i+\tau(\theta-\mu)}{\sqrt{\tau(1+\tau)}}\right). \end{aligned}$$

Note that $J(\theta)$ is decreasing in θ, which can be shown using (8) and (9), and that $H(\theta)$ is quasi-monotone in θ and

$$\int_{-\infty}^{+\infty} H(\theta)\, d\theta = 0.$$

Hence:

$$\int_{-\infty}^{+\infty} H(\theta)\, J(\theta)\, d\theta = \int_{-\infty}^{+\infty} H(\theta)\left[J(\theta) - J\left(\frac{\tau\mu+\theta_i}{1+\tau}\right)\right] d\theta =$$

$$= \left\{ \begin{array}{c} \int_{-\infty}^{\frac{\tau\mu+\theta_i}{1+\tau}} H(\theta)\left[J(\theta) - J\left(\frac{\tau\mu+\theta_i}{1+\tau}\right)\right] d\theta + \\ \int_{\frac{\tau\mu+\theta_i}{1+\tau}}^{+\infty} H(\theta)\left[J(\theta) - J\left(\frac{\tau\mu+\theta_i}{1+\tau}\right)\right] d\theta \end{array} \right\} < 0,$$

which implies that

$$\int_{-\infty}^{+\infty}\left(\int_{-\infty}^{\gamma_s} g_s(\gamma_i,\theta,\theta_n)\, d\Phi_{\gamma_i}(\gamma_i) - \int_{-\infty}^{\gamma_n} g_n(\gamma_i,\theta,\theta_n)\, d\Phi_{\gamma_i}(\gamma_i)\right) d\Phi_{\theta|\theta_i}(\theta)$$

is decreasing in θ_i.

Proof of existence and uniqueness of a solution to

$$\Phi\left(\frac{\sqrt{\tau}\left(\theta^*-\mu\right)}{\sqrt{1+\tau}}-\frac{\Phi^{-1}\left(\frac{\theta^*}{k}\right)}{\sqrt{1+\tau}}\right)=\frac{k}{n+Q\left(\theta^*\right)} \tag{6}$$

Recalling the definition of $Q\left(\theta^*\right)$ in (5), we know that $Q\left(\theta^*\right)\in[0,s-n]$. In (6), if $\theta^*\to 0$, the LHS converges to 1, while the RHS is smaller than $\frac{k}{n}<1$. When $\theta^*\to k$, the LHS converges to 0, while the RHS is larger than $\frac{k}{s}>0$. Since LHS and RHS are continuous functions of θ^*, a solution exists. To show uniqueness, the additional complication with respect to the proof of Theorem 1 is to show the behavior of $\frac{dQ(\theta^*)}{d\theta^*}$ when τ is small. Using (5),

$$\frac{dQ\left(\theta^*\right)}{d\theta^*}=$$
$$\left(\begin{array}{c}
\int_{-\infty}^{+\infty}\int_{-\infty}^{\gamma_s} g_s\left(\gamma_i,\theta,\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)d\Phi_{\gamma_i}\left(\gamma_i\right)d\phi_{\theta|\theta_i=\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)}\left(\theta\right)+\\
+\int_{-\infty}^{+\infty}\int_{-\infty}^{\gamma_s}\frac{d\left(g_s\left(\gamma_i,\theta,\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)\right)}{d\theta^*}d\Phi_{\gamma_i}\left(\gamma_i\right)d\Phi_{\theta|\theta_i=\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)}\left(\theta\right)+\\
-\int_{-\infty}^{+\infty}\int_{-\infty}^{\gamma_n}\frac{d\left(g_n\left(\gamma_i,\theta,\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)\right)}{d\theta^*}d\Phi_{\gamma_i}\left(\gamma_i\right)d\Phi_{\theta|\theta_i=\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)}\left(\theta\right)+\\
-\int_{-\infty}^{+\infty}\int_{-\infty}^{\gamma_n} g_n\left(\gamma_i,\theta,\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)d\Phi_{\gamma_i}\left(\gamma_i\right)d\phi_{\theta|\theta_i=\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)}\left(\theta\right).
\end{array}\right)$$

We denote

$$\begin{aligned}
\vec{\theta}_n^* &= \lim_{\tau\to 0}\theta_n^*\\
\vec{\theta}^* &= \lim_{\tau\to 0}\theta^*\\
\vec{\gamma}^* &= \lim_{\tau\to 0}\gamma^*,
\end{aligned}$$

and when $\tau\to 0$, we have

$$\begin{aligned}
g_n\left(\gamma_i,\theta,\theta_n\right) &\to \Phi_{\gamma|\gamma_i}\left\{k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right\}n-k\\
g_s\left(\gamma_i,\theta,\theta_n\right) &\to \Phi_{\gamma|\gamma_i}\left\{k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right\}s-k\\
p &= 1-\Phi_{\theta_j|\theta}\left(\vec{\theta}_n^*\right),
\end{aligned}$$

Using the definition of g_s and g_n, calculating their derivative with respect to θ^* and taking limits we obtain:

$$\lim_{\tau\to 0}\frac{d\left(g_s\left(\gamma_i,\theta,\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)\right)}{d\theta^*}=\phi_{\gamma|\gamma_i}\left(k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right)\phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\frac{k^3\left(s-n\right)}{n}\phi_{\theta_j|\theta}\left(\vec{\theta}^*\right)>0$$

$$\lim_{\tau\to 0}\frac{d\left(g_n\left(\gamma_i,\theta,\theta^*+\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)\right)\right)}{d\theta^*}=\phi_{\gamma|\gamma_i}\left(k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right)\phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\frac{k^3\left(s-n\right)}{s}\phi_{\theta_j|\theta}\left(\vec{\theta}^*\right)>0$$

$$\lim_{\tau\to 0}\frac{d\gamma_s}{dp}=\lim_{\tau\to 0}\frac{d\gamma_n}{dp}=\lim_{\tau\to 0}\frac{d\gamma^*}{dp}=-k\left(k\frac{s-n}{ns}\right)<0,$$

and $\frac{dQ(\theta^*)}{d\theta^*}$ converges to

$$\begin{aligned}&\left(s-n\right)\int_{-\infty}^{+\infty}\int_{-\infty}^{\vec{\gamma}^*}\Phi_{\gamma|\gamma_i}\left\{k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right\}d\Phi_{\gamma_i}\left(\gamma_i\right)d\phi_{\theta|\theta_i=\vec{\theta}_n}\left(\theta\right)+\\&+\tfrac{k^3(s-n)^2}{ns}\int_{-\infty}^{+\infty}\int_{-\infty}^{\vec{\gamma}^*}\phi_{\gamma|\gamma_i}\left(k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right)\phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\phi_{\theta_j|\theta}\left(\vec{\theta}^*\right)d\Phi_{\gamma_i}\left(\gamma_i\right)d\Phi_{\theta|\theta_i=\vec{\theta}_n}\left(\theta\right)>0,\end{aligned}\tag{10}$$

when τ is going to zero. Note that the first integral in (10) is positive and smaller than $\frac{s-n}{\sqrt{2\pi}}$. Indeed,

$$\begin{aligned}&\left(s-n\right)\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{\vec{\gamma}^*}\Phi_{\gamma|\gamma_i}\left\{k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right\}d\Phi_{\gamma_i}\left(\gamma_i\right)\right]d\phi_{\theta|\theta_i=\vec{\theta}_n}\left(\theta\right)=\\&-\left(s-n\right)\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{\vec{\gamma}^*}\Phi_{\gamma|\gamma_i}\left\{k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right\}d\Phi_{\gamma_i}\left(\gamma_i\right)\right]x\phi\left(x\right)dx,\end{aligned}$$

where

$$x=\frac{\left(\theta-\theta^*\right)+\tau\left(\theta-\mu\right)-\sqrt{\tau}\Phi^{-1}\left(\frac{\theta^*}{k}\right)}{\sqrt{\tau\left(1+\tau\right)}}.$$

Since

$$\int_{-\infty}^{\vec{\gamma}^*}\Phi_{\gamma|\gamma_i}\left\{k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right\}d\Phi_{\gamma_i}\left(\gamma_i\right)$$

is positive, smaller than 1, and decreasing in x, and since

$$\int_{-\infty}^{+\infty}x\phi\left(x\right)dx=0$$

then

$$\left(s-n\right)\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{\vec{\gamma}^*}\Phi_{\gamma|\gamma_i}\left\{k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right\}d\Phi_{\gamma_i}\left(\gamma_i\right)\right]d\phi_{\theta|\theta_i=\vec{\theta}_n}\left(\theta\right)>0.$$

Moreover, note that

$$\begin{aligned}&\left(s-n\right)\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{\vec{\gamma}^*}\Phi_{\gamma|\gamma_i}\left\{k\Phi_{\gamma_j|\gamma}\left(\vec{\gamma}^*\right)\right\}d\Phi_{\gamma_i}\left(\gamma_i\right)\right]d\phi_{\theta|\theta_i=\vec{\theta}_n}\left(\theta\right)<\\&<\left(s-n\right)\int_{0}^{+\infty}x\phi\left(x\right)dx=\frac{s-n}{\sqrt{2\pi}}.\end{aligned}$$

We can bound the second integral in (10) as follows:

$$\frac{k^3 (s-n)^2}{ns} \int_{-\infty}^{+\infty} \int_{-\infty}^{\gamma^*} \phi_{\gamma|\gamma_i} \left\{ k\Phi_{\gamma_j|\gamma} (\gamma^*) \right\} \phi_{\gamma_j|\gamma} (\gamma^*) \, d\Phi_{\gamma_i} (\gamma_i) \, d\Phi_{\theta|\theta_i=\vec{\theta}_n} (\theta)$$
$$< \frac{k^3 (s-n)^2}{ns} \frac{1}{2\pi}.$$

Since the derivative of the LHS of (6) converges to $-\frac{1}{k}$ when $\tau \to 0$, a sufficient condition for uniqueness is

$$\frac{1}{k} > \lim_{\tau \to 0} \left(\frac{k}{(n + Q(\theta^*))^2} \frac{dQ(\theta^*)}{d\theta^*} \right). \tag{11}$$

Note that

$$\lim_{\tau \to 0} \frac{dQ(\theta^*)}{d\theta^*} \leq \frac{s-n}{\sqrt{2\pi}} \left(\frac{k^3 (s-n)}{ns} \frac{1}{\sqrt{2\pi}} + 1 \right), \tag{12}$$

and that the RHS of (12) is an increasing function of $s-n$, in the relevant range where $s-n>0$. Hence there exists a $b>0$ such that, if $(s-n)<b$, (11) is satisfied. This shows existence and uniqueness of a solution to $\pi(\theta^*, \theta^*) = 0$.