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A NOTE ON THEORETICAL ISSUES OF RESOURCE DEPLETION

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Social Science Working Paper
Number 91
August 1975

RESOURCE DEPLETION

Abstract

This note considers the optimal management of a non-renewable resource subject to depletion effects. It is demonstrated that even with depletion effects, if there are no non convexities in production it always pays to exhaust the resource, if it pays to exploit it at all.

A NOTE ON THEORETICAL ISSUES OF RESOURCE DEPLETION [1]

In recent studies of the economics of non-replenishable resources, Salant, (1975) Schmalensee, (1975) Smith, (1975) and Vousden (1973) describe optimal extraction policies when the resource is subject to "depletion effects."[2] Generally the presumption is that the costs of mining or pumping out the resource increase as reserves are diminished. Furthermore, positive amounts of the resource may be left unexploited if the depletion effect is sufficiently strong so that further utilization of the resource becomes unprofitable. In this note we establish the somewhat surprising result that even with depletion effects if there are no non-convexities in production it always pays to exhaust the resource, if it pays to exploit it at all.

Let E be the rate of resource extraction and X be the resource stock available for exploitation. Then the flow of net benefits from the resource is given by

$$B = B(E, X)$$

with B(E, X) concave in E and X, and

$$B_E > 0, \quad B_{EE} < 0 \quad (1a), \qquad B(0, X) = 0 \quad (1b)$$

The function B presumably measures variable profits, consumers' and producers' surplus, or whatever else we want to maximize. Condition (1b) indicates that the resource only yields positive benefits when it is being consumed. In previous analyses the depletion effect is captured in the assumption:

$$B_X, B_{EX} > 0 \qquad (2)$$

However, Eq. (1b) and the concavity of B imply $B_{EX}(0,X) = 0$ which establishes that resource exhaustion is always optimal, as we show below.

For X^1, X^2, and $E \geq 0$, the concavity of B implies:

$$B(0, X^1) + B_X(0,X^1)\,(X^2 - X^1) + B_E(0,X^1)\,E \geq B(E, X^2) \quad (3)$$

By Eq. (1b) $B_X(0, X) = 0$. Thus for $E > 0$, Eq. (3) can be rewritten as:

$$B_E(0, X^1) \geq \frac{B(0, X^2)}{E} \quad (4)$$

Taking the limit as E approaches zero of both sides of Eq. (4), assuming it exists,[3] we obtain:

$$B_E(0, X^1) \geq B_E(0, X^2) \quad (5)$$

Since this is true for all X^1, $X^2 > 0$, it follows that Eq. (5) holds with equality, implying that $B_{EX}(0, X) = 0$. Thus for concave functions B, the depletion effect vanishes as $E \to 0$.

Clearly it will pay to leave some amount of the resource, $\bar{X}$, unexploited if $B_E(0, \bar{X}) \leq 0$. However, since $B_{EX}(0, X) = 0$, this implies $B_E(0, X) \leq 0$ for all X. On the other hand, if it is optimal to extract some amount of the resource then $B_E(0, X^o) > 0$ for some X^o, which implies $B_E(0, X) > 0$ for all X and that therefore, exhaustion is optimal. It is interesting to note that both Smith (1975, pp. 395) and Vousen (1973, pp. 139) suggest that exhaustion may be non-optimal even though the benefit functions they work with are concave.

Consequently, incomplete exhaustion occurs only when there are non-convexities in the extraction of natural resources. As an example, Schmalensee (1975) shows that for cases where there are non-convexities in the form of a fixed cost associated with any positive level of extraction, incomplete exhaustion may be optimal.[4]

FOOTNOTES

1. The author wishes to thank D.V.T. Bear, R. Robert Russel, Richard Schmalensee, and an anonymous referee for helpful suggestions in preparing this note.

2. To my knowledge, these are the only analyses that deal with depletion effects for non-replenishable resources.

3. If the limit doesn't exist, then $B_E(0,X) = \infty$ and complete exhaustion is optimal.

4. Vousden (1973) also allows for fixed costs (benefits) of a sort by assuming society receives some fixed negative (positive) level of utility (independent of the level of resource use) during the entire planning horizon. With this formulation, he obtains the perplexing result that preference ordering for different resource consumption paths are not preserved under positive linear transformations of the social welfare function. The optimal extraction strategy may differ for utility functions that vary only by an additive constant (the size of the fixed cost). However, this peculiarity arises only because the time horizon during which society is assumed to exist is itself a choice variable in Vousden's analysis.

REFERENCES

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4. N. Vousden, "Basic theoretical issues of resource depletion," J. Econ. Theory, 6, no. 2 (1973), 126-143.