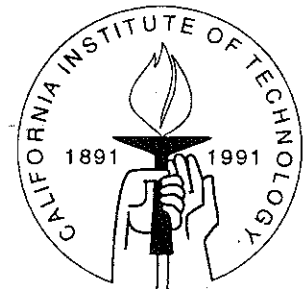DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

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AN EXPERIMENTAL EXAMINATION OF THE ASSIGNMENT PROBLEM

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ABSTRACT

The problem of optimally assigning individuals to heterogeneous objects so that each individual is allocated at most one object (the assignment problem) has a long history. Algorithms based on ordinal preferences have been developed and several auctions using monetary transfers have been proposed. The performance of two auction mechanisms to solve the assignment problem is examined in an experimental setting. One of the auctions is a sealed-bid variant of the Vickrey auction for homogeneous objects and the other auction is an extension of the English auction. The auctions are tested in two diverse competitive environments (high and low contention). The experimental results show that the English auction generates higher revenues and efficiencies than its sealed-bid counterpart especially if there is a high level of contention. However, the efficiency gains of the English auction are at the expense of consumers' surplus. Indeed, a random assignment creates greater consumers' surplus relative to either auction outcomes in the high contention environment.

1. Introduction

We consider the problem of allocating a fixed and heterogeneous set of goods, which we will generically call *slots*. This problem is presented from the point of view of a planner or institution designer, who wishes to implement a social welfare maximum such that each demander is assigned at most one slot. The formulation supposes that the planner himself attaches no value to any assignment.

This problem appears in a variety of settings; computer scheduling, the administration of office space, the assignment of students to dormitory rooms or courses, and the disbursement of social services. The problem encountered by the Jet Propulsion Laboratory of allocating antenna time on NASA's Deep Space Network (DSN) to spacecraft outside the earth's orbit has motivated this project. The DSN problem is an example of the allocation of a set of services in fixed supply within a given time period to a group of agents, a scheduling problem. In its most abstract and generic form the scheduling problem can be modeled as an assignment or one-sided matching problem.

In the assignment problem, if the planner knows the values agents place on slots then an optimal assignment of agents to slots can be found by solving an integer programming problem. However, true values are known only to the agents so that any mechanism, which the planner uses, must work with revealed rather than true valuations. Several auction processes (Barr and Shaftel (1976); Leonard (1983); Demange et al. (1986)) have been proposed to solve the coordination and incentive issues posed by the assignment problem when the planner is allowed to use monetary transfers.[1]

To date, there is very little empirical evidence on the ability of such auctions to solve the assignment problem. Much evidence exists for single-unit and multiple-unit versions of the Vickrey and English Auctions for homogeneous goods (see Cox et al. (1982), McCabe et al. (1990) and Coppinger et al. (1980)). However, when the goods to be allocated are heterogeneous, the only evidence available is that of Rassenti et al. (1982), who present a combinatorial version of a "Vickrey" auction to allocate goods with severe complementarities (*e.g.* airline landing slots) and Banks et al. (1989), who use an English auction for multi-dimensional bundles of services (*e.g.* weight and volume in the Space Shuttle). One purpose of this paper is to provide some experimental evidence on the performance of a sealed-bid auction and a variant of the

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[1] Olson (1991a) and a companion piece to this study (Olson and Porter (1991)) contain discussions of the assignment problem when the planner is not allowed to use transfers.

English auction to solve the standard assignment problem.

In Section 2 we formally define the assignment problem. In Section 3 two proposed auctions to solve the assignment problem are described. In Section 4 the experimental design is presented. Section 5 provides a detailed description of the mechanisms tested and their implementation. In Section 6 we present the experimental results. Section 7 contains a summary and some concluding remarks.

2. Formal Description of the Problem

In this section, we describe the classic assignment problem as a planner's problem. It is assumed throughout that the planner's goal is to maximize the total welfare of the system by assigning a set of slots to a group of agents. Each agent attempts to maximize utility (acquire their most valued slot). Since several agents may place their highest value on the same slots, the planner must know the relative value of the slots to each agent. However, depending on the mechanism used, it may be in the agent's best interest to overstate or understate relative preferences for slots.

The environment consists of n agents and k slots to be allocated. Let $N = \{1, ..., n\}$ index the set of agents, and let $K = \{1, ..., k\}$ index the set of slots. It is assumed that both N and K are finite and nonempty. Let $\mathcal{A}$ be the set of feasible deterministic, allocations of K to N, including the zero allocation, wherein no agent receives a slot. An element in $\mathcal{A}$ is an $n \times k$ matrix consisting of at most a single 1 in each row and column, where $a_{ij} = 1$, if agent i is assigned slot j, and $a_{ij} = 0$, if he is not. We also define $a^i = (a_{i1}, ..., a_{ik})$.

The payoff of each agent depends upon the slot allocated, any monetary payment, and the agent's type. An agent's type parameterizes the value he places on the goods being allocated. Let $\Theta^i \subset \Re^k$ be a set of possible types for agent i, $\forall i \in N$. Let $\Theta^N = \underset{i \in N}{\mathsf{X}} \Theta^i$. A $\theta \in \Theta^N$ will be called a *profile*. The number of agents and slots is fixed, so the feasible set is independent of the profile. Each agent i, evaluates each assignment $x \in \mathcal{A}$ (or assignment) through a valuation function $v(x, \theta^i) = \sum_j x_{ij} \theta^i_j$. The quantity $v(x, \theta^i)$ represents the willingness to pay of agent i of type θ^i for assignment x. The utility of agent i is quasi-linear and is given by $U(x, t, \theta^i) = v(x, \theta^i) + t^i$, where t^i is any monetary transfer to (or from) agent i.

We note that in the above definition agents may be indifferent between distinct assignments since they are selfish; that is, they care only about the slots allocated to them. When the outcome space is $\mathcal{A}$, and agents are selfish, there is no loss of

generality in the linear description of utility, since there are a finite number of slots. That is, when the outcome space is $\mathcal{A}$, given any utility function $\hat{U}(x)$, there is a θ^i such that $U(x, \theta^i) = \sum_j x_{ij} \theta_j^i = \hat{U}(x)$. The planner's objective is to assign the agents in N to the slots in K such that total system welfare is maximized. We can describe this problem as follows:

Given a profile $\theta \in \Theta^N$, $\underset{x \in \mathcal{A}}{\text{Max}}\ W = \sum_{i \in N} \sum_{j \in K} \theta_j^i x_{ij}$;   (A)

such that   A1) $\sum_{j \in K} x_{ij} \leq 1, \quad \forall i \in N$;

            A2) $\sum_{i \in N} x_{ij} \leq 1, \quad \forall j \in K$;

            A3) $x_{ij} \in \{0, 1\}, \quad \forall i \in N, \forall j \in K$.

Koopmans and Beckmann (1957) were the first to consider this problem in an economic context. They showed that there always exists a solution to the problem but that it is not necessarily unique, that there always exists a competitive equilibrium set of prices $\{p_j \geq 0\}$ $j \in K$, which may not be unique. A further observation concerned the additive invariance of the parameter θ^i. That is, if a positive constant is added to every element in the vector θ^i, then the solution remains the same. If an allocation solves (A), then we say that it is *outcome efficient*, and we call W the *total (or social) welfare* of the system.

3. Description of Allocation Mechanisms

Given the environment described above, several mechanisms are available to implement the outcome-efficient allocation in weakly dominant strategies. These mechanisms are multi-object generalizations of the "second-price" auction first described by Vickrey (1961). In these mechanisms the allocation is outcome-efficient and the prices paid by each agent are the minimum competitive equilibrium prices. We shall call these the *Vickrey prices.*

Leonard (1983) proposed a sealed-bid auction to obtain the optimal allocation and Vickrey prices, which we shall call the Vickrey-Leonard auction. The Vickrey-Leonard auction requires each participant to submit a sealed bid listing his valuation of each of the slots. The planner then determines the allocation by solving the assignment problem (A) using each participant's submitted bids in place of true valuations. There

are two equivalent methods to find the prices that the agents pay in this auction. One way is to compute the impact of a second slot of similar type. This entails the solution of k additional assignment problems.[2] A computationally simpler solution is to find the minimum dual prices.[3] Given a profile $\theta = (\theta^1, ..., \theta^n)$, prices are determined by solving the dual program:

$$\operatorname*{Min}_{p_j} \sum_{j \in K} p_j \qquad (A')$$

such that

$$w_i + p_j \geq \theta^i_j \qquad \forall j \in K, \forall i \in N$$

$$\sum_{j \in K} p_j + \sum_{i \in N} w_i = W$$

$$w_i, p_j \geq 0, \qquad \forall j \in K, \forall i \in N,$$

where w_i are slack variables.

Leonard (1983) and Demange and Gale (1985) have shown that it is a dominant strategy for agents to reveal their true valuations in this auction. However, in some environments truthful revelation is not a strong dominant strategy, in the sense that there may be many bids which generate the same assignment for an agent, a fact that we make precise in the following theorem. One of the environments ($n = k = 6$) we investigate satisfies the conditions for a weakly dominant strategies, while the other environment ($n = 8 > k = 6$) does not satisfy these conditions when agents are allowed to bid on only k slots.

Theorem 1: For the Vickrey-Leonard auction, when $n \leq k$ the strategy $b^i_j = \theta^i_j + c$, $c \in \Re$ is weakly dominant, i.e., revelation up to a constant.

proof. Recall that the optimal solution to the assignment problem does not change when a constant is added to any row of the valuation matrix (see, e.g., Koopmans and Beckman (1957)) as long as $n = k$ (for the case $n < k$ we can use imaginary bidders who always bid zero). That is, suppose for some fixed $i^* \in N$, $\hat{\theta}^{i^*}_j = \theta^{i^*}_j + c$, $\forall j \in K$; then $\sum_i \sum_j \hat{\theta}^i_j x_{ij} = \sum_i \sum_j \theta^i_j x_{ij} + c$. The (x_{ij}) that maximizes $\sum_i \sum_j \hat{\theta}^i_j x_{ij}$ also maximizes $(\sum_i \sum_j \theta^i_j x_{ij} + c)$. So two vectors of bids that differ by a constant will assign an agent

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[2] Prices can be computed directly by setting $p_j = (W^{K+j}_N - W^K_N)$, where $W^K_N \equiv$ largest sum of bids on slots $K = \{1, ..., k\}$ assigned to agents in N, and $W^{K+j}_N \equiv$ maximum of the sum of bids on slots $K \cup \{j\}$ to agents in N; that is, add another slot j and solve the assignment program and obtain W^{K+j}_N. This is how the price calculation was described to the subjects--see Appendix B for instructions.

[3] A similar approach was used by Rassenti, Smith, and Bulfin (1982).

to the same slot, and since the price an agent pays is independent of his reported valuations, the price he pays is the same price when his bids differ by a constant. Since the allocation and the price of the slot are the same when an agent's bids differ by only a constant, he is indifferent between submitting the two bids. □.

Demange, Gale and Sotomayor (1986), hereafter DGS, proposed two variations of an English auction to obtain the Vickrey prices.[4] When there is only one slot to allocate, an English clock auction is conducted by first setting an arbitrarily low asking price for the slot; each bidder then announces whether he wants the slot at the announced price. If only one bidder demands the slot, he is awarded it at the announced price and the auction ends. If more than one bidder demands the slot, the price is increased by a fixed amount. The auction continues by increasing the price until only one bidder demands the slot. When a set of heterogeneous slots are to be allocated the pricing algorithm becomes more complicated.

DGS developed an "exact" and an "approximate" auction where bids are requests to buy slots at the announced price. The two variations differ in the procedure used to determine which slots will have their prices increased in the next step. We only examine the approximate auction case in this paper. The DGS auction begins with the planner announcing a set of prices, one for each slot. Each agent submits a request to purchase the slot at the announced price; selecting more than one slot implies that the agent is indifferent among the slots selected for assignment. Slots that are contained in the largest, pure-overdemanded set have their prices increased.[5] A set of slot is *overdemanded* if the number of bidders demanding only slots in this set is greater than the number of slots in the set. The largest pure-overdemanded set contains all over-demanded sets. The auction ends when there are no overdemanded slots.

DGS rely on the implicit assumption that agents will act honestly,[6] that is, request only those slots that maximize value minus current price. This assumption requires a bidder whose maximum net value is zero to place bids on all slots that have a net value of zero, if he places a bid on any such slot. In particular, when $k < n$ this requires the existence of null items with a value and price of zero. If this assumption is

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[4] Barr and Shaftel (1976) propose a generalization of a second-price descending-bid auction to obtain the Vickrey prices; it is discussed in detail in Olson (1991b).

[5] This variation is due to Mo (1988).

[6] A bid $B \subset K$ is *honest* if $\forall\ l \in B$, $v_l - p_l = \max_{j \in K}(v_j - p_j)$ and if $l \in K/B$, $v_l - p_l < \max_{j \in K}(v_j - p_j)$; if $\forall j \in K\ (v_j - p_j) < 0$, then $B = \emptyset$ and the agent does not submit a bid, where p_j is the price of item j, and v_j is the value of slot j to the bidder.

met, then the outcome-efficient allocation and Vickrey prices will be obtained in the DGS auction. However, for the DGS auction process, honesty is not a dominant strategy (Olson (1991b) provides an example) but it is a Nash equilibrium.

Theorem 2: In the DGS exact auction, it is a Nash equilibrium for each agent to select the slot(s) that maximize utility at each price announcement (truthful revelation); i. e., given $(p_1^t,\ldots,p_k^t)$, agent i selects the slots $l \in K$, such that $(\theta_l^i - p_l^t) = \max_j\{\theta_j^i - p_j^t\}$, and $(\theta_l^i - p_l^t) \geq 0$.

Before we supply the proof, some notation and behavior implied by honest bidding is provided. At each iteration t given prices p^t, an agent places a bid $b^t \subseteq K$, which is a selection of slots. At iteration $t+1$, $b_p^{t+1} \subseteq b^t$ is the set of slots that the agent bids for at iteration t that had a price increase at iteration $t+1$. At iteration $t+1$, $b_c^{t+1} \subseteq b^t$ is the set of slots that the agent bid for at iteration t that did not have a price increase at iteration $t+1$. Thus:

1. If $b^t = \emptyset$ at t, then $b^{t+1} = \emptyset$ at $t+1$. If an agent does not bid for a slot at iteration t, she will not bid for a slot at iteration $t+1$.

2. If $b^t \subseteq K$ at t and $b_p^{t+1} = b^t$, then either $b^{t+1} = b^t \cup b$, or $b^{t+1} = \emptyset$, where $b \subseteq K \backslash b^t \cup \emptyset$. If an agent bids for a set of slots at iteration t and each slot has a price increase at iteration $t+1$, then either the agent will bid on the same slots and possibly an additional slot, or will not bid on any slot.

3. If $b^t \subseteq K$ at t and $b_p^{t+1} \subset b^t$ (a proper subset), then $b^{t+1} = b_c^{t+1}$. If the price increases on some of the slots that the agent has bid for at iteration t, then he will drop his bids on those slots and keep his bids on the slots that did not have an increase in price.

We now provide the proof:

proof: In the DGS auction, if all agents are honest, then the vector of prices p^e that results from the auction is the vector of minimum core prices (or competitive equilibrium; see DGS). By definition of the core, an agent cannot increase his net value in any slot other than the slot assigned to him. So once the equilibrium prices are reached, there is no advantage for an agent to be dishonest. If $p^t \geq p^e$ and all agents are honest, the auction stops; if an agent is not honest, the auction will stop and he will be no better off at another slot. If the auction does not stop, then there will be a price

increase on some slot, and the agent cannot be made better off, since p^e gives him his highest net value.

Also, by definition of the core, if $p_j^t < p_j^e$ at some iteration t and some slot j there will be overdemanded slots. If all agents are honest, prices will increase at $t + 1$. So an auction will end with $p_j^t < p_j^e$ only if an agent is dishonest. But that would imply removing a bid from slot j or adding a bid to another slot. In either case the agent is worse off if the auction stops (since bidding on his highest net-valued slot is the honest bid). $\square$.

An agent cannot directly make himself better off by being dishonest; only if other agents bid dishonestly is it possible to be made better off by a dishonest bid. We note that honesty is not a weakly dominant strategy in our environment, and that there are many Nash equilibria that do not correspond to the social optimum.

Theoretically, both auctions yield the same assignment (outcome-efficient assignment and Vickrey prices), assuming that bidders act honestly. In the experimental literature there has been much success with the use of progressive (English) auctions in obtaining efficient allocations, especially relative to their sealed-bid counterparts (examples of this literature are Banks et al. (1989) and McCabe et al. (1990)). The ability of the English auction to provide feedback to participants concerning where they stand and how to improve their current standing appears helpful.

4. Experimental Design

The experimental design consists of two fixed factors: type of mechanism (sealed-bid (Vickrey-Leonard) and progressive (DGS) auctions), and parameter set (high and low contention). We begin by discussing the parameters of the environment and then describe the payment conditions. We end this section with a summary list of the experiments we have conducted.

4.1. Parameters of the Environment

The environment under consideration consists of 6 slots, $K = \{1, 2, \ldots, 6\}$, which must be allocated to a set of six or eight subjects. Preferences over slots are induced using monetary payoffs for each slot provided to each subject (see Smith (1976)). Each participant could be assigned one of 10 possible payoff sheets or types. An abbreviated list of payoffs is provided below in Table 1 (the complete listing of the payoffs used in

our experiment can be found in Appendix A). For example, given the payoff list in Table 1, if a subject were provided with sheet 2 and assigned item 3, he would obtain a value of 800. At the beginning of each period each subject is assigned a payoff sheet that is drawn uniformly from the set with replacement, i.e., the fact that priors over types are uniform was given as *common information* to the subjects.

Table 1:
An Example of a Payoff List

Payoff Sheet Number	Item Number					
	1	2	3	4	5	6
1	800	600	400	200	400	600
2	400	600	800	600	400	200
. .						
10	300	300	300	300	300	900

Given the payoff tables and number of subjects, we can solve for the optimal assignments and the set of competitive equilibrium prices (*the core*). Let p_j denote the *Vickrey prices* (minimal dual prices in the core) determined from (A'). If v_j^* is the value of slot j from the optimal assignment determined in (A), then the closer p_j is to v_j^*, the higher is the level of competition for the slot (more of the buyers' surplus is transferred to the planner). Competition for a slot is a function of both the profile and the number of agents wanting a slot allocation. In our experiments we created two alternative competitive environments based on the following ratio we call the *contention index* ($\mathbb{C}$):

$$\mathbb{C} = \mathbb{E}_L \frac{\sum_j p_j}{W^*},$$

where j indexes the slot, p_j is its Vickrey price, W^* is the outcome-efficient welfare for a profile of payoff sheets from the payoff list, and $\mathbb{E}_L$ is the expectation operator defined over the possible profiles from a given payoff list. Notice that $\mathbb{C} \in [0,1]$. A realization of $\mathbb{C} = 1$ implies that all the surplus in the system is paid out at the "competitive" equilibrium prices, and a realization of $\mathbb{C} = 0$ implies that the profiles are diverse and all the surplus is retained by the subjects.

Varying $\mathbb{C}$ in the experiments provides us with a check on the robustness of

potential allocation mechanisms, so that we may explore the hypothesis that the surplus and efficiency of the tested mechanisms are sensitive to the expected contention. For our experiments two environments are considered: one with a "low-contention index" and one with "high-contention index." The low-contention environment utilizes six subjects and six slots, with values that, on average, provide contention for only one or two of the slots. In the high-contention environment, there are six slots and eight subjects and the values were such that almost all slots would have a high-contention index. Figure 1 in Appendix A supplies a graph of the actual realization of contention levels used for the low-contention and high-contention treatments. The individual draws and associated core prices for each slot for the experiments we conducted can be found in Appendix A.

Table 2 shows our 2x2 design. The number of experiments for each cell is listed. A summary list of each experiment is provided in Table 3. All of our experiments were conducted at the California Institute of Technology using graduate and undergraduate subjects. Each experimental session consisted of 20 periods where at the beginning of each period, each subject was given a payoff sheet. All communication was done through computer terminals, and a history of prices and personal selections was provided by the software so that subjects could review past periods. Each experimental session consisted of only one allocation mechanism and one set of payoff parameters (high-contention or low-contention parameters). A partial set of subject instructions can be found in Appendix B.

Table 2

2x2 Design Factors
(numbers in cells are the number of experiments conducted)

Mechanism   *Environment Parameters*

	Low contention	High Contention
Vickrey-Leonard	3	2
DGS-Progressive	2	2

Given the environment defined above, the planner's objective is to design allocation mechanisms to assign slots to subjects, which result in the maximum social welfare. We consider this design question next.

5. Implementation of the Allocation Mechanisms Tested

In the VL auction subjects submitted a sealed bid for each of the six slots to be allocated (if no bid was entered for an item, it was assumed to be 0). Each subject's bid consisted of a vector of monetary bids $(b_1^i, \ldots, b_k^i)$ over the slots with the restriction that $b_j \in [0, 9999]$, $\forall j \in K$. The allocation is determined by solving the integer program described in (A), replacing θ_j^i with b_j^i (using bidder's submitted bids in place of their valuations).[7] The prices were determined by solving the dual program. Once the allocation and prices were determined, they were transmitted to the subjects, profits were then calculated and histories updated, after which a new period was started.

Implementation of the DGS auction was more involved. The process proceeded as follows: First, at the beginning of a period (iteration $t = 0$) initial prices were set at zero for each slot. Given these prices individuals selected the slots they would like at those prices. Given the selections, an algorithm determined which slots were overdemanded.[8] If a slot was overdemanded, its price would increase for the next iteration and the period would continue with the updated prices. For each overdemanded slot its price at the next iteration was increased by 50 francs.[9] The process stopped when there were no overdemanded slots; an assignment was then made. Those assigned to the slots paid the current price, except in an instance that is described below.

To implement the DGS auction process, we imposed two additional rules that were based on our experience with the DGS pilot experiment and with single-object English auctions. First, we imposed a commitment rule. If a subject selected a slot at an iteration and the slot was *not* overdemanded, then he was *committed* to select that slot at the next iteration (*i.e.*, subjects could not renege on selections if the price of those selections did not increase). Second, the auction does not elicit bids when a subject's maximum net value is zero, thus we used a back-tracking rule: if at the end of a period a slot is unassigned, then the slot is randomly allocated among the last *unassigned* bidders who placed a bid on the slot at a previous iteration.

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[7] If there were ties in the bids to determine allocations, they were broken randomly. If a slot was not demanded in the auction, it was assigned randomly to those who were not previously assigned a slot.

[8] The algorithm is a variation of the Ford-Fulkerson procedure (see, *e.g.*, Franklin (1980) and Gale (1960)).

[9] In a pilot we tried increments of 10 and 25 francs but found that 20 periods could not be completed in a reasonable amount of time (less than 2 hours).

Table 3 lists the experiments we have conducted along with pertinent information about each session.

Table 3. Experiment History

name	#	contention	payoff	time
VL1	1	high	$15.80	1.6hr
VL2	2	low	$15.20	1.2hr
VL3	3	high	$14.20	1.5hr
VL4	4	low	$15.00	1.1hr
DGS1	5	low	$18.70	1hr
DGS2	6	low	$18.00	50min
DGS3	7	high	$15.40	1.5hr
DGS4	8	high	$16.00	1.5hr
VL5	9	low	$15.20	55min

Notes:

All experiments had 6 slots and 20 periods. High-contention experiments had 8 subjects; low-contention experiments had 6 subjects. The name describes the type of experiment: VL = Vickrey-Leonard sealed-bid, DGS = Demange et al., progressive auction.

We also ran 2 pilot VL sealed-bid auctions (1 low-contention and 1 high-contention) and one pilot DGS auction (high-contention).

6. Experimental Results

For each mechanism and environment, we measure two aspects of performance: efficiency and consumer surplus. Efficiency measures overall performance relative to the optimal allocation (as defined by (A)); that is, it measures the ability of the mechanism to maximize total welfare. Consumer surplus measures the distribution of system surplus to the subjects. These measurements are normalized by the outcome-efficient allocation and the Vickrey prices. We also measure the revenue generated from each auction and individual choice behavior.

6.1 Efficiency

Efficiency is measured as the total observed welfare divided by the total welfare that would have been realized if the optimal allocation had been implemented. That is, for each period,

$$E = (\sum_{i,j} x_{ij} v_{ij})/(\sum_{i,j} x^*_{ij} v_{ij}),$$

where (x^*_{ij}) is the optimal allocation, and x_{ij} is the allocation that was actually realized. We divide by the total welfare in order to normalize the data of each trial so that we can compare relative efficiencies across trials. Notice that if the allocation (x_{ij}) is outcome efficient then $E = 1$.

In Table 4a we display the average efficiencies and standard deviations achieved with the Vickrey-Leonard auction and the DGS auction for low and high contention. The table includes the expected efficiency from assigning slots randomly to subjects with the restriction that every slot is assigned. Efficiencies are averaged using three different restrictions on an experiment session: (all periods, the first ten periods, and the last ten periods of a session) to see if "learning has occurred."[10] The time series of efficiencies for each mechanism can be found in appendix C.

Table 4a: Mean Efficiencies
(observed relative to predicted VL assignment)

	Periods		
	All (σ)	First10 (σ)	Last10 (σ)
Low Contention			
Vickrey-Leonard	0.97 (0.04)	0.97 (0.04)	0.97 (0.04)
DGS Auction	0.95 (0.11)	0.92 (0.15)	0.98 (0.03)
Random	0.75 (0.10)	0.75 (0.10)	0.76 (0.09)
High Contention			
Vickrey-Leonard	0.95 (0.07)	0.94 (0.08)	0.97 (0.04)
DGS Auction	0.99 (0.03)	0.99 (0.04)	0.99 (0.03)
Random	0.62 (0.08)	0.61 (0.08)	0.62 (0.08)

From Table 4a we can make a number of observations:

1. Both the VL and DGS auctions yield higher efficiencies than the expected random assignment.

2. In the low-contention DGS and high-contention VL, the efficiencies for the first 10 periods are lower than the efficiencies for the last ten periods and the variances are higher in the first ten periods than in the last ten periods.

3. In the high-contention DGS and low-contention VL there does not appear to be any difference in the first and last ten periods.

4. Except for the first periods of the low-contention DGS and high-contention VL

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[10] We will discuss the issue of learning in the section on individual behavior.

treatment, all the observations are close to predicted efficiency of 1.0.

5. Except in the first ten periods of the low-contention DGS treatment, mean efficiency of DGS is greater than VL.

To determine if the difference in efficiencies is significant we estimate the following model:

$$y_{ep} = \beta_0 + \beta_{1m_e} + \beta_2 f(p) + \beta_3 c_{m_e p} + \overline{c}_{m_e}(\gamma_0 + \gamma_{1m_e} + \gamma_2 f(p) + \gamma_3 c_{m_e p}) + \beta_e I(e) + \epsilon_{ep},$$

where:

$p \equiv$ period number, $p \in \{1,...,20\}$; $e \equiv$ experiment index.

$I(e) \equiv$ Indicator of experiment.

$m_e \equiv$ mechanism used in experiment e, $m_e \in \{$VL, DGS$\}$.

$y_{ep} = (y_{ep}^{\text{ef}}, y_{ep}^{\text{cs}})$; $y_{ep}^{\text{ef}} \equiv$ efficiency of experiment e in period p,

$\qquad\qquad y_{ep}^{\text{cs}} \equiv$ consumer surplus of experiment e in period p.

$f(p) \equiv$ a monotonic function of the period, in the following estimation we used: $f(p) = 1/p$.

$c_{m_e p} \equiv$ contention of mechanism m_e in period p.

$\beta_0 \equiv$ constant.

$\beta_{1m_e} \equiv$ indicates mechanism effect.

β_2, $\beta_3 \equiv$ coefficients of period variate and contention, respectively.

$\overline{c}_{m_e} \equiv 1$ if contention is high and 0 if contention is low.

$\gamma_0, \gamma_{1m_e}, \gamma_2, \gamma_3 \equiv$ change due to high contention.

$\beta_e \equiv$ indicates experiment effect.

$\epsilon_{ep} = (\epsilon_{ep}^{\text{ef}}, \epsilon_{ep}^{\text{cs}}) \equiv$ error term; and we assume the ϵ_{ep}'s are independent and $\mathbb{E}[\epsilon_{ep}] = (0,0)$, $\mathbb{V}[\epsilon_{ep}] = \Sigma > 0$.

Table 4b: Efficiencies[11]

| Variable | -------- All periods -------- | | -------- Last ten periods -------- | |
	Coef.	P-Value	Coef.	P-Value
Constant	0.9850	0.0000[†]	0.9683	0.0000
P1	-0.0803	0.0044[†]	0.0920	0.8393
Contention	0.0290	0.6180	0.0222	0.6808
DGS	-0.0130	0.4621	0.0275	0.5113
DGS*P1	0.0829	0.0280[†]	-.3813	0.5108
C01	0.0236	0.7229	0.0046	0.9505
P1*C01	-0.0054	0.8874	0.3024	0.6050
Cont*C01	-0.0720	0.4690	0.0079	0.9319
DGS*C01	0.0245	0.2867	0.0108	0.6263
VL1(high)	-0.0119	0.4654	-.0224	0.1563
VL2(low)	-0.0018	0.9126	-.0172	0.2761
VL3(low)	-0.0046	0.7768	0.0014	0.9306
DGS1(low)	0.0098	0.5821	0.0008	0.9586
DGS3(high)	0.0011	0.9482	-.0187	0.2359
DGS1(per ≤ 5)	-0.2685	0.0000[†]	na	na

Variables: Main effects:: P1=1/period, DGS=I(DGS auction), C01=I(high contention), Cont=Contention index. VL1, VL2, VL3, DGS1, DGS3 are experimental indicators with their contention in parenthesis. Interaction effects are denoted by the symbol *. DGS1(per ≤ 5) indicates the first 5 periods of experiment DGS1, in which the subjects had difficulty during the first 5 periods. The symbol [†] indicates a probability level ≤ 0.10; these variables will be considered significant in the discussions below.

The estimation indicates:

1. When all periods are included there is a significant period effect; when only the last ten periods are included there are no significant effects. This would indicate that there is no difference in the efficiencies generated by the DGS and VL auctions.

2. The Vickrey-Leonard and DGS auctions result in efficiencies close to the theoretical prediction.

3. The significant positive coefficient for DGS*P1 indicates that there is a greater increase of efficiency over periods for the DGS treatment.

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[11] The effects on efficiency and consumer surplus were measured simultaneously and we present them in full in Appendix D. The results for consumer surplus are presented in the next section.

6.2 Consumers' Surplus, Prices and Revenue

We measure relative consumers' surplus as the sum of the surplus realized by all subjects divided by the sum of the surplus that would have been realized if the optimal allocation and the Vickrey prices had been implemented. That is:

$$S = (\textstyle\sum_{i,j} x_{ij} v_{ij} - \sum_j p_j)/(\sum_{i,j} x_{ij}^* v_{ij} - \sum_j p_j^*),$$

where (x_{ij}^*) is the optimal allocation and p_j^* are the Vickrey prices, and x_{ij} is the allocation and p_j are the prices that are actually realized. These are listed in Table 5a below. The time series of consumer surplus for each mechanism can be found in appendix C.

Table 5a: Mean Consumers' Surplus
(observed relative to predicted VL assignment)

	Periods		
	All (σ)	First10 (σ)	Last10 (σ)
Low Contention			
Vickrey-Leonard	1.01 (0.10)	1.01 (0.11)	1.01 (0.09)
DGS Auction	0.88 (0.16)	0.83 (0.20)	0.92 (0.10)
Random	0.88 (0.01)	0.89 (0.01)	0.88 (0.01)
High Contention			
Vickrey-Leonard	1.46 (0.97)	1.53 (1.15)	1.39 (0.75)
DGS Auction	1.03 (0.61)	0.89 (0.18)	1.18 (0.84)
Random	3.09 (0.58)	3.03 (0.56)	3.15 (0.60)

From Table 5a we observe that:

1. The DGS auction did not result in consumer surplus that is much different from from the random allocation in the low contention environment. The random allocation results in much higher consumer surplus in the high contention environment.

2. Except for the the low-contention VL, there appears to be a difference in the first and last ten periods.

3. Only the consumer surplus in the low-contention VL treatment appears to be consistently (both first and last periods) close to the predicted consumer surplus of 1.0.

4. In both the high-contention and low-contention treatment, the VL gives higher

relative consumers' surplus than the DGS. Note that the only way that relative consumers' surplus can be over 1 is for participants to under-reveal.

To determine if the difference in consumers' surplus is significant, we estimate the same model as for efficiency except that:

$y_{ep} \equiv$ relative consumers' surplus of experiment e and period p.

Table 5b: Consumers' Surplus

Variable	------- All periods ---------		-------- Last ten periods --------	
	Coef.	P-Value	Coef.	P-Value
Constant	0.9612	0.0000	0.8398	0.0638
P1	-0.0514	0.8556	1.5531	0.8070
Contention	0.3319	0.5746	0.4139	0.5847
DGS	-0.0859	0.6306	0.3290	0.5746
DGS*P1	-0.1244	0.7438	-6.1225	0.4513
C01	-1.2655	0.0625[†]	-0.9757	0.3462
P1*C01	0.1285	0.7382	-0.5533	0.9461
Cont*C01	2.0964	0.0390[†]	1.8228	0.1612
DGS*C01	-0.5123	0.0292[†]	-0.4810	0.1246
VL1(high)	-0.2515	0.1288	-0.5455	0.0147[†]
VL2(low)	0.0352	0.8311	0.0417	0.8497
VL3(low)	0.0085	0.9589	0.0233	0.9156
DGS1(low)	0.0265	0.8840	0.0284	0.8973
DGS3(high)	0.1356	0.4116	0.1420	0.5192
DGS1(per $\leq$ 5)	-0.1669	0.5803	na	na

Variables: Main effects:: P1=1/period, DGS=I(DGS auction), C01=I(high contention), Cont=Contention index. VL1, VL2, VL3, DGS1, DGS3 are experimental indicators with their contention in parenthesis. Interaction effects are denoted by the symbol *. DGS1(per $\leq$ 5) indicates the first 5 periods of experiment DGS1, in which the subjects had difficulty during the first 5 periods. The symbol [†] indicates a probability level $\leq$ 0.10; these variables will be considered significant in the discussions below.

The estimation indicates:

1. The period effect is not significant.

2. When the first ten periods are included, contention is significant with low-contention consumers' surplus (relative to optimal) being lower than the high-contention consumer surplus for the same mechanism. When the first ten periods are excluded, contention is not significant.

3. In terms of consumer's surplus we have VL-DGS > 0, but this difference becomes smaller with time.

We conclude that when contention is low both of the mechanisms tested will give the subjects the same level of consumer surplus. But when contention is high consumers' surplus is significantly affected.

6.3 Revenue

In Table 6 we present the revenue generated by the two transfer mechanisms: the Vickrey-Leonard sealed-bid auction and the DGS progressive auction. Even though revenue is total welfare less consumer surplus, we present this information for two reasons. First, the efficiency and consumer surplus measures may be confounding and the effect on revenue generation may not be apparent; that is, if one mechanism has both lower efficiency and lower consumer surplus than another mechanism, then the revenue from one mechanism could be either higher or lower than the revenue from the other mechanism. Second, in most auction studies, progressive auctions tend to generate more revenue than sealed-bid auctions (see, *e.g.*, Banks et. al (1989)).

Table 6: Revenue
(observed vs. predicted)

Mechanism\Experiments	$1\,(\sigma)$	$2\,(\sigma)$	$3\,(\sigma)$	All (σ)	Predicted (σ)
Low Contention					
Vickrey-Leonard	314 (277)	375 (236)	437 (269)	375 (254)	490 (335)
DGS Auction	613 (367)	768 (602)		690 (500)	490 (335)
High Contention					
Vickrey-Leonard	2821 (780)	2658 (693)		2739 (731)	3253 (289)
DGS Auction	3098 (731)	3225 (727)		2993 (943)	3253 (289)

From Table 6 we observe that in both the high-contention and low-contention environments, the DGS auction generated higher levels of revenue. In the low-contention environment the DGS auction generated higher than predicted levels of revenue and a very high variance in the second experiment. The high variance in the second DGS auction appears to be from the first 5 periods, where it appears some subjects may have been confused. Nonetheless, the results we find here are consistent with the results found in other studies (see, Banks et. al (1989)).

6.4 Individual Behavior

In this section, we look at individual behavior to see if the mechanisms are robust to individual deviations from predicted behavior and if the behavior assumptions we applied were appropriate.

We do not formally model an individual's "learning" or the events that determine a subject's behavior, but only inquire if we can measure the direction of the difference in our treatment effects. This means that if we can measure a difference in subjects' bids between the first and last periods of an experiment, then we cannot announce that we have found learning, but only that there is a difference in bidding.

Auction Behavior

Smith (1980) and Coppinger, Smith, and Titus (1980) report the results of experiments in which the Vickrey (second-price) auction was used to allocate a single slot. It was found that many subjects played their dominant strategy fairly rapidly, but that violations of single-period, dominant strategy behavior were common, especially in the "early" trials of an experiment session. Miller and Plott (1985) study an auction for multiple homogeneous units in which price is set at the highest rejected bid (a uniform price auction). For their parameters (many units on the margin), the uniform-price auction is demand-revealing, and they find that after replication, bidders report their true valuation. Our experiments attempt to see if this behavior continues when the goods to be allocated are heterogeneous.

We note that unlike previous experiments with uniform price auctions we do not restrict our subjects to bids below or equal to their slot valuations. The rationale given by some for restricting bids is that overbidding can result in negative profits (bankruptcy) and thus lead to a subject "sabotaging" an experiment if there is only a small chance that he may obtain positive profits. Not being able to extract payment

from the subjects allows them to be indifferent between a payoff of zero and any negative amount. We had very low priors on this happening so we chose to allowed bidding to be in $[0, 9999]$ interval. We did not observe "sabotaging" to happen in any of our experimental trials.

Vickrey-Leonard sealed-bid auction

In the VL auction we hypothesized that subjects would play a weakly dominant strategy, and when it existed, their strong dominant strategy. Subjects in the low-contention environment have a set of weakly dominant bidding strategies; each bid differs from the subject's slot valuations by the addition of a constant. In the high-contention environment, subjects have a unique, strong dominant strategy to bid their slot valuations. The lack of a larger set of weak strategies in the high-contention environment is the result of eight subjects and only six slots; this creates two implied slots that have zero value for all the subjects. For weakly dominant strategies to exist, the subjects must be able to place bids on all the slots, but in the high-contention case, they are permitted to place bids on only six of the slots.

In Appendix E we display bidding behavior for each subject and each VL sealed-bid experiment. Each graph shows three series, which are based on the difference between a subject's bid and his slot values. The three series are:

$$\text{1) } \max_j\{\text{bid}_j - \text{val}_j\},$$
$$\text{2) } \text{mean}_j\{\text{bid}_j - \text{val}_j\}, \text{ and}$$
$$\text{3) } \min_j\{\text{bid}_j - \text{val}_j\},$$

where $\text{bid}_j \equiv$ bid slot j, and $\text{val}_j \equiv$ value of slot j.
So high values on the graph indicate overbidding and low values indicate underbidding. The plotted variables were truncated so that all graphs were in the range $[-1000, 1000]$. The low-contention experiments were also adjusted by a constant for each period, depending on the valuations and bids, since in the low-contention environment subjects had many weakly dominant strategies that varied only by a constant. The adjustment was accomplished as follows:
Let $\text{bid}^*_j = \text{bid}_j - \min_j\{\text{bid}_j\}$, and let $\text{val}^*_j = \text{val}_j - \min_j\{\text{val}_j\}$. Then the three series are:
$$\text{1) } \max_j\{\text{bid}^*_j - \text{val}^*_j\},$$
$$\text{2) } \text{mean}_j\{\text{bid}^*_j - \text{val}^*_j\}, \text{ and}$$
$$\text{3) } \min_j\{\text{bid}^*_j - \text{val}^*_j\}$$
From these displays we can observe that:

1. The high-contention environment has a higher variance of (bid - value) and more consistent overbidding. Often the overbidding occurs in the earlier periods and disappears in latter periods.

2. The low-contention environment has more consistent underbidding, and rarely is overbidding observed.

These results can be explained by the intense competition for slots in the high contention case pushing subjects to initially overbid for slots. However, when such overbidding resulted in a loss it was rarely repeated.

Table 7a: Vickrey-Leonard Sealed-bid
(bid-value summary statistics)*

High Contention	$\bar{x}$	σ	min	max	$\tilde{x}$
Min (bid-val)	-167.1	660.3	-900.0	9099	-200
Mean (bid-val)	88.5	820.3	-416.6	9649	-50.0
Max (bid-val)	652.5	1983.8	-300.0	9999	1.0

Low Contention	$\bar{x}$	σ	min	max	$\tilde{x}$
Min (bid-val)	-160.9	174.7	-600.0	0	-100
Mean (bid-val)	-51.0	93.7	-299.8	241.7	-16.7
Max (bid-val)	57.9	142.4	0	800	0.0

High Contention (truncated)	$\bar{x}$	σ	min	max	$\tilde{x}$
Min (bid-val)	-208.5	266.7	-900.0	1000	-200
Mean (bid-val)	2.3	300.0	-416.6	1000	-50.0
Max (bid-val)	190.0	382.3	-300.0	1000	1.0

* $\bar{x}$ is the mean value, σ is the standard deviation, and $\tilde{x}$ is the median.

Table 7a shows that for high-contention environments the means and standard deviation of the bid — value difference are much larger than for low-contention environments. The last part of the table contains the summary statistics for the high-contention environment when the bid — value observations are truncated above at 1000. This was done because a few very high differences above 1000 skew the summary statistics except for the median (there were 4 truncations in the minimum observations,

16 for the mean observations, and 51 for the maximum observations).

The second measure of subjects' behavior is found by substituting an individual's bid with his valuations to determine if there is a gain, and hence if a subject's deviations were costing him. If deviations from truthful reporting do not cost the subject, then we cannot argue that it is in his best interest to play the dominant strategy. The descriptive statistics below indicate that on average the gain for truthful revelation was largest in the high-contention environment.

Table 7b: Vickrey-Leonard Sealed-bid
(net gain statistics)*

Experiment	contention	$\bar{x}$	$\tilde{x}$	$\bar{\sigma}$
1	high	82.4	61.6	288.9
2	low	30.6	26.0	63.9
3	high	53.3	37.6	188.4
4	low	29.2	30.0	80.9
9	low	16.7	22.3	84.9

* $\bar{\sigma}$ is the mean of the standard deviations for the subjects.

In addition, an analysis of covariance was performed, wherein subjects were considered to be random effects, type (payoff values) were considered to be a fixed effect, and time over periods was measured as (1/period). The following period-effect results are presented:

Table 7c: Vickrey-Leonard Sealed-bid
(period effect)
Experiment type

Variable	Low contention	High Contention	High Truncated
Min (bid-val)	-65.2 (0.08)	286 (0.09)	-14.4 (0.002)
Mean (bid-val)	-32.0 (0.09)	786 (0.00)	131.0 (0.028)
Max (bid-val)	80.0 (0.004)	2010 (0.00)	213.0 (0.003)

A Hausman (1978) χ^2–test specification test was performed, and the null hypothesis of correct specification could not be rejected with probability $p = 0.99$ on all the models except Min (val-bid) for high truncation.

From the above model we observe:

1. The period effect is found to be significant in all cases, especially for $\text{Max}(\text{bid} - \text{val})$.

2. In the high-contention treatment, the $(\text{bid} - \text{value})$ measurements are higher than average in earlier periods (as observed from the positive coefficient).

3. In the low-contention treatment, Min(bid-val) and Max(bid-val) are lower than average in the earlier periods. This would indicate that subjects underbid more on their least favorable slots in the earlier periods.

DGS Progressive Auction

To study individual behavior in the DGS progressive auction experiments we construct three measures of bidding behavior. In the description of the DGS experiments, $B_i(p) \subseteq K$ refers to set of slots selected by subject i given the vector of prices p over the slots. $B_i(p)$ is subject i's bid and subject i bids on slot j if $j \in B_i(p)$. Net value is the subject's value for a slot minus the price for that slot $(v_{ij} - p_j)$. We now make the following categories of bid types in the DGS auction.

1. A bid $B_i(p)$ is ***revealing*** if $h \in B_i(p)$ then $v_{ih} - p_h \geq 0$ and if $h^* = \underset{j \in K}{argmax} \ v_{ij} - p_j$ then $h^* \in B_i(p)$. That is, at least one of the bids is on a slot that maximizes net value, and there are no bids on slots that have negative net value. A bid may be placed on a nonmaximizing slot, and there may be maximizing slots that do not receive a bid (if there is more than one maximizing slot).

2. A bid $B_i(p)$ is ***positive nonrevealing*** if $h \in B_i(p)$ then $v_{ih} - p_h \geq 0$ and if $h^* = \underset{j \in K}{argmax} \ v_{ij} - p_j$ then $h^* \notin B_i(p)$. That is, there are no bids on slots that maximize net value and there are no bids on slots that have negative net value. If there are slots that have positive net value, there is at least one bid.

3. A bid $B_i(p)$ is ***negative nonrevealing*** if $\exists h \in B_i(p)$ such that $v_{ih} - p_h < 0$ or $B_i(p) = \emptyset$ when $\exists h \in K$ such that $v_{ih} - p_h \geq 0$. That is, there is a bid on a slot that has negative net value, or there are no bids when there is a slot with positive net value.

These categories are mutually exclusive and exhaustive; that is, a bid falls in one and only one of the three categories. The following table presents summary statistics of these measures for the four DGS auction experiments.

Table 8: DGS

Variable	Contention	N	Mean	Std Dev
Reveal	low	40	0.9297	0.0766
	high	40	0.8604	0.0813
Positive	low	40	0.0447	0.0519
nonrev	high	40	0.1037	0.0553
Negative	low	40	0.0256	0.0586
nonrev	high	40	0.0359	0.0492

In table 8 we observe that there is a high percentage of revealing bids for both the low- and high-contention treatments, and that there is a low percentage of negative nonrevealing bids for both the low- and high-contention treatments. In Appendix F we present graphs by contention for these three measures. From these graphs we observe that in the low-contention treatment there is a tendency for more revelation in the later periods, and that in the high-contention treatment there is a tendency for less revelation in the later periods. We conjecture that this is due to the different number of iterations necessary to complete an allocation (an average 7.5 in the low-contention treatment and 21 in the high-contention treatment). In the high-contention treatment some subjects tended to place a "quick" bid in the the early iterations; they were not as careful in their bidding when the likelihood that the period would end was low.

There is a noticeable spike at period 15 in the high-contention treatment, which occurs in both of the high-contention experiments. These spikes can be explained by observing the individual behavior. After period 10 in the high-contention experiments subjects began to sit out of the early iterations of a period. In most of the periods there was considerable contention for slots, and a lot of bidding in the early iterations, so that waiting did not affect the outcome. In period 15 there is relatively less contention, and when two subjects sat out, there was an immediate (after 2 iterations) allocation omitting the subjects who sat out. In subsequent periods the subjects no longer sat out the early iterations.

7. Concluding Remarks

In this paper the allocation problem of assigning (or matching) a set of slots to a set of agents is considered. Two auction processes were tested (sealed-bid and progressive auctions) in two different environments (low and high contention). As in single-object auctions, the progressive auction generated higher revenues and higher efficiencies than the sealed-bid auction, though the efficiency difference was not significant. The net effect is that the small gain in efficiency from using a progressive auction instead of its sealed-bid counterpart is at the expense of consumers' surplus. An examination of the individual subject data in the sealed-bid auction revealed that subjects tended to overbid in the high-contention environment, especially in the early periods, but that in the low-contention environment, underbidding was more prevalent. In the progressive auction subjects tended to bid "honestly," and deviations from honest behavior had little effect on the outcome.

In general, subjects tended to behave differently in the high and low contention environments. There was more variance in behavior in the earlier periods and less variance in the later periods particularly in the high-contention environment. This behavioral difference was due to the amount of competition in the high-contention environment and the higher likelihood that a nonrevealing strategy would result in lost profits.

The main ingredient that is used by the auctions examined in this paper to overcome the incentive problem confronted in the assignment problem is the use of money transfers from agents to the planner. This of course begs the question of what the planner should do with the transfer; this is a different game than the one analyzed here. In addition, the use of money transfers reduces consumers' surplus in the high contention environments where these auctions produce relatively high efficiencies. The natural question to ask is whether these mechanism can be extended to the case where money transfer cannot be used or that all revenue generate by the auction is "rebated" back to the participants. This is the avenue of our current research efforts.

Appendix A.

LOW-CONTENTION PAYOFF LIST

	Unit Number					
Set of Values	1	2	3	4	5	6
1	800	600	400	200	400	600
2	400	600	800	600	400	200
3	400	200	400	600	800	600
4	850	350	350	850	350	350
5	750	400	400	750	400	400
6	900	300	300	300	300	300
7	300	300	900	300	300	300
8	500	500	500	500	500	500
9	550	550	550	550	550	550
10	300	300	300	300	300	900

HIGH-CONTENTION PAYOFF LIST

	Unit Number					
Set of Values	1	2	3	4	5	6
1	900	450	400	350	300	250
2	400	600	800	600	400	200
3	800	600	400	200	400	600
4	100	100	900	400	300	200
5	400	800	400	200	0	200
6	900	600	300	200	100	0
7	300	300	300	300	300	900
8	750	250	250	750	400	400
9	400	200	400	600	800	600
10	850	350	350	650	150	150

Realized Payoff Lists

The next pages contain the realized payoff lists the used in the experiments. Each list (one for high and low contention) contains a matrix of numbers for each period. These matrices contain the potential slot values for each subject. Each row indicates a subject and each column indicates a slot. There are 8 rows (8 subjects) for the high contention list, and 6 rows (6 subjects for the low contention list. Each matrix contains 6 columns (6 slots) for both the low and high contention lists.

Low-Contention Payoff Lists

period 1
400 600 800 600 400 200
800 600 400 200 400 600
400 200 400 600 800 600
400 600 800 600 400 200
550 550 550 550 550 550
400 600 800 600 400 200

period 2
900 300 300 300 300 300
900 300 300 300 300 300
400 600 800 600 400 200
800 600 400 200 400 600
300 300 900 300 300 300
800 600 400 200 400 600

period 3
500 500 500 500 500 500
800 600 400 200 400 600
300 300 300 300 300 900
300 300 300 300 300 900
300 300 300 300 300 900
750 400 400 750 400 400

period 4
750 400 400 750 400 400
400 200 400 600 800 600
900 300 300 300 300 300
500 500 500 500 500 500
550 550 550 550 550 550
750 400 400 750 400 400

period 5
400 200 400 600 800 600
900 300 300 300 300 300
750 400 400 750 400 400
750 400 400 750 400 400
800 600 400 200 400 600
850 350 350 850 350 350

period 6
500 500 500 500 500 500
500 500 500 500 500 500
750 400 400 750 400 400
850 350 350 850 350 350
850 350 350 850 350 350
750 400 400 750 400 400

period 7
900 300 300 300 300 300
550 550 550 550 550 550
400 600 800 600 400 200
750 400 400 750 400 400
900 300 300 300 300 300
900 300 300 300 300 300

period 8
500 500 500 500 500 500
800 600 400 200 400 600
850 350 350 850 350 350
800 600 400 200 400 600
750 400 400 750 400 400
800 600 400 200 400 600

period 9
300 300 900 300 300 300
550 550 550 550 550 550
400 200 400 600 800 600
850 350 350 850 350 350
900 300 300 300 300 300
500 500 500 500 500 500

period 10
850 350 350 850 350 350
800 600 400 200 400 600
400 600 800 600 400 200
550 550 550 550 550 550
800 600 400 200 400 600
900 300 300 300 300 300

period 11
850 350 350 850 350 350
750 400 400 750 400 400
850 350 350 850 350 350
750 400 400 750 400 400
500 500 500 500 500 500
550 550 550 550 550 550

period 12
550 550 550 550 550 550
400 600 800 600 400 200
750 400 400 750 400 400
500 500 500 500 500 500
500 500 500 500 500 500
800 600 400 200 400 600

period 13
500 500 500 500 500 500
550 550 550 550 550 550
850 350 350 850 350 350
900 300 300 300 300 300
400 200 400 600 800 600
900 300 300 300 300 300

period 14
900 300 300 300 300 300
400 600 800 600 400 200
800 600 400 200 400 600
300 300 900 300 300 300
800 600 400 200 400 600
400 200 400 600 800 600

period 15
550 550 550 550 550 550
400 600 800 600 400 200
750 400 400 750 400 400
800 600 400 200 400 600
850 350 350 850 350 350
900 300 300 300 300 300

Low-Contention cont.

period 16
800 600 400 200 400 600
500 500 500 500 500 500
850 350 350 850 350 350
800 600 400 200 400 600
750 400 400 750 400 400
800 600 400 200 400 600

period 17
850 350 350 850 350 350
300 300 900 300 300 300
550 550 550 550 550 550
400 200 400 600 800 600
900 300 300 300 300 300
500 500 500 500 500 500

period 18
900 300 300 300 300 300
850 350 350 850 350 350
800 600 400 200 400 600
400 600 800 600 400 200
550 550 550 550 550 550
800 600 400 200 400 600

period 19
750 400 400 750 400 400
850 350 350 850 350 350
750 400 400 750 400 400
850 350 350 850 350 350
750 400 400 750 400 400
550 550 550 550 550 550

period 20
400 600 800 600 400 200
550 550 550 550 550 550
750 400 400 750 400 400
500 500 500 500 500 500
500 500 500 500 500 500
800 600 400 200 400 600

High-Contention Payoff
Lists

period 1
```
400 600 800 600 400 200
800 600 400 200 400 600
400 200 400 600 800 600
400 600 800 600 400 200
400 800 400 200   0 200
400 600 800 600 400 200
100 100 900 400 300 200
900 600 300 200 100 0
```

period 6
```
900 600 300 200 100 0
900 600 300 200 100 0
750 250 250 750 400 400
850 350 350 650 150 150
850 350 350 650 150 150
750 250 250 750 400 400
400 600 800 600 400 200
800 600 400 200 400 600
```

period 11
```
850 350 350 650 150 150
750 250 250 750 400 400
850 350 350 650 150 150
750 250 250 750 400 400
900 600 300 200 100 0
400 800 400 200 0 200
400 800 400 200 0 200
850 350 350 650 150 150
```

period 2
```
900 450 400 350 300 250
900 450 400 350 300 250
400 600 800 600 400 200
800 600 400 200 400 600
100 100 900 400 300 200
800 600 400 200 400 600
400 800 400 200 0 200
400 800 400 200 0 200
```

period 7
```
900 450 400 350 300 250
400 800 400 200 0 200
400 600 800 600 400 200
750 250 250 750 400 400
900 450 400 350 300 250
900 450 400 350 300 250
800 600 400 200 400 600
850 350 350 650 150 150
```

period 12
```
400 800 400 200 0 200
400 600 800 600 400 200
750 250 250 750 400 400
900 600 300 200 100 0
900 600 300 200 100 0
800 600 400 200 400 600
850 350 350 650 150 150
400 800 400 200 0 200
```

period 3
```
900 600 300 200 100 0
800 600 400 200 400 600
300 300 300 300 300 900
300 300 300 300 300 900
300 300 300 300 300 900
750 250 250 750 400 400
900 600 300 200 100 0
100 100 900 400 300 200
```

period 8
```
900 600 300 200 100 0
800 600 400 200 400 600
850 350 350 650 150 150
800 600 400 200 400 600
750 250 250 750 400 400
800 600 400 200 400 600
900 450 400 350 300 250
400 600 800 600 400 200
```

period 13
```
900 600 300 200 100 0
400 800 400 200 0 200
850 350 350 650 150 150
900 450 400 350 300 250
400 200 400 600 800 600
900 450 400 350 300 250
400 200 400 600 800 600
850 350 350 650 150 150
```

period 4
```
750 250 250 750 400 400
400 200 400 600 800 600
900 450 400 350 300 250
900 600 300 200 100 0
400 800 400 200 0 200
750 250 250 750 400 400
900 450 400 350 300 250
750 250 250 750 400 400
```

period 9
```
100 100 900 400 300 200
400 800 400 200 0 200
400 200 400 600 800 600
850 350 350 650 150 150
900 450 400 350 300 250
900 600 300 200 100 0
750 250 250 750 400 400
900 600 300 200 100 0
```

period 14
```
900 450 400 350 300 250
400 600 800 600 400 200
800 600 400 200 400 600
100 100 900 400 300 200
800 600 400 200 400 600
400 200 400 600 800 600
400 600 800 600 400 200
900 600 300 200 100 0
```

period 5
```
400 200 400 600 800 600
900 450 400 350 300 250
750 250 250 750 400 400
750 250 250 750 400 400
800 600 400 200 400 600
850 350 350 650 150 150
850 350 350 650 150 150
400 200 400 600 800 600
```

period 10
```
850 350 350 650 150 150
800 600 400 200 400 600
400 600 800 600 400 200
400 800 400 200 0 200
800 600 400 200 400 600
900 450 400 350 300 250
400 200 400 600 800 600
100 100 900 400 300 200
```

period 15
```
400 800 400 200 0 200
400 600 800 600 400 200
750 250 250 750 400 400
800 600 400 200 400 600
850 350 350 650 150 150
900 450 400 350 300 250
750 250 250 750 400 400
750 250 250 750 400 400
```

High-Contention cont.

period 16
400 800 400 200   0 200
400 600 800 600 400 200
100 100 900 400 300 200
900 600 300 200 100 0
400 600 800 600 400 200
800 600 400 200 400 600
400 200 400 600 800 600
400 600 800 600 400 200

period 17
100 100 900 400 300 200
800 600 400 200 400 600
400 800 400 200 0 200
400 800 400 200 0 200
900 450 400 350 300 250
900 450 400 350 300 250
400 600 800 600 400 200
800 600 400 200 400 600

period 18
300 300 300 300 300 900
750 250 250 750 400 400
900 600 300 200 100 0
100 100 900 400 300 200
900 600 300 200 100 0
800 600 400 200 400 600
300 300 300 300 300 900
300 300 300 300 300 900

period 19
400 800 400 200 0 200
750 250 250 750 400 400
900 450 400 350 300 250
750 250 250 750 400 400
750 250 250 750 400 400
400 200 400 600 800 600
900 450 400 350 300 250
900 600 300 200 100 0

period 20
800 600 400 200 400 600
850 350 350 650 150 150
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400 200 400 600 800 600
400 200 400 600 800 600
900 450 400 350 300 250
750 250 250 750 400 400
750 250 250 750 400 400

Contention
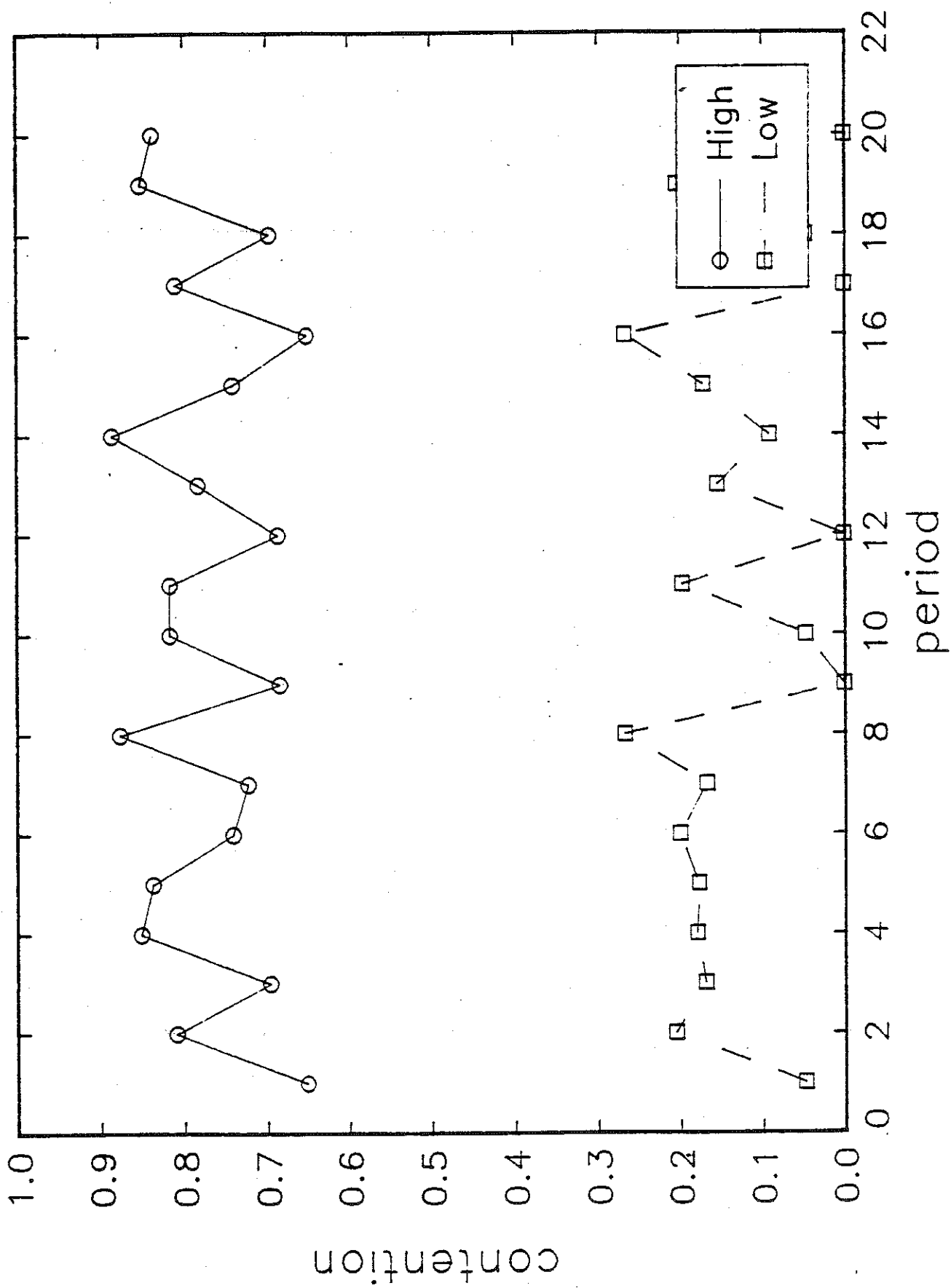
Core Price Range
(low contention)
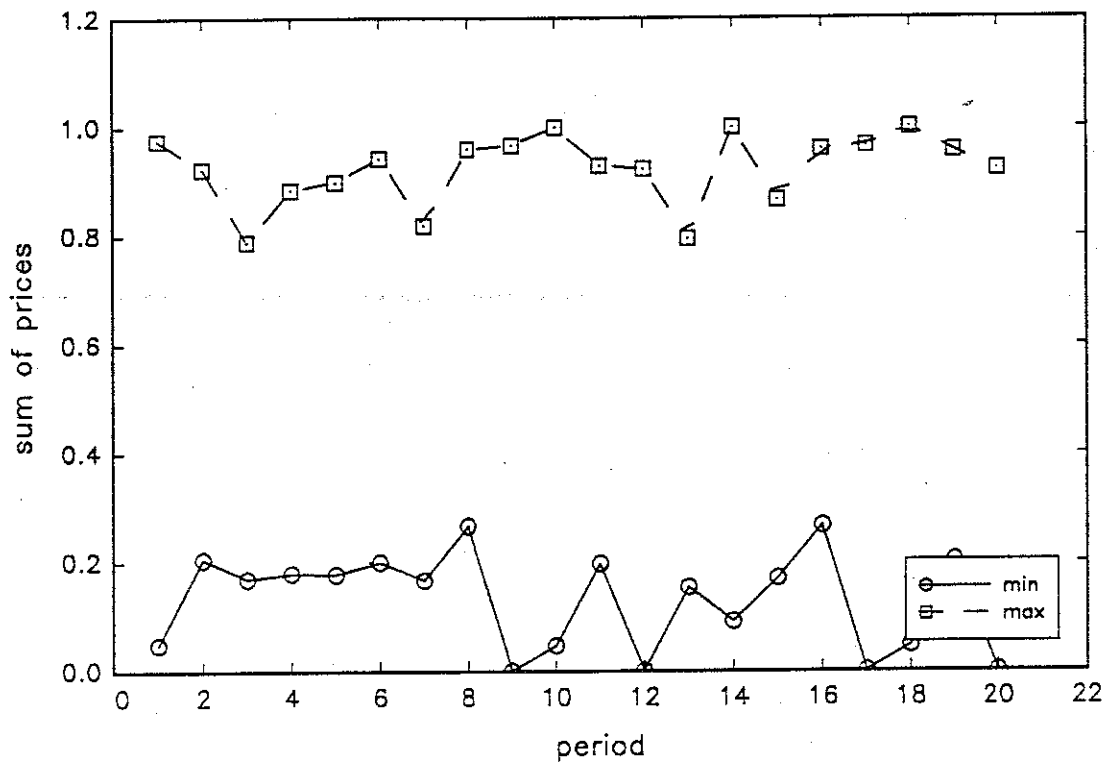
Core Price Range
(high contention)
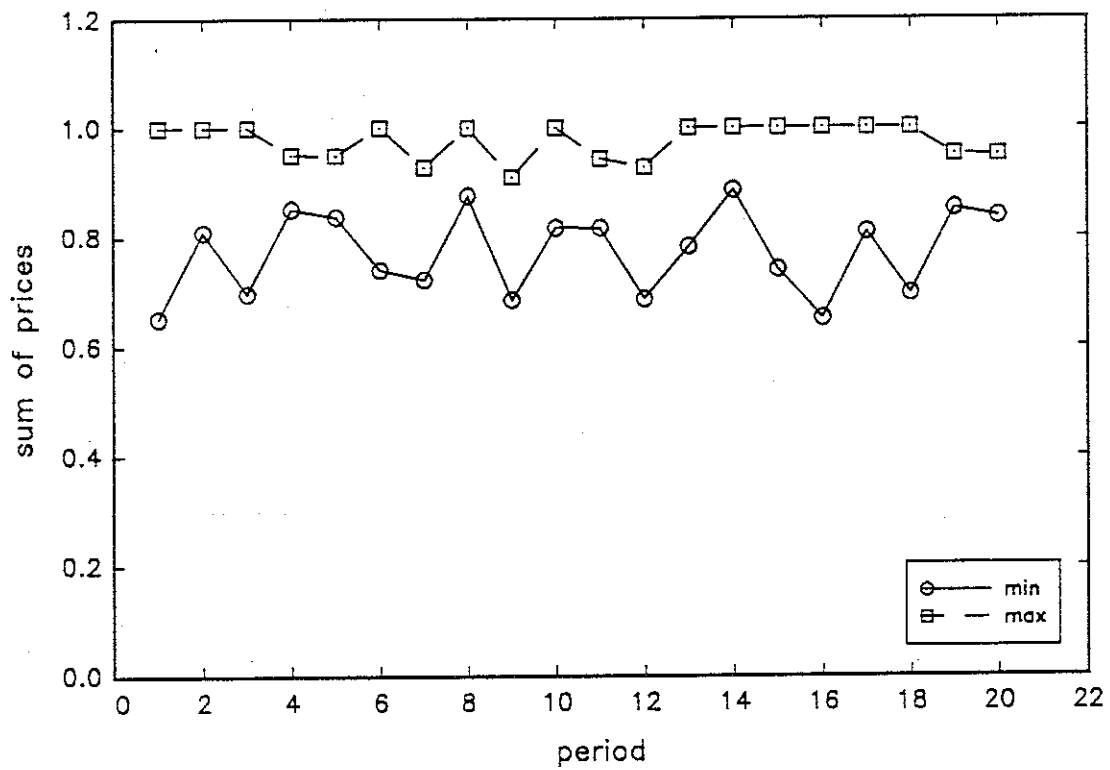
Appendix B.

INSTRUCTIONS FOR EXPERIMENT

You are about to participate in an experiment in which you will make decisions in a market. Your profits from the experiment will be in terms of francs. You can convert your franc earnings into U.S. dollars at a conversion rate of 600 francs to 1 U.S. dollar. Any profits you make in the experiment are yours to keep. You will be paid at the end of the experiment.

The experiment will be divided up into a series of "periods." At the beginning of each period you will be given redemption values on your terminal screen. The redemption values are the franc values to you of six different items. Your redemption values are known only to you, and you should not reveal them to any other participants. Your profit each period is equal to the redemption value of the unit you receive minus the price for the unit. For example, suppose the redemption value for the unit you bought is 700 francs and its price is 400. Then your profit for that period is 300 francs.

In our market you will be one of _8_ participants to be assigned units. There will be six units, which will be numbered from one to six, allocated simultaneously each period. These units are not the same; that is, they do not necessarily have the same redemption values to a participant. They will be allocated through a procedure that will be described later.

In Table 1 you will find the ten possible sets of redemption values. The table lists the number of the unit and the corresponding value. The sheet has eleven rows. The first row, labeled unit, indicates the number of the unit being allocated (in the experiment there will be six units assigned, which will be referred to as units 1,2,...,6). The second through the eleventh row give the possible participants' redemption values.

For the first set of redemption values, unit 1 is worth 900
francs to the participant, whereas unit 4 is worth 350.

Each participant in the experiment is given one of the ten
possible sets of redemption values at the beginning of each
period. The sets of redemption values other participants happen
to receive do not affect the redemption values you receive.

TABLE 1.

Set of Values	Unit Number					
	1	2	3	4	5	6
1	900	450	400	350	300	250
2	400	600	800	600	400	200
3	800	600	400	200	400	600
4	100	100	900	400	300	200
5	400	800	400	200	0	200
6	900	600	300	200	100	0
7	300	300	300	300	300	900
8	750	250	250	750	400	400
9	400	200	400	600	800	600
10	850	350	350	650	150	150

THE ALLOCATION PROCESS

Each period you will see a display on your terminal like the one shown below. The top row gives the number of the unit to be allocated. The third row, labeled value, gives your redemption values for that period. The second row indicates your bids on the corresponding units. You may enter a bid on the unit by selecting the correct box and typing in your bid. You must enter a bid that is greater than or equal to zero for each unit. In the following figure, the buyer has bid 0 for unit 1, 500 for unit 2, and 600 for unit 3.

Unit	1	2	3	4	5	6
Bid	0	500	600	199	205	22
Value	800	600	400	200	400	600

Once you have entered all of your bids and pressed the **END** key, you will be asked to confirm them. After checking them and making sure they are the bids you want, press the **Y** key to send the bids. A market program determines the recipient of each unit. By this procedure you can receive at most one unit, and all six units will be assigned.

The Allocation:

1) Once the bids are received from all the participants, the six units are allocated by the following method. It finds the combination of "assignments" for which the total of the winning bids for all six units is the greatest. That is, it gives units to buyers (recall, however, that one buyer can get at most one unit) so that the total of the bids of the buyers on the units they actually receive is the highest possible. An example with three participants and three units is given below (example 1).

Example 1:   **Three Buyers and Three Units**

Buyer	Units and Bids		
	1	2	3
1	800	700	200
2	700	500	400
3	400	400	400

Here, buyer 1 has bid 800 for unit 1, 700 for unit 2, and 200 for unit 3. Buyer 2 has bid 700 for unit 1, 500 for unit 2, and 400 for unit 3. Buyer 3 has bid 400 for each unit.

The allocation is:

Buyer 1 receives unit 2.

Buyer 2 receives unit 1.

Buyer 3 receives unit 3.

The total of the winning bids from this assignment is 1800, and the total of winning bids from any other assignment is less than 1800. Notice from the example, that the buyer who bids the most on a unit does not necessarily receive that unit. If two or more assignments yield the same maximum total, the assignment is chosen randomly.

Prices

In addition to allocating the slots, the market program computes a price for each slot. They are calculated as follows:

2) After the allocation is made, the program calculates the total of the bids of the buyers on those units that they are allocated.

3) The following total is calculated for unit 1. The program supposes that there was an extra unit 1 available and therefore a total of seven units to be sold. It then finds the combination of assignments for which the total amount bid for units received is the greatest possible (as in step 1). The total of the bids of the buyers on the units they would receive is calculated (as in step 2). Notice this is always greater than or equal to the amount in step 2 because there are more combinations available, and all of the combinations previously available are still available.

4) The difference between the two-bid total is calculated. This difference is the price charged for unit one.

5) Steps 3 and 4 are repeated for units 2-6. The example below works out the process for a case when there are two units to be sold and two buyers.

Example 2:

	Units and Bids	
Buyer	1	2
1	1000	600
2	800	100

The combination of assignments where the total amount bid on the units is the greatest possible is the following: Buyer 1 receives unit 2 and buyer 2 receives unit 1. The total amount bid is 600 + 800 = 1400. If there were another unit 1 available, however, each buyer would receive a unit 1, and the total amount bid would be 100 + 800 = 1800. Therefore, the price charged for one unit is 1800 - 1400 = 400. If there was another unit 2 available, the allocation would be unchanged. Therefore, the price of unit 2 is zero.

After this process is completed, the terminal will indicate the unit you received. The redemption value of the unit you receive is your profit for the period. If you do not receive a unit, your profit is zero for the period.

You can press the H key at any time to see the history screen. The screen shows your redemption values for each unit during the past periods in the rows labelled values, and the bids you submitted on each unit during the past periods in the rows labelled bid. The units which you have already received and the payoffs you have earned are in the rows labelled payoffs, which are highlighted.

This concludes the instruction for the experiment. If you have any questions, please raise your hand and a monitor will answer your questions. We will have a practice period to help familiarize you with the experiment.

Appendix C: Graphical displays of efficiencies and consumer surplus.

Efficiencies (low contention)

Consumer Surplus (low contention)
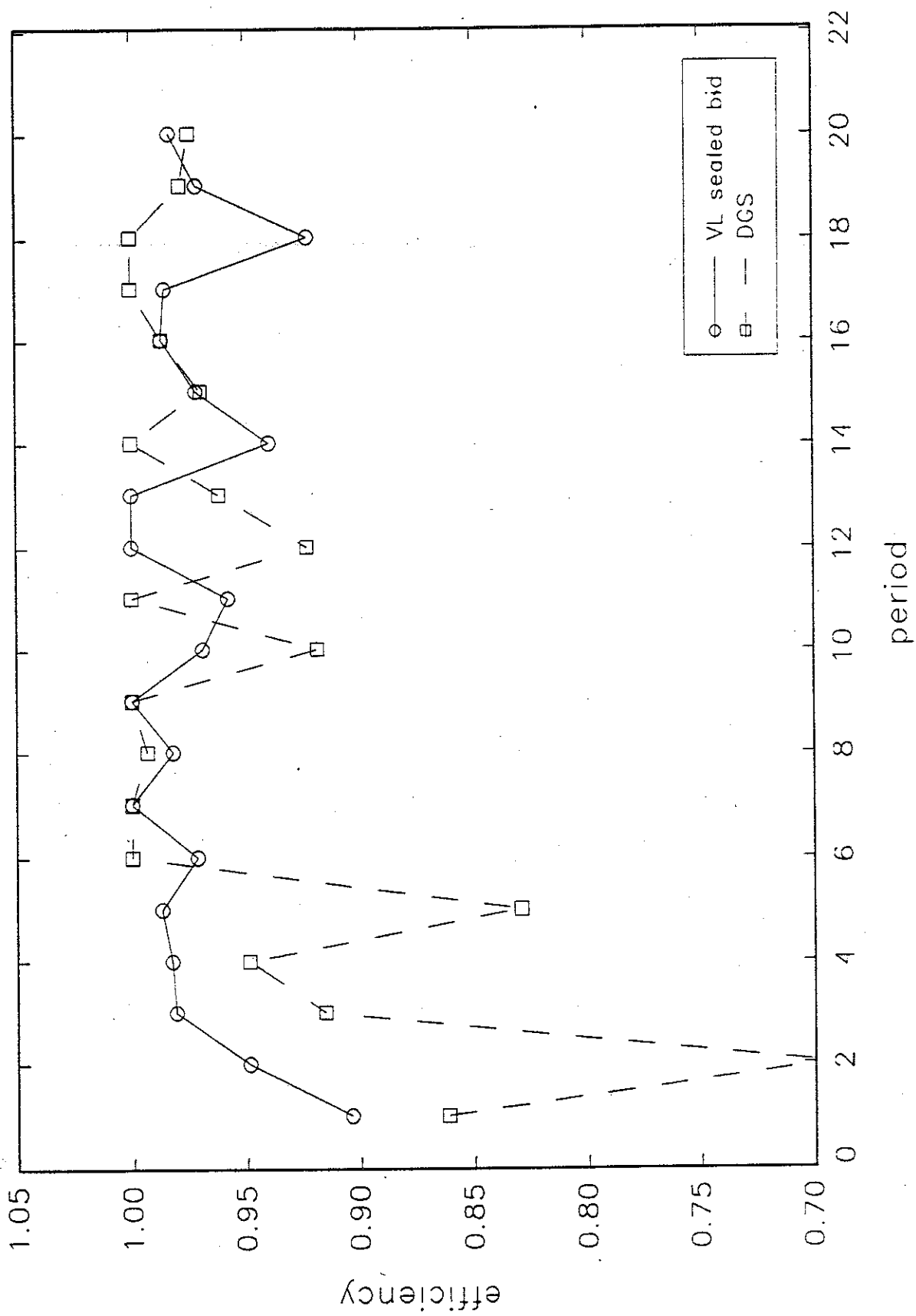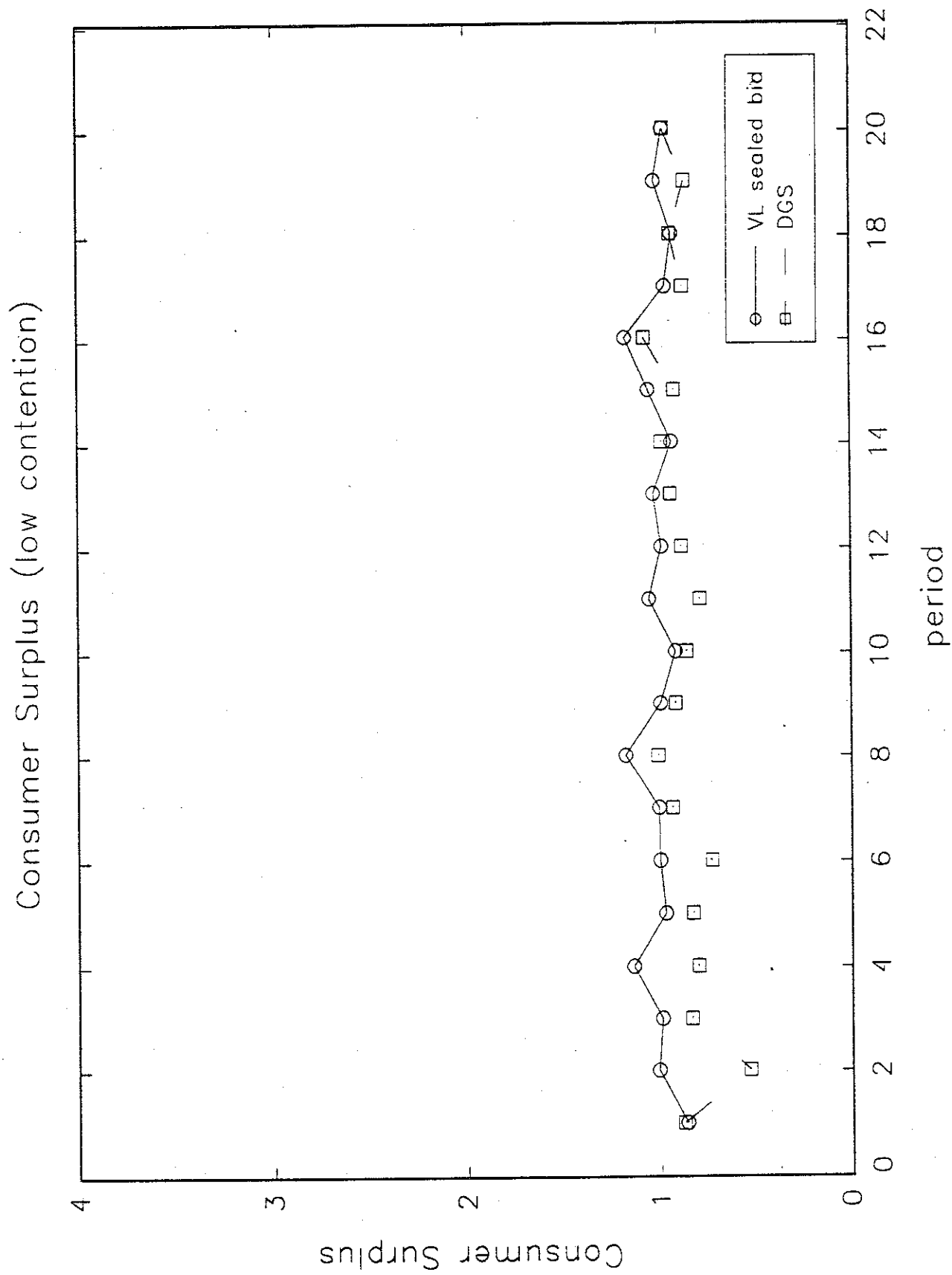
Efficiencies (high contention)
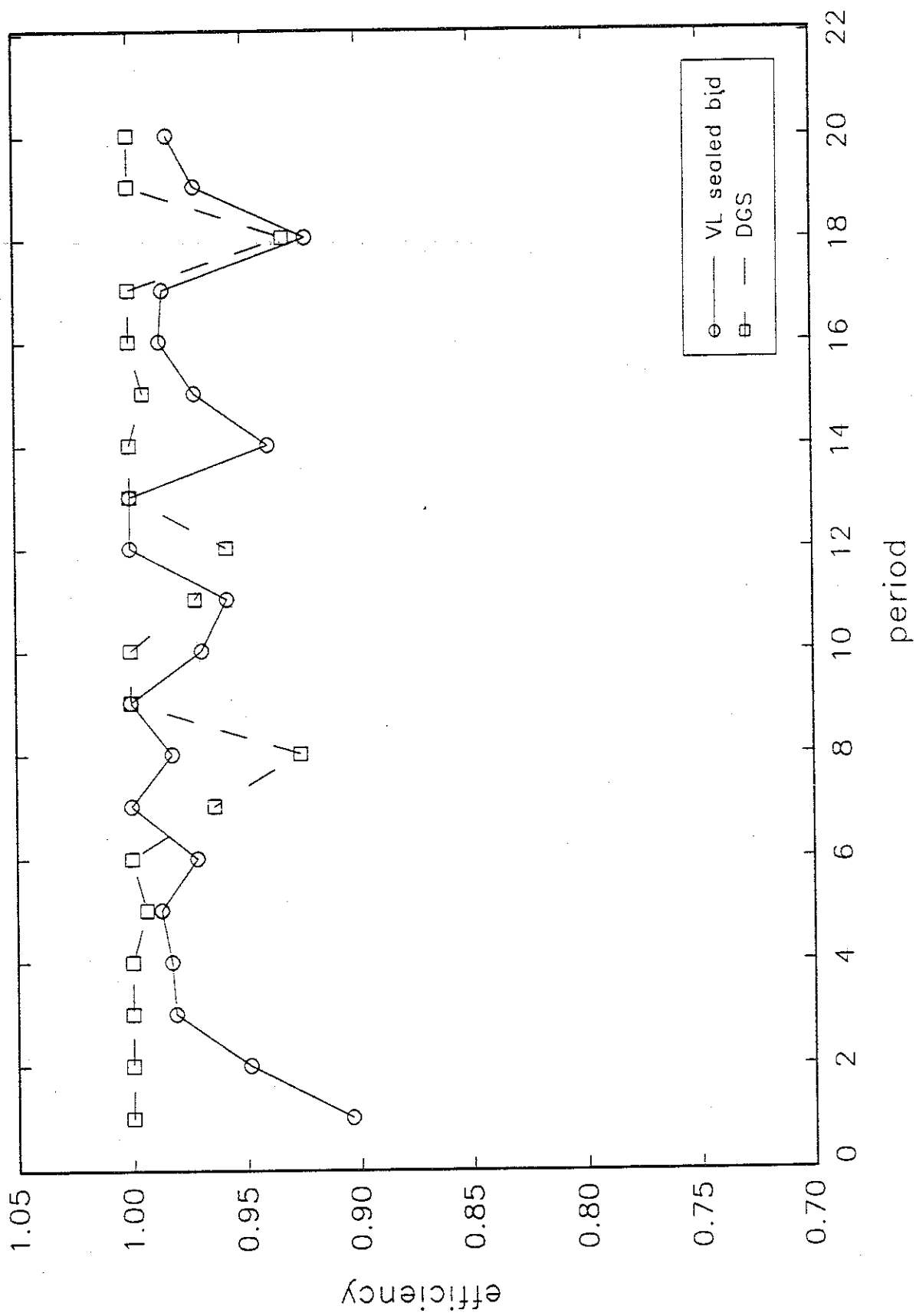
Consumer Surplus (high contention)
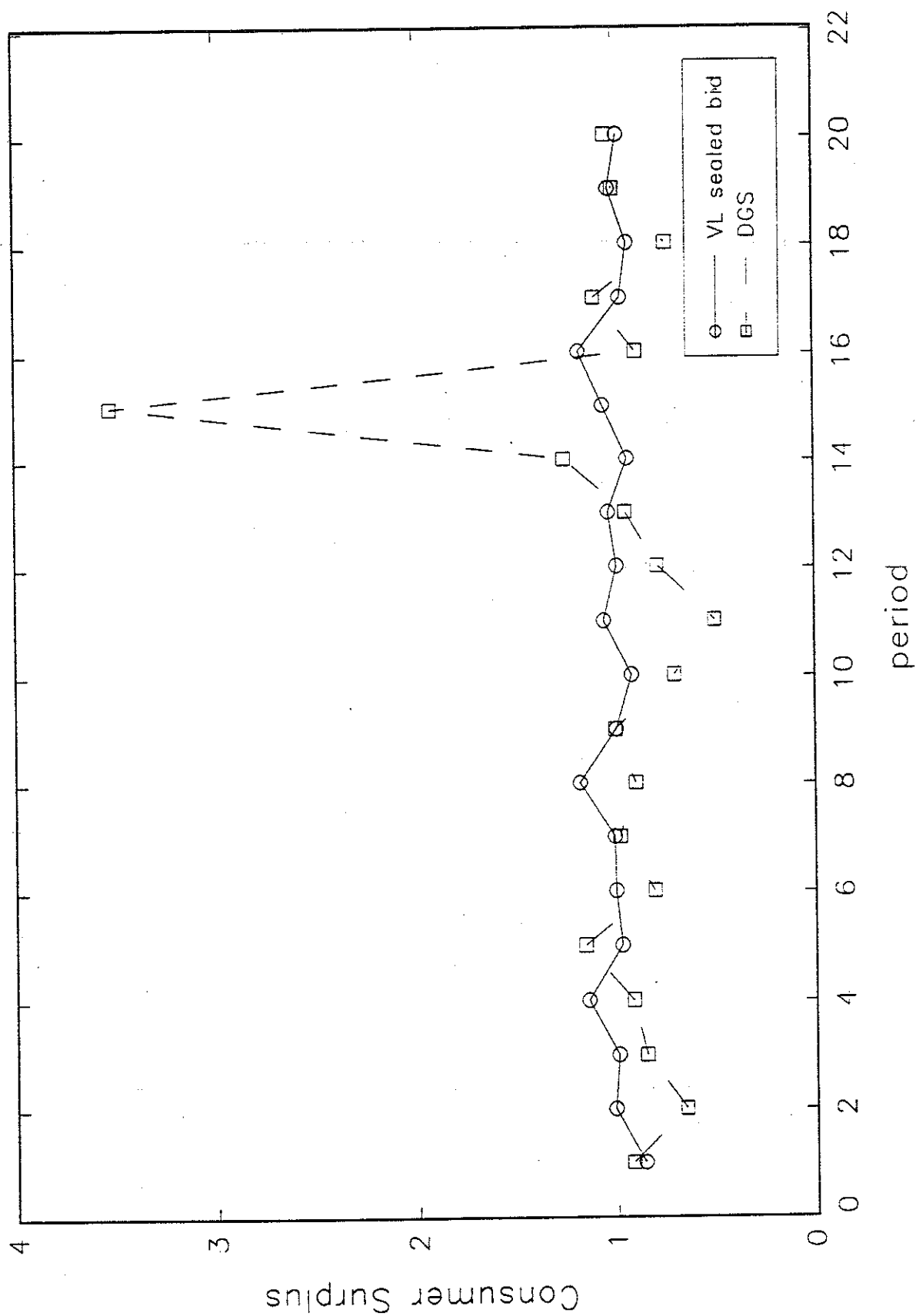
Appendix D: Estimation of experimental effects.

------------------- SURE RESULTS: All periods --------------------------

Log of Likelihood function =   144.5338

                    Equation   1
             Dependent variable: EFF

Valid cases: 180   Missing cases: 0
Total SS: 0.8468   Degrees of freedom: 165
R-squared: 0.4398   Rbar-squared: 0.4429
Residual SS: 0.4744   Std error of est: 0.0513

Var	Coef	Std. Error	t-Stat	P-Value
Constant	0.9850	0.0147	67.1169	0.0000
P1	-0.0803	0.0278	-2.8875	0.0044
Contention	0.0290	0.0581	0.4995	0.6180
DGS	-0.0130	0.0176	-0.7370	0.4621
DGS*P1	0.0829	0.0374	2.2150	0.0280
C01	0.0236	0.0665	0.3552	0.7229
P1*C01	-0.0054	0.0378	-0.1418	0.8874
Cont*C01	-0.0720	0.0993	-0.7256	0.4690
DGS*C01	0.0245	0.0230	1.0686	0.2867
VL1(high)	-0.0119	0.0162	-0.7316	0.4654
VL2(low)	-0.0018	0.0162	-0.1099	0.9126
VL3(high)	-0.0046	0.0162	-0.2840	0.7768
DGS1(low)	0.0098	0.0178	0.5514	0.5821
DGS3(high)	0.0011	0.0162	0.0651	0.9482
DGS1(per $\leq$ 5)	-0.2685	0.0297	-9.0463	0.0000

```
                         Equation    2
               Dependent variable:          CS
        ------------------------------------------

   Valid cases:              180   Missing cases:                0
   Total SS:              60.4543  Degrees of freedom:          165
   R-squared:             0.1912   Rbar-squared:             0.1957
   Residual SS:          48.8926   Std error of est:         0.5212

Var              Coef        Std. Error      t-Stat      P-Value
-----------------------------------------------------------------
Constant         0.9612        0.1490         6.4516      0.0000
P1              -0.0514        0.2823        -0.1822      0.8556
Contention       0.3319        0.5902         0.5623      0.5746
DGS             -0.0859        0.1784        -0.4817      0.6306
DGS*P1          -0.1244        0.3801        -0.3274      0.7438
C01             -1.2655        0.6753        -1.8741      0.0625
P1*C01           0.1285        0.3837         0.3348      0.7382
Cont*C01         2.0964        1.0081         2.0796      0.0390
DGS*C01         -0.5123        0.2331        -2.1981      0.0292
VL1(high)       -0.2515        0.1648        -1.5257      0.1288
VL2(low)         0.0352        0.1648         0.2136      0.8311
VL3(high)        0.0085        0.1648         0.0517      0.9589
DGS1(low)        0.0265        0.1812         0.1462      0.8840
DGS3(high)       0.1356        0.1648         0.8229      0.4116
DGS1(per ≤ 5)   -0.1669        0.3013        -0.5539      0.5803
```

Log of Likelihood function =   110.8225

Equation	1
Dependent variable:	EFF

Valid cases:	90	Missing cases:	0
Total SS:	0.1219	Degrees of freedom:	76
R-squared:	0.0953	Rbar-squared:	0.1054
Residual SS:	0.1103	Std error of est:	0.0350

Var	Coef	Std. Error	t-Stat	P-Value
Constant	0.9683	0.0319	30.3164	0.0000
P1	0.0920	0.4523	0.2034	0.8393
Contention	0.0222	0.0539	0.4127	0.6808
DGS	0.0275	0.0417	0.6594	0.5113
DGSP1	-0.3813	0.5775	-0.6603	0.5108
C01	0.0046	0.0735	0.0623	0.9505
P1*C01	-0.3024	0.5826	-0.5190	0.6050
Con*C01	0.0079	0.0921	0.0857	0.9319
DGS*C01	0.0108	0.0221	0.4886	0.6263
VL1	-0.0224	0.0157	-1.4296	0.1563
VL2	-0.0172	0.0157	-1.0958	0.2761
VL3	0.0014	0.0157	0.0874	0.9306
DGS1	0.0008	0.0157	0.0520	0.9586
DGS3	-0.0187	0.0157	-1.1934	0.2359

```
                        Equation    2
                 Dependent variable:            CS
      -------------------------------------------
```

Valid cases:		90	Missing cases:	0
Total SS:		27.3210	Degrees of freedom:	76
R-squared:		0.2074	Rbar-squared:	0.2162
Residual SS:		21.6543	Std error of est:	0.4905

Var	Coef	Std. Error	t-Stat	P-Value
Constant	0.8398	0.4476	1.8764	0.0638
P1	1.5531	6.3376	0.2451	0.8070
Contention	0.4139	0.7546	0.5485	0.5847
DGS	0.3290	0.5840	0.5634	0.5746
DGS*P1	-6.1225	8.0925	-0.7566	0.4513
Cont*C01	-0.9757	1.0305	-0.9468	0.3462
P1*C01	-0.5533	8.1643	-0.0678	0.9461
Cont*C01	1.8228	1.2902	1.4128	0.1612
DGS*C01	-0.4810	0.3102	-1.5504	0.1246
VL1	-0.5455	0.2194	-2.4869	0.0147
VL2	0.0417	0.2194	0.1901	0.8497
VL3	0.0233	0.2194	0.1063	0.9156
DGS1	0.0284	0.2194	0.1294	0.8973
DGS3	0.1420	0.2194	0.6472	0.5192

Appendix E: Graphical displays of Bid - Value.
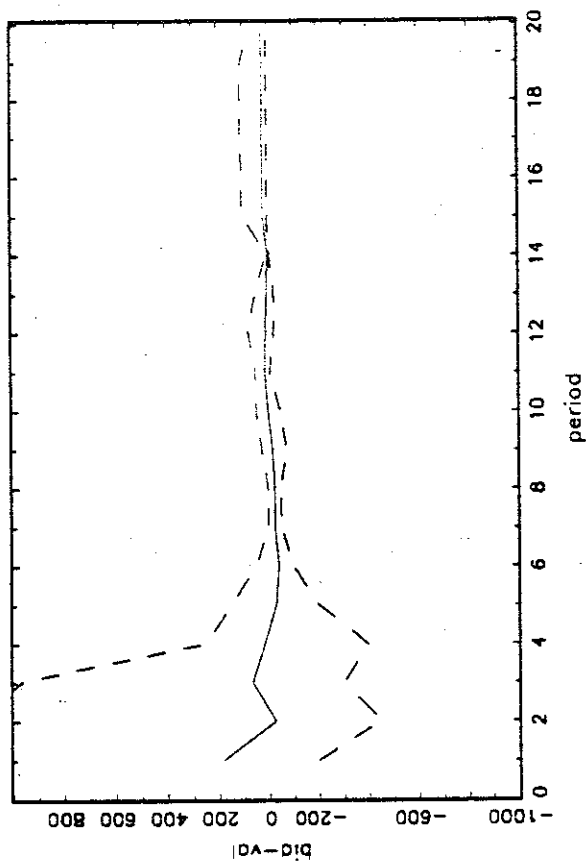
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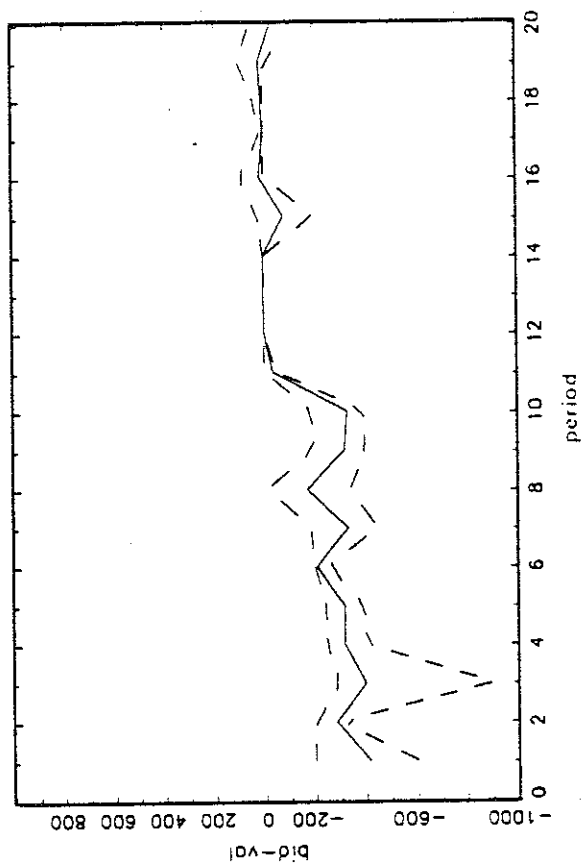
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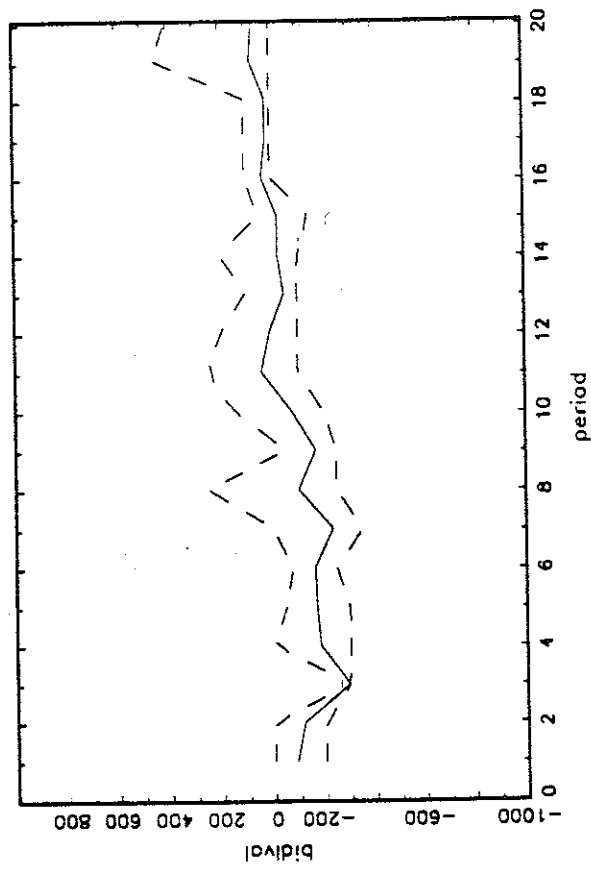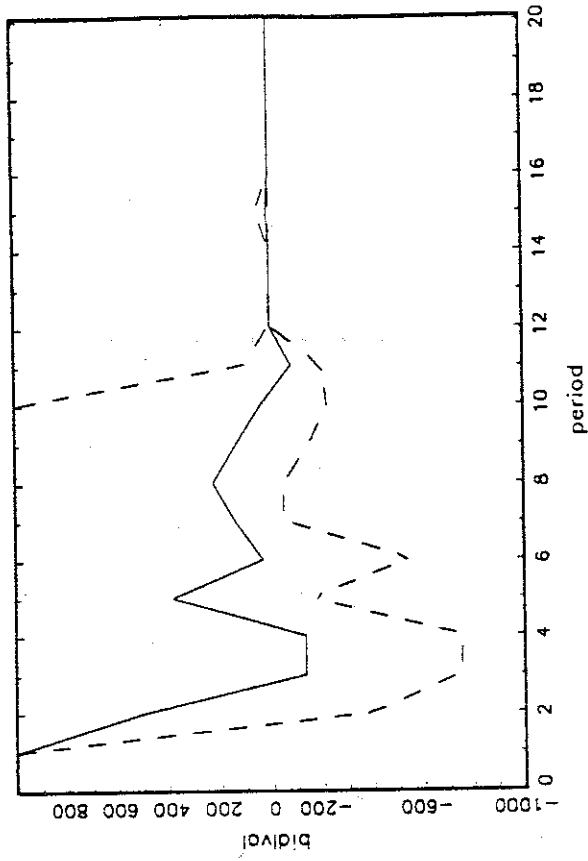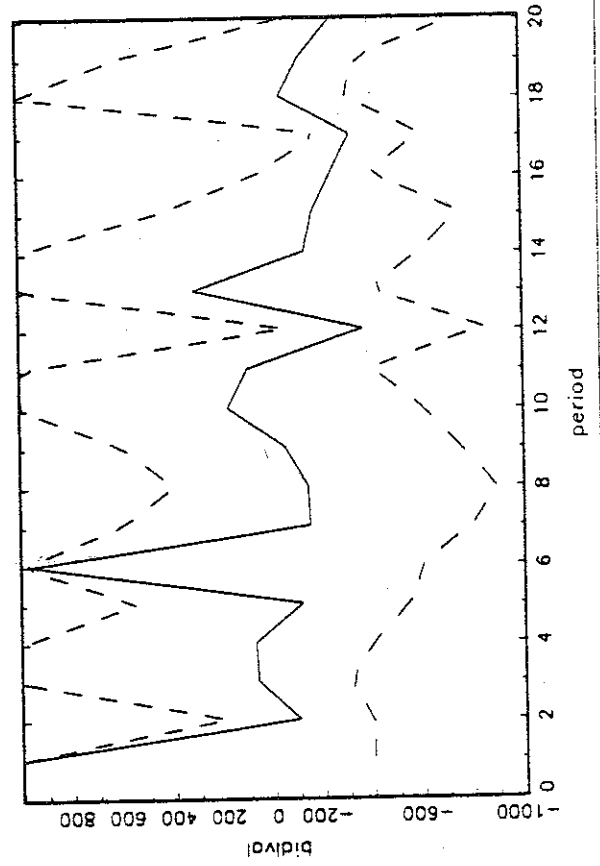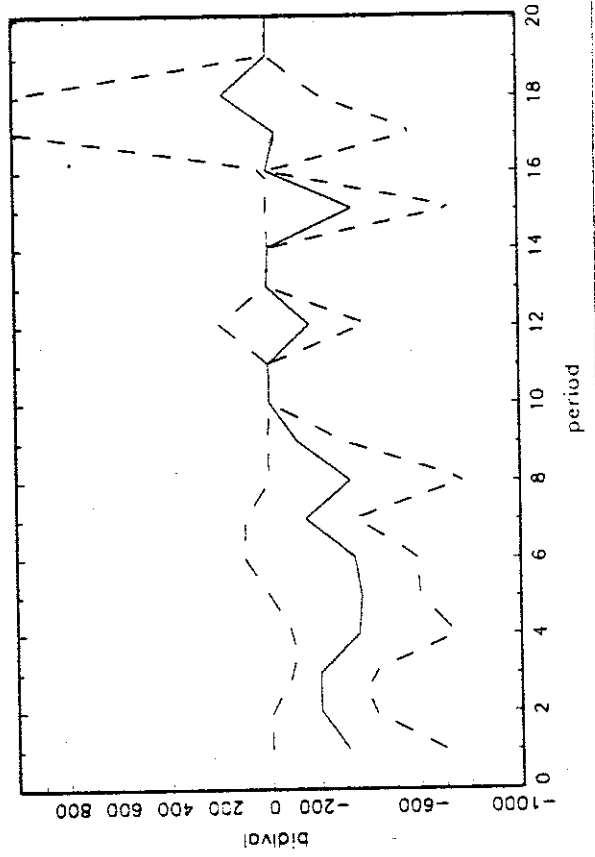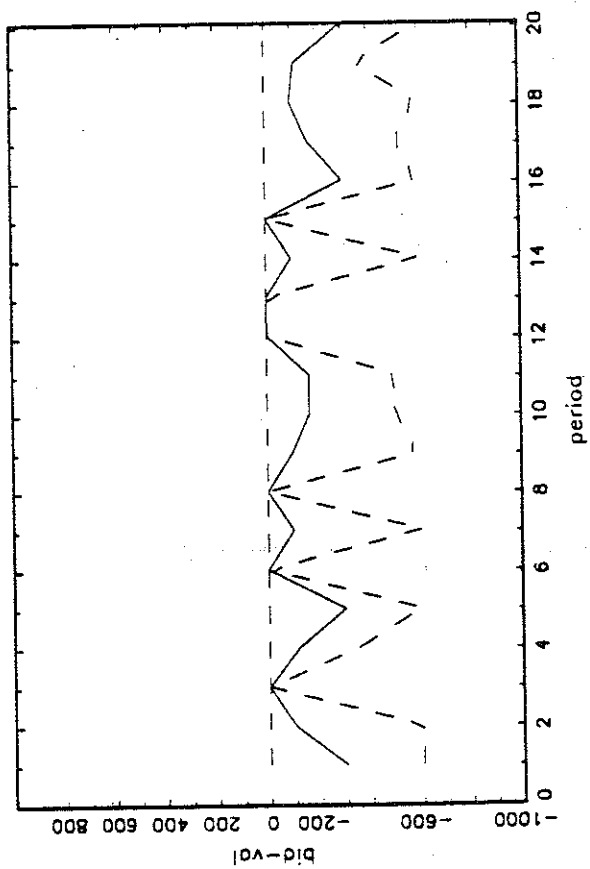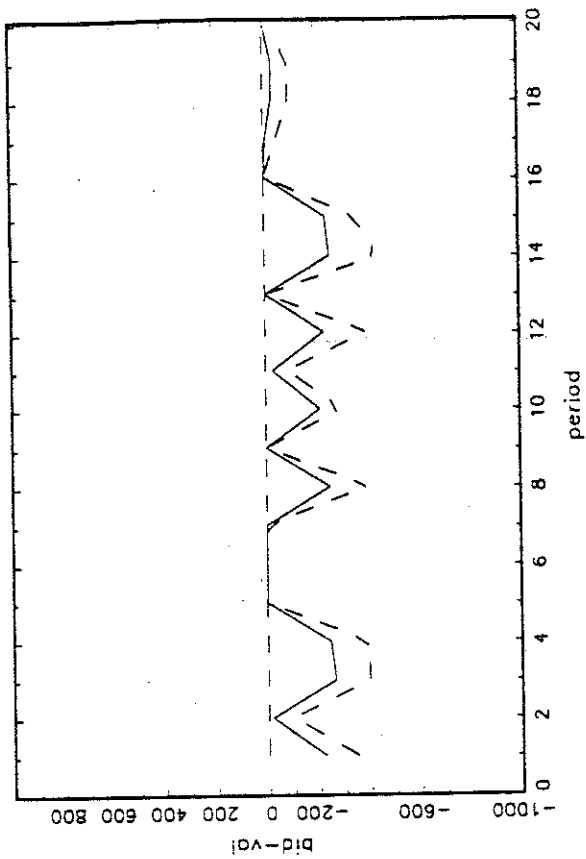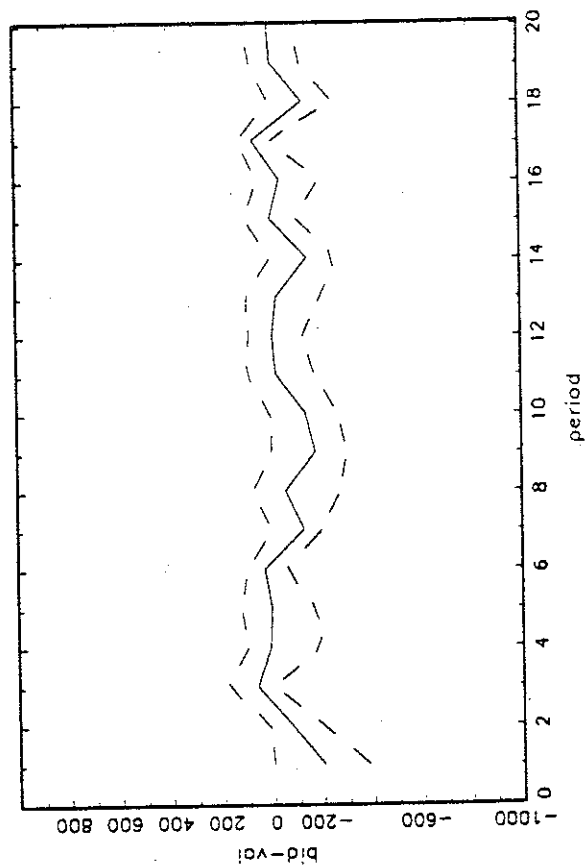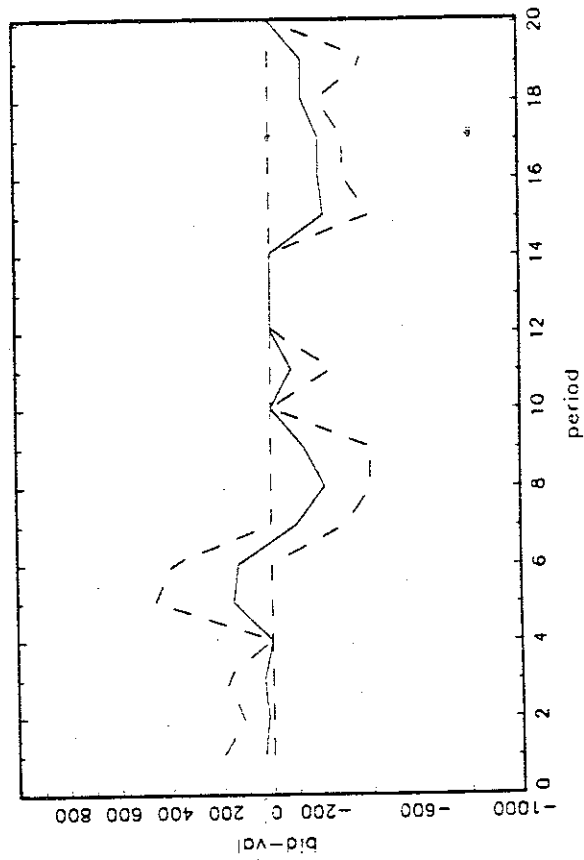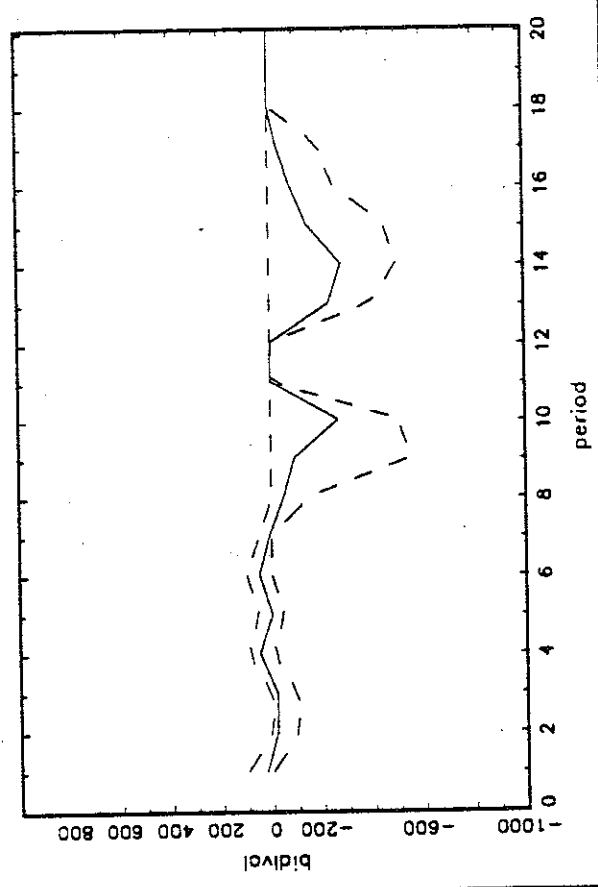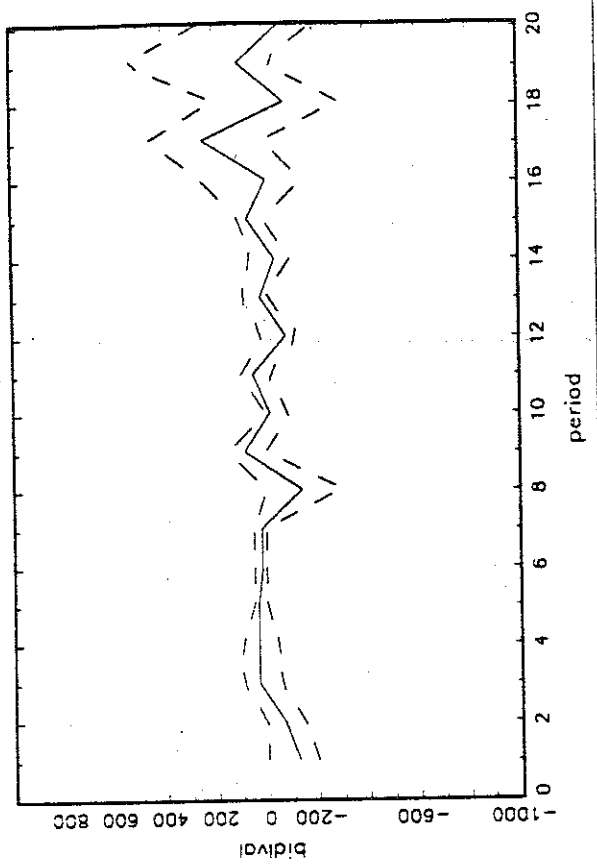
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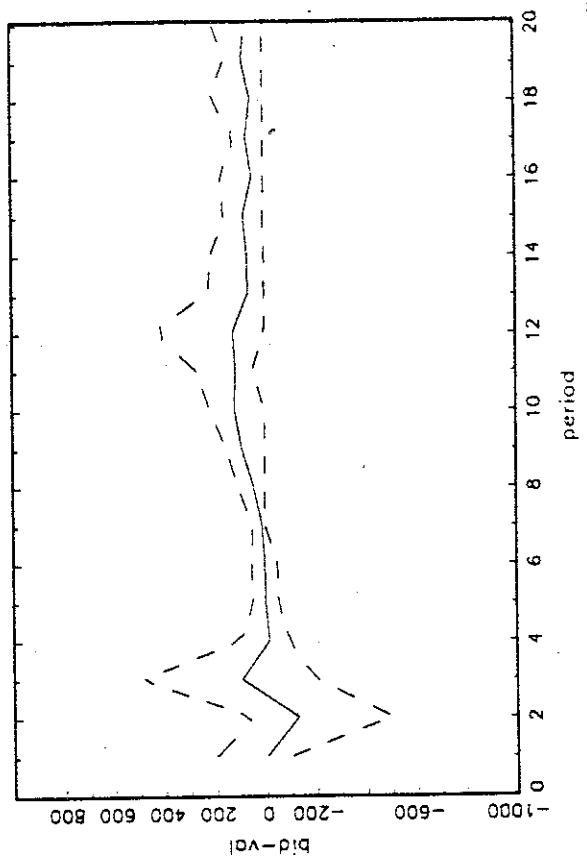
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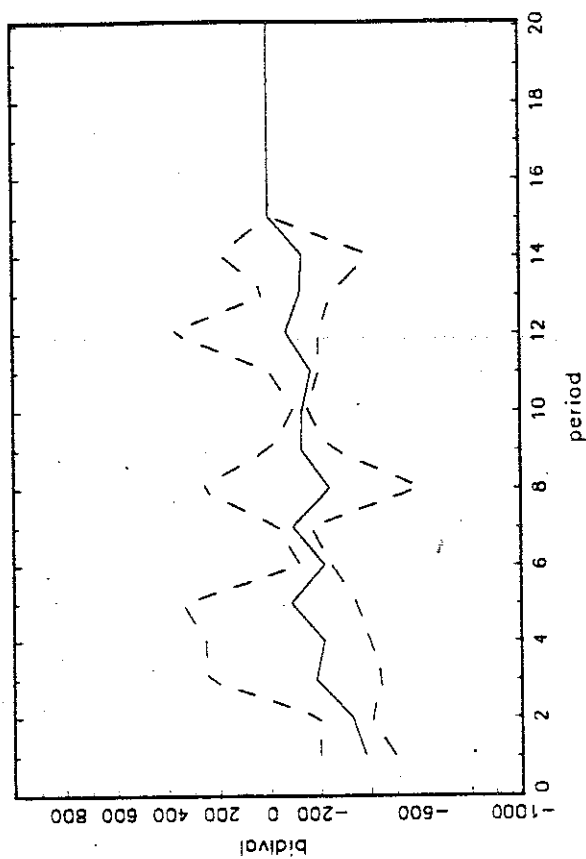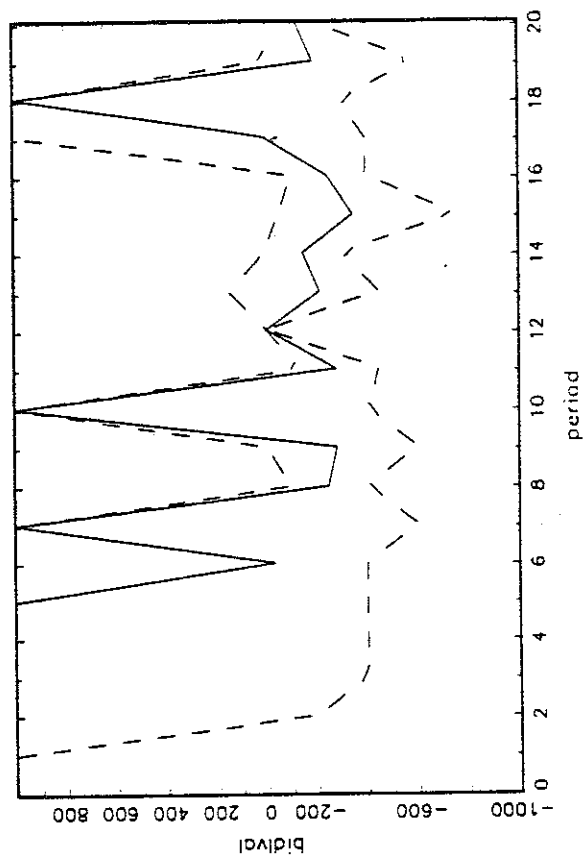
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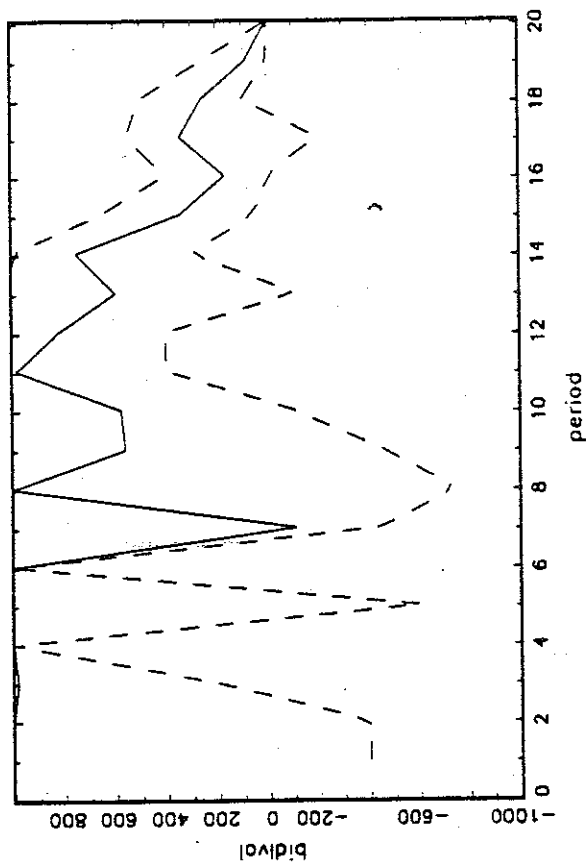
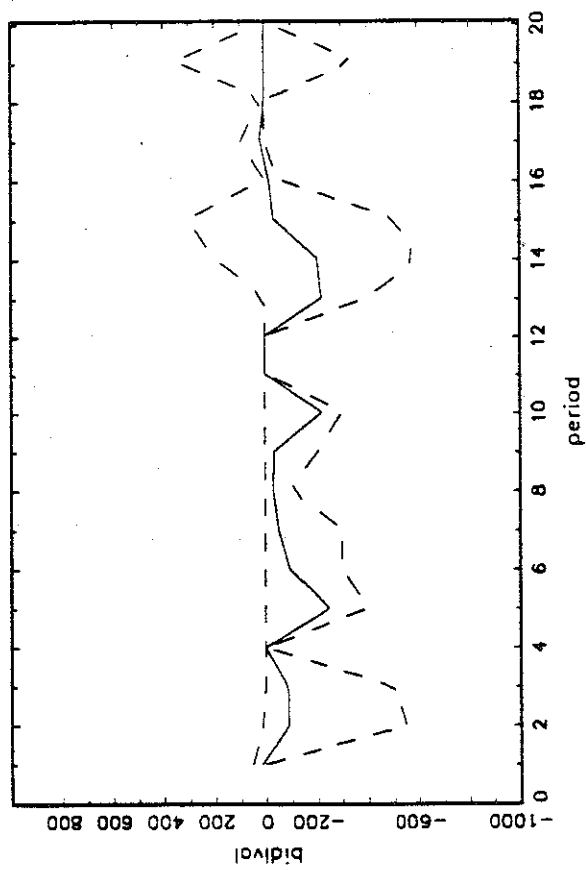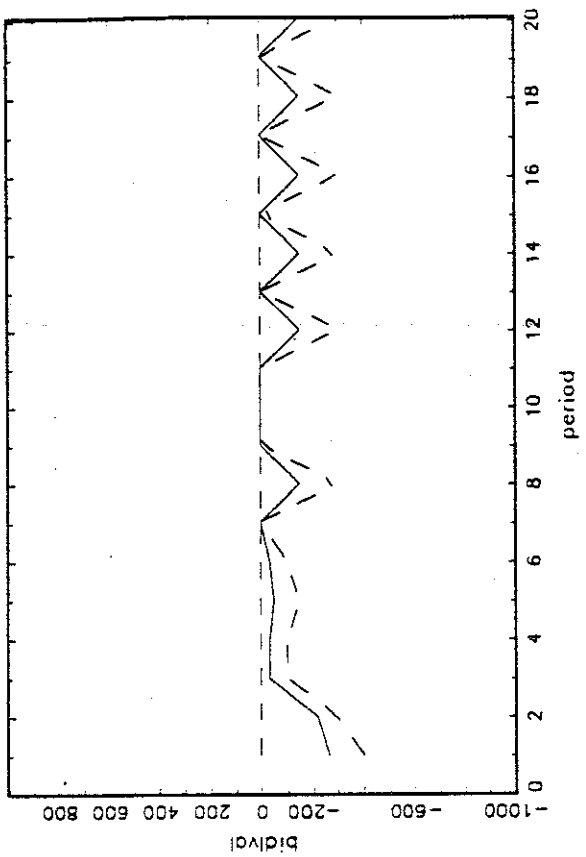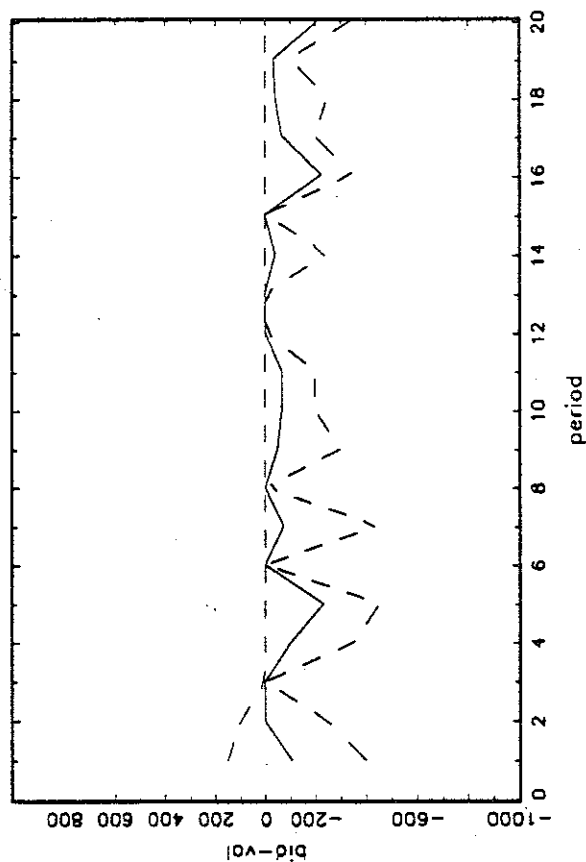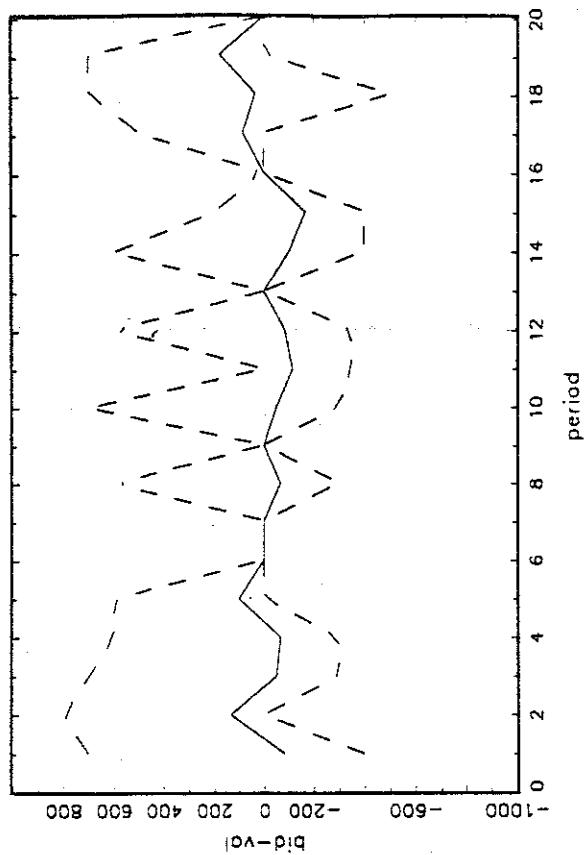exper = 4 :subject = 7
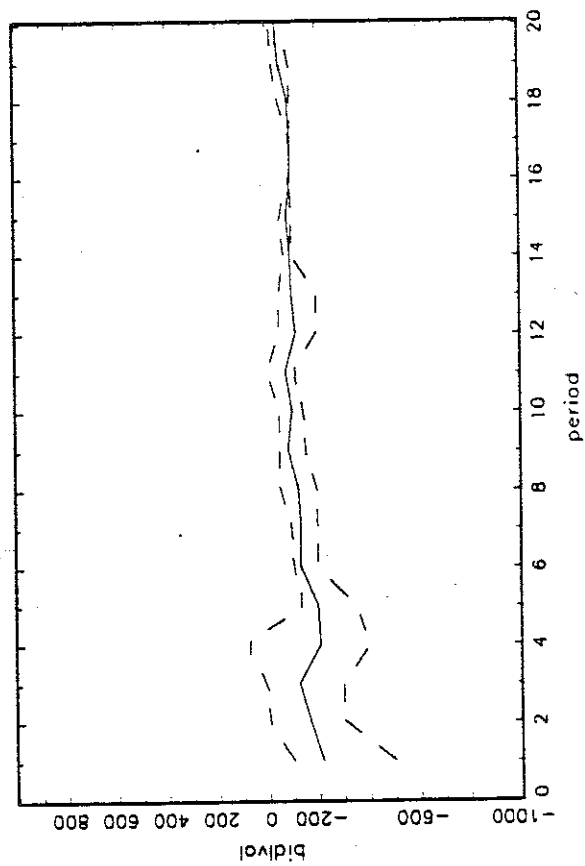
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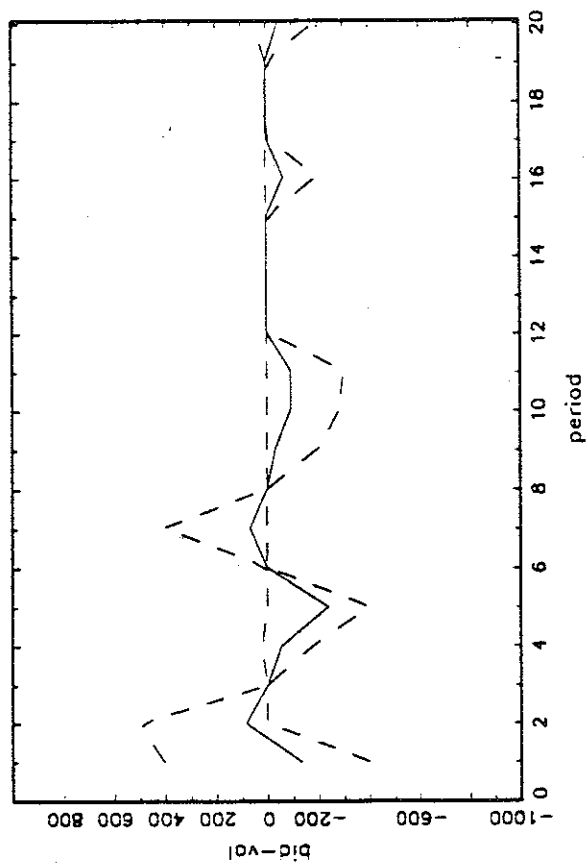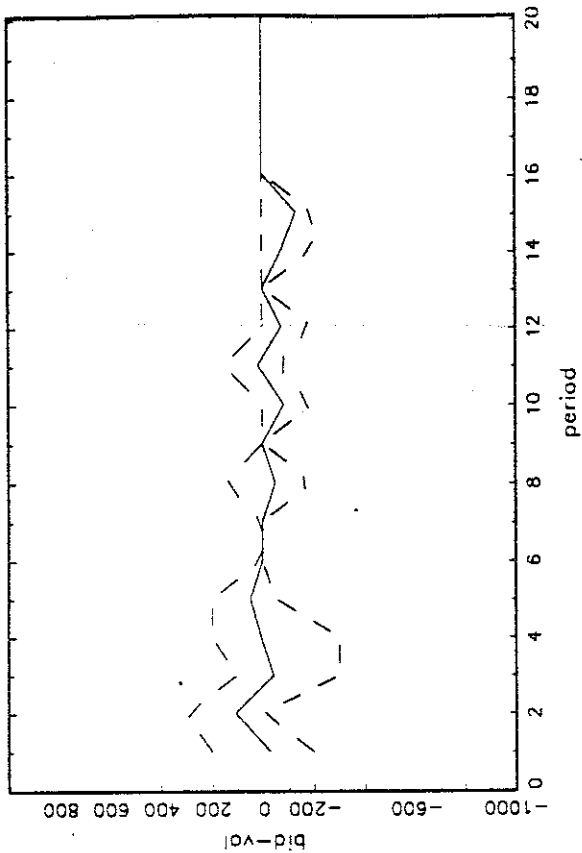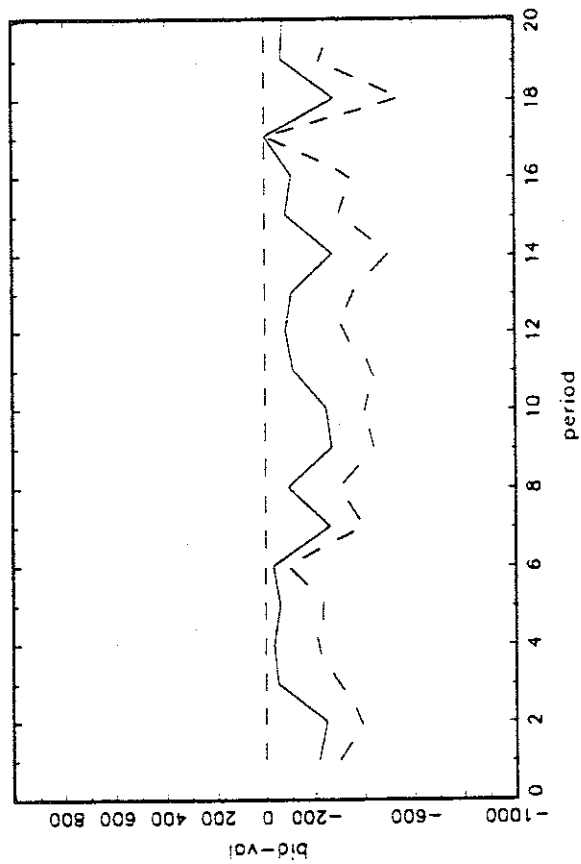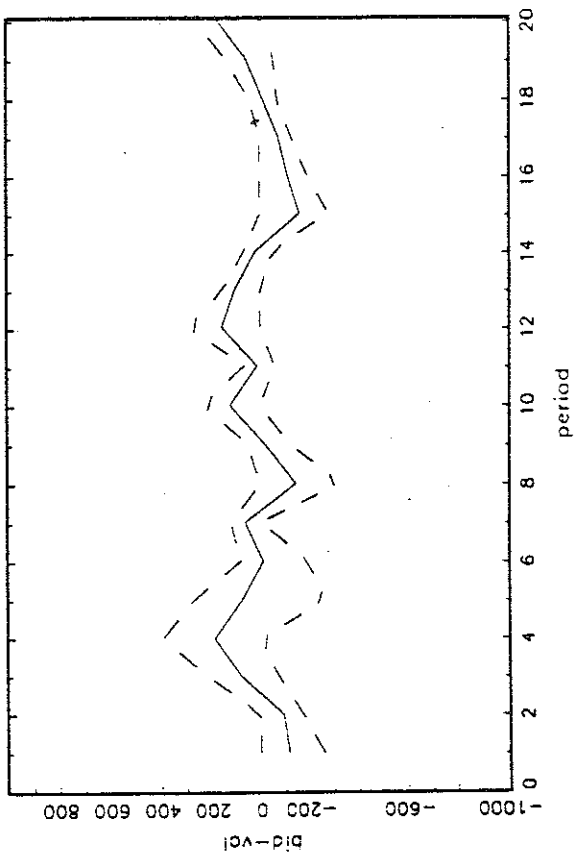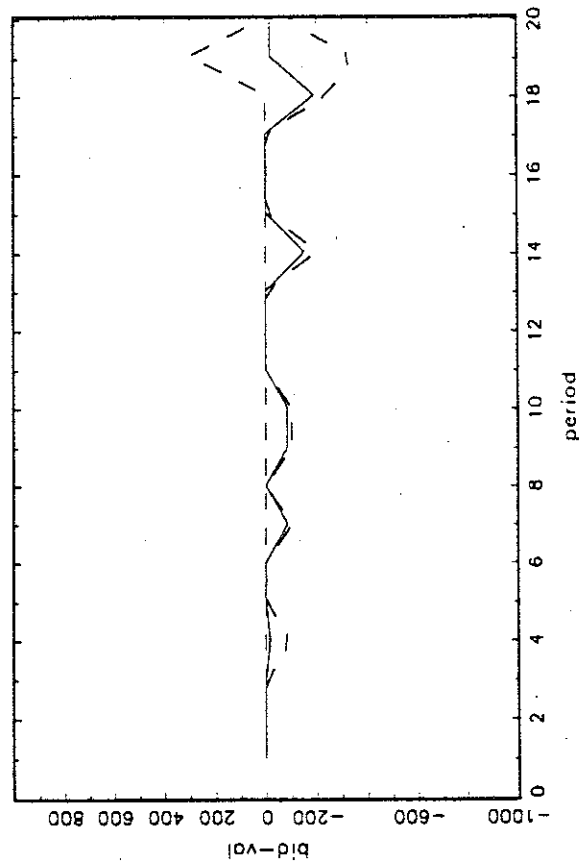
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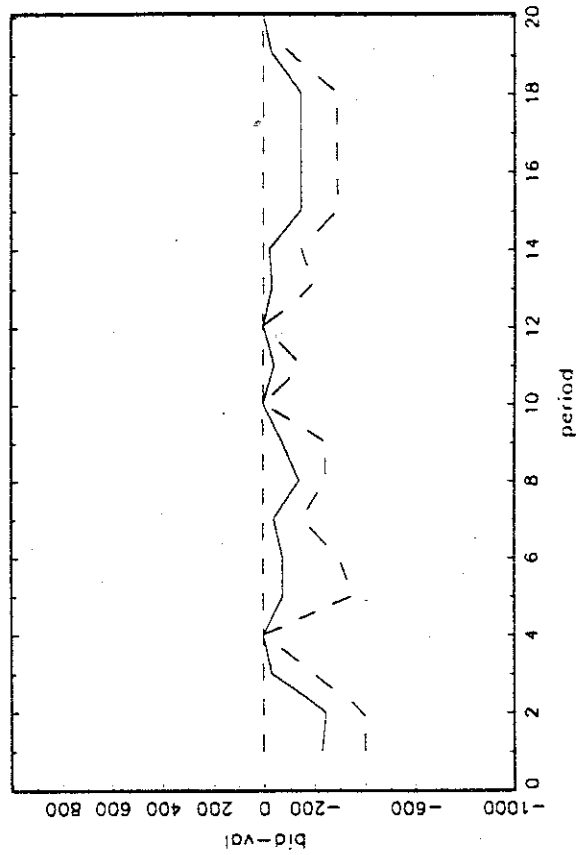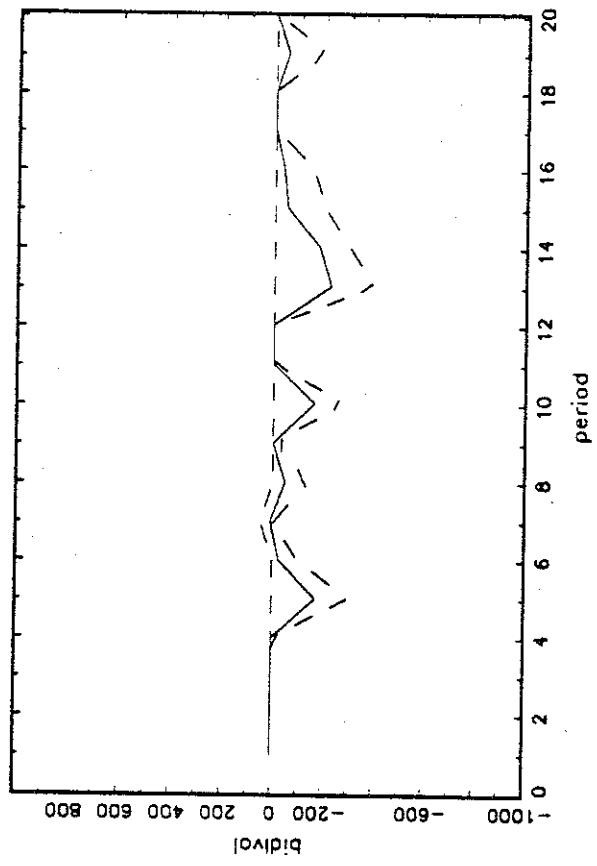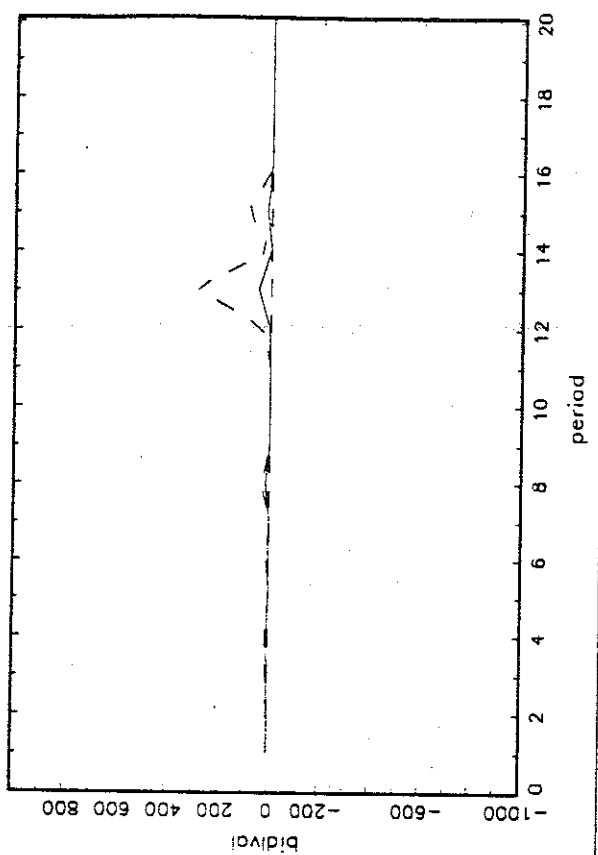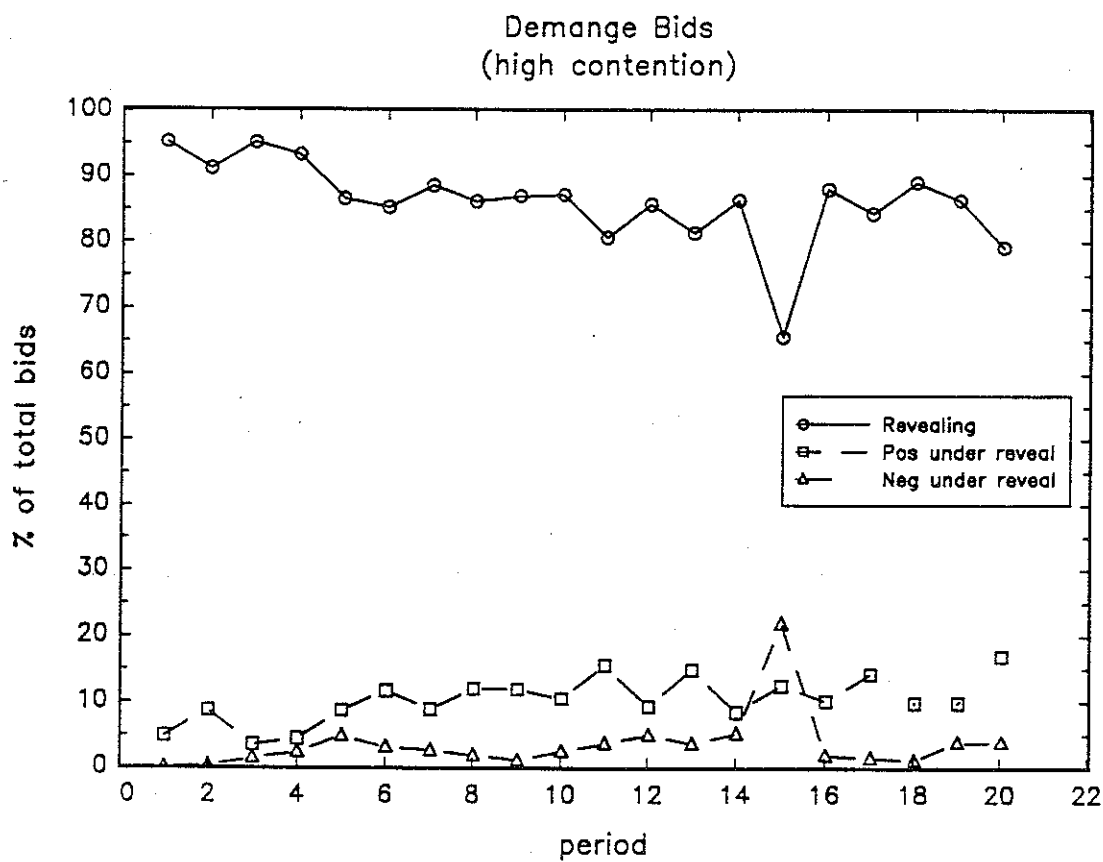
exper = 19 :subject = 6

exper = 19 :subject = 5

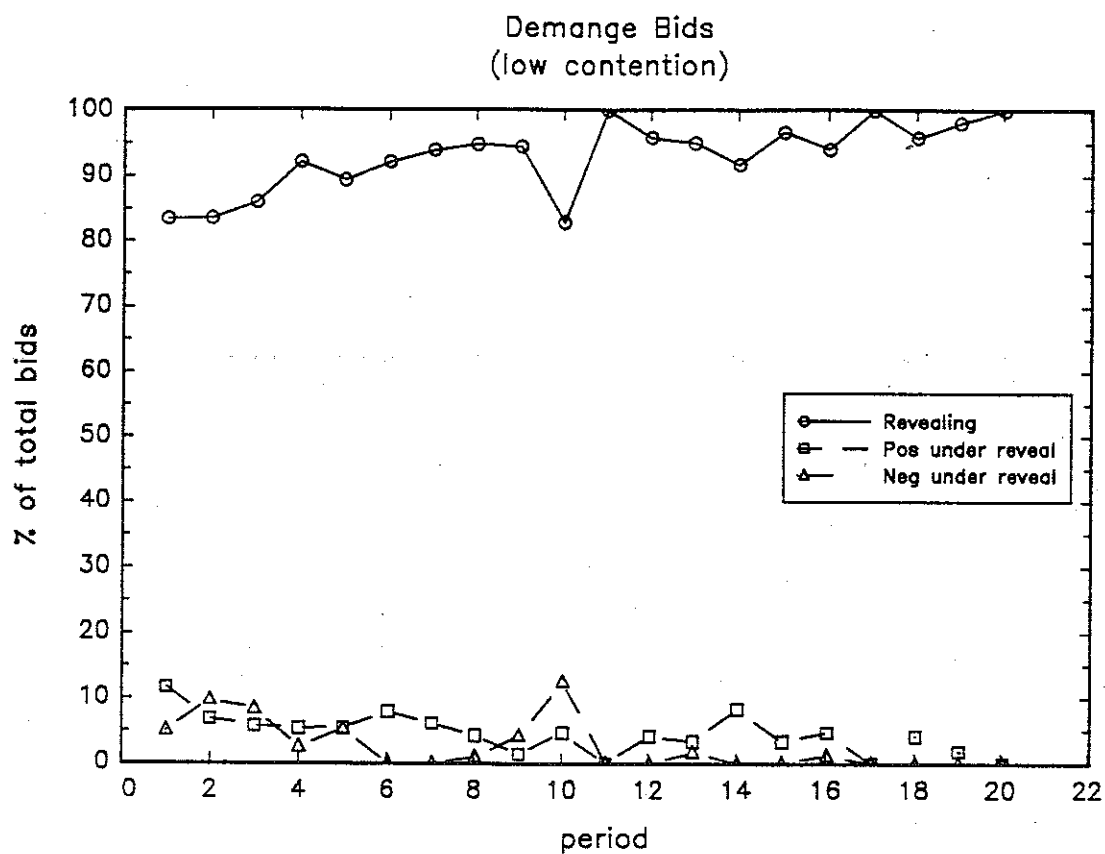Appendix F: Graphical displays of DGS bidding types.

Demange Bids
(low contention)

Demange Bids
(high contention)

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