DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA 91125

NONLINEAR BEHAVIOR IN SEALED BID FIRST PRICE AUCTIONS

Kay-Yut Chen

Charles R. Plott

SOCIAL SCIENCE WORKING PAPER 774

Revised August 1995

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ABSTRACT

The extent to which the behavior of people is consistent with game theoretic principles is investigated in a first price sealed bid auction environment. Three linear rules of thumb with increasing complexity are used as benchmarks to gauge the accuracy of the Constant Relative Risk Aversion Model (CRRAM). In addition, the CRRAM is tested against the relaxation of the rational expectation hypothesis.

Existing competitive bidding experiments cannot clearly distinguish between game theoretic models and linear markdown rules on an individual level. Within the parametric environments studied and reported in the experimental literature, game theoretic solutions are linear over the range of private values in which bid functions are estimated. In this study, agents drew values from nonuniform distributions. As a result, the game theoretic bidding behavior is nonlinear.

Due to the nonlinearity, special econometric and numerical techniques are applied to solve the model and obtain the estimates. The CRRAM exhibits good fit of the data. The pseudo R^2 is greater than 0.8 in 90 percent of the subjects. The CRRAM is more accurate than the Markdown Model (MM) and the Simple Ad hoc Model (SIMAM) but not as accurate as the Sophisticated Ad hoc Model (SOPAM). The data also supports the relaxation of the rational expectation hypothesis and suggests that substantial increases in the predictive power of game theoretic models can be gained from improvements in the theory of belief formation.

Nonlinear Behavior in Sealed Bid First Price Auctions[1]

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1 Introduction

This paper investigates the extent to which the behavior in first price sealed bid auctions is consistent with the principles of rationality that form the foundations of game theory. Two different sets of rationality principles are of interest. The first set can be described as principles of maximizing behavior. Roughly speaking, it is as if people are maximizing expected utility conditioned on their opponents' strategies and their beliefs about the state of the world. The second set of principles deals with belief formation and imply the rational expectations hypothesis which states that in equilibrium all of the beliefs of all of the agents are consistent with experience/reality.

The research strategy is to conduct a series of first price auction experiments in an environment in which game theory predicts the existence of substantial nonlinearities in behavior. The predictions of game theory are then compared and tested against a family of alternative models. The class of linear decision rules (including piece wise linear decision rules) is used as alternate behavior models.

The reasons to choose the class of linear decision rules as alternatives are three-fold. Firstly, linear decision rules are often advocated as replacements for the rationality postulates of game theory because they are easy to implement and yet they still capture aspects of strategic considerations. Secondly, in a substantial amount of previous work linear decision rules were investigated and cannot be ruled out as the explanation of the data. Thus, a theory of linear rules of thumb competes with game theory as an explanation of existing data. The third reason reflects the fact that almost any decision rule can be described in terms of (perhaps piece wise) linear rules - at least within the limits of existing measurement technology. Sometimes this feature of the models is an extreme disadvantage since it implies that the models cannot be rejected when they become complex enough. However, in this study, this flexibility is used as a tool to identify the "degree" to which subjects exhibit rational behavior.

[1]The financial support of the National Science Foundation and the Caltech Laboratory for Experimental Economics and Political Science is gratefully acknowledged. We also wish to thank John Ledyard, Mahmoud El-Gamal, John Kagel, Dan Levin, and James Walker for many helpful suggestions.

The authors are indebted to Caltech student Ralph Wolf who conducted initial experimentation with first price auctions under nonuniform distributions. The authors are also indebted to Kemal Guler whose analysis of the Wolf data had a substantial impact on the research reported here.

Previous experimental studies have focused on uniformly distributed individual private values. This results in linear game theoretic models (except near a boundary of the support of private values) and enables researchers to track the models with ease. However, under these environments, it is hard to separate the predictions of game theory from the predictions of theories that hold that human decisions are governed by linear decision rules such as a constant percentage markdown. In contrast to previous studies, by applying numerical techniques and related econometric methods we are able to study basic game theory in an environment that was previously not so accessible and where the behavior predicted has substantial nonlinearities. Thus, we are able to separate the predictions of game theory from the predictions of theories that hold that human decisions are based on simple linear decision rules.

In a seminal paper, Cox, Smith, and Walker (1988), the (asymmetric) constant relative risk averse model, referred to as CRRAM, was developed to explain experimental data from first price sealed bid auctions. In essence, Cox, Smith, and Walker (CSW) concluded that the data are very consistent with CRRAM (with the exception that 63% of the observed bidding functions have negative intercepts while CRRAM predicts zero intercepts). Thus, they demonstrated that CRRAM explains a long history of sealed bid auction experiments better than any other model. On the other hand, Kagel, Harstad, and Levin (1987) (KHL), compared ad hoc discounting rules to Nash equilibrium theories. They concluded from an analysis of the effects of the changes in public information on average revenue and on the directional changes in individual bids that the Risk Averse Symmetric Nash Equilibrium (RASNE) was the best of the theories. However, when the analysis was applied directly to individual bids alone, they could not statistically rule out a competing discounting model which assumes the bidders bid a discounted amount of their private value according to some rule of thumb. In the KHL study of the English auction and the second price auction, overbidding is observed in the latter but not the former. The overbidding is consistent with ad hoc reasoning but not consistent with game theoretic models. A third study by Guler, Plott, and Vuong (1987) (GPV) strongly rules out the model that people use general linear decision rules in favor of a modified game theoretic model. However, the data from the GPV study cannot easily be applied to an evaluation of CRRAM.

These three patterns of results set the stage for our investigation. It is not conclusive that people are using optimal decisions rules when involved in competitive situations such as bidding. Could the CSW results be due to a general tendency for individuals to use linear decision rules? KHL cannot rule out the possibility that the answer is "yes." Furthermore, a controversy has blossomed about the econometric methodology used by CSW (see Kagel and Roth, 1991 and CSW, 1991).

In our study, new data are obtained with the goal of resolving some of the issues mentioned above. We focus on how well models of rationality (game theoretic models) can explain the data in comparison to a number of ad hoc alternate models. A new method is provided to compare a Nash equilibrium model to linear bidding rules. In previous studies of the first price auction, experimenters used private values that were drawn from a uniform distribution.[2] Such an environment will result in linear Nash equilibrium bidding strategies over a substantial interval of possible values.[3] Such Nash linear strategies, however, are consistent with a set of ad hoc linear rules. Because of this correspondence, investigators have resorted to the study of special independent variables (number of agents, auction rules, information structures) in order to separate models of Nash equilibrium and ad hoc linear behavior. In the experiments reported below, private values are drawn from non uniform distributions. Under CRRAM, the equilibrium bidding behavior is nonlinear and the non linearity depends on individual risk behavior and beliefs of aggregate risk behavior.

In the first price auction, the rational expectation hypothesis states that all the agents in an auction have beliefs about the aggregate risk behavior of the other agents that reflect the truth. Under the CRRAM, risk behavior of an individual is characterized by a utility function that has one parameter called the risk parameter. The bidding function under the CRRAM of a player depends on his/her own risk parameter as well as his/her beliefs about the aggregate distribution of the risk parameter in the population. The beliefs about aggregate distribution of the risk parameter can be parameterized and estimated from bids of an individual. The individual risk parameter can also be estimated from bids of an individual. Thus, by testing whether all the individuals have the same beliefs and whether their beliefs are consistent with the estimated aggregate distribution of the risk parameter, we are able to test the reliability of the rational expectation hypothesis.

Thus, we are able to achieve two things. The first is to distinguish between CRRAM and linear bidding rules.[4] The second is to test the rational expectation hypothesis by comparing the beliefs of aggregate risk behavior to the estimated aggregate risk behavior.

A subtle but important distinction should be emphasized. Previous studies[5] have tested game theoretic formulations against ad hoc linear decision rules where the latter included a hypothesis of how linear behavior would change in response to changes in the parameters of the economic environment. The ad hoc rules were rejected because they failed to track behavior across changing environments as well as the behavior was tracked by game theoretic models. In some sense, it is the assumption of how linear behavior changes across different economic environments instead of the assumption of linearity that

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[2]A paper by Palfrey (1985) is a possible exception. He studied the bundling of values each of which is drawn from a rectangular distribution. Thus, the value of a bundle would be drawn from a nonuniform distribution. Palfrey studied only risk neutral models.

[3]In CSW (1988) a portion of the bidding function is actually nonlinear but they did not utilize the nonlinear data in their estimates.

[4]In CSW (1984) in which multiple unit discriminatory auctions were conducted, and Kagel and Levin (1988) in which third price auctions were conducted, subjects' behavior was not consistent with risk aversion under the Nash equilibrium. Subjects appear to be risk loving if Nash equilibrium theory was applied. These facts create a type of paradox because they suggest that people sometimes behave as if they are risk averse and other times behave as if they are risk loving. The existence of such inconsistencies casts doubt on the whole theory.

[5]Kagel, Harstad, and Levin, 1987.

is rejected. And, since the assumption about how the use of ad hoc rules might change with changes in the economic environment is itself ad hoc, the methodology has an inherent weakness that is difficult to overcome. By contrast, this study focuses on those same ad hoc linear rules within a given economic environment and asks if the linear aspect of the rules generates statistical models that are as good as those derived from game theory. Thus, the methodology supplements other studies in an important way. The three linear models (the MM, the SIMAM, and the SOPAM) studied in this paper are designed with this fact in mind. Each model assumes a different set of characteristic parameters in a different economic environment.

2 Experimental Design

A series of six experiments were conducted.[6] All the experiments were carried out in Caltech's Laboratory for Experimental Economics and Political Science with help from networking software. The instructions read to the subjects are in Appendix A.

Twelve subjects were in each experiment. Some subjects participated in more than one experiment. The experiments were conducted in periods. In each period, the subjects were randomly divided into groups of three who would bid against each other in a sealed bid auction. Then the private values of a subject were revealed (only to that subject) on the subject's computer screen. Then a first price auction was conducted among each group of three.

In considering and discussing the models some care must be exercised to avoid confusion among the various units used in theory, observation, payoffs, etc. Subjects operated in a space characterized by units called francs. The information that subjects received and the decisions that subjects made, the entire message space, was defined in these franc units. Each franc could be converted to U.S. dollars at a rate (privately) known to the subject.

Each private value was generated by the following scheme: let v be a random variable where $v \in [0,1]$ and with distribution

$$H(v) \begin{cases} 2av & \text{if } 0 \leq v < \frac{1}{2} \\ 2(1-a)v + 2a - 1 & \text{if } \frac{1}{2} \leq v \leq 1 \end{cases} =$$

where a is a parameter.

[6]Actually, 8 experiments were conducted. The first of the two experiments not reported was a pilot experiment. Instead of the computers, it was conducted on a chalkboard by an auctioneer. In the second experiment that is not reported, only five data points were collected.

Again, as a reminder, we note that the private franc value V, received by a subject, is $V = T + 1000v$ where T is the "offset."

Each subject was given the distribution described above. Furthermore, each was given a table in which the probability of each value is listed. This table was part of the instructions. The subjects were also told that the amount of dollars they would be paid at the end of the experiment would be a conversion rate times their franc earnings in the experiment.

Table 1 summarizes the parameters used in the experiments. Parameters include the subject pool, the franc conversion rate, number of subjects, etc. The important parameters are those that determine the distribution function from which private values were drawn.

There are two important parameters to consider in the generation of private values. Both parameters define types of controls in the experiments. First, the parameter a dictates the slopes and the slope change of the distribution function at the midpoint of the support. In our experiments, $a = 0.8$ and $a = 0.2$ were used.

Second, the offset T is the lowest possible private value that can occur. Since the length of the support of private values is fixed to 1000, the support is the interval [offset, offset + 1000] = $[T, T + 1000]$. The bidding functions from CRRAM go through the point (T,T). In our experiments, two values of the variable offset were chosen, $T = 0$ and $T = 500$. See Figures 5A and 5B for graphs of the distribution functions.

As an illustration of the importance of the two parameters, consider how they can be used to distinguish between behavior generated by a naive markdown rule as opposed to behavior generated by a game theoretic model. Consider the case in which the offset is 500. If the subjects are following a naive markdown rule, then their bid functions go through (0,0) but not through (500,500) as is predicted by game theoretic models. Therefore, whether the bidding behavior goes through (500,500) in the experiments where offset = 500 can be used as an extra test of the CRRAM and the ad hoc rule of behavior.

3 Overview of Models

Theory is discussed in normalized units. By normalizing key variables to take values in the interval [0,1] all derivations and discussions are simpler. In the experiments, it is not necessary that the private values are in the interval of [0,1]. Lemma 3 proves the invariance of the theory under linear transformations for the most general case considered in this paper.

The central variables are bids and private values. When franc units which are used in the experiments are intended the notations will be upper case B and V for bids and private values, respectively, and in normalized units that will typically be used in theoretical derivations, the script b and v will be used for the two variables, respectively.

For all experiments V takes values in some interval of length 1000. Specifically, $V \in [T, T + 1000]$ where the parameter T will be called the "offset" which can vary across experiments. The variable v will take values in the interval [0,1] and thus $V = T + 1000v$. All figures containing data and some of the econometrics will be presented in terms of V but the technical theoretical discussions will be in terms of v.

Because the models are complex, a brief overview of the issues and the models might be useful. The following are all of the models studied in the literature. A brief discussion of the models is also provided. From this discussion an overview of our experimental design can also be inferred.

In all the models, the following assumptions are made:

1. Subjects are expected utility maximizers with increasing utility for money.

2. Subjects have the following information before making a bid:

   (a) his/her private value;

   (b) number of subjects he/she is bidding against;

   (c) the fact that all values drawn are i.i.d.;

   (d) the distribution from which the values are drawn is publicly known.

3. Recall from last section, distribution of normalized private values are of the following form:

$$H(v) = \begin{cases} 2av & \text{if } 0 \le v < \tfrac{1}{2} \\ 2(1-a)v + 2a - 1 & \text{if } \tfrac{1}{2} \le v \le 1 \end{cases}$$

Recall that the private value seen by a subject is $V = T + 1000v$. So, both V and v can be called the "private value" without confusion. The parameter a adds control to the experiment as does the parameter T.

Notice that the distribution $H(.)$ is not uniform over its support. In previous experiments that have studied bidding in nonaffiliated environments, the distribution of private values has always been uniform. This departure from previous studies is the key to the interpretation of the experiments.

The following models are studied:

1. The Markdown Model (MM)

This model holds that people follow a rule of thumb based on their private values. The bid will be a proportion of the value. Where v = private value randomly drawn, the model has the form

$$\text{bid} = \beta v.$$

Without tracking the implications of other variables such as the number of bidders or information, etc., MM is not distinguishable from the Nash equilibrium behavior under uniformly distributed private values. An example is contained in Figure 1.

2. The Simple Ad hoc Model (SIMAM)

This model is slight generalization of MM to allow for the possibility that the bidding function might not go through the origin. When values are sufficiently low people, may simply bid zero. The function is

$$\text{bid} = \alpha + \beta v.$$

Measured bidding functions (CSW, 1988) have the property that $\alpha \neq 0$. An example is in Figure 1.

3. The Sophisticated Ad hoc Model (SOPAM)

This model assumes that subjects use a more sophisticated rule of thumb. The model has the form

$$\text{bid} = \begin{cases} \alpha + \beta v & \text{if } 0 \leq v < \frac{1}{2} \\ \alpha + \beta v + \gamma(v - 0.5) & \text{if } \frac{1}{2} \leq v \leq 1 \end{cases}$$

An example is in Figure 2. This is essentially a two-piece linear rule where the subject follows different linear rules when his private value is in [0,1/2] and [1/2,1]. The strategic implication of this will be more clear in the experimental design part of this paper.

4. The Risk Neutral Nash Equilibrium Model (RNNE)

Subjects are identical and risk neutral utility maximizers (i.e., $u(x) = x$). This model is not consistent with CSW's experimental data because their data uniformly lie above the risk neutral bid function. Generally, this fact has been interpreted as a manifestation of risk aversion. For the parametric environment used in our experiments, examples of the RNNE bid function are contained in Figures 3A and 3B for the cases where $a = 0.8$ and $a = 0.2$ respectively.

5. The Risk Aversion Symmetric Nash Equilibrium Model (RASNE)

Subjects are identical utility maximizers. Each subject has a one-parameter concave/linear utility function in the form $u(x) = x^r$. This model is also not consistent with CSW's data because the hypothesis that agents are identical can be rejected. An example of RASNE bid function is contained in Figures 3A and 3B for $a = 0.8$ and $a = 0.2$, respectively. The RASNE bid function has a property that it lies above the RNNE bid function for both values of a. This property is the reason that RASNE provides a better account of the CSW data than does the RNNE.

6. The Constant Relative Risk Aversion Model (CRRAM)

Subjects do not have identical utility functions. Each subject has a one-parameter utility in the form of $u(x) = x^{r_i}$. The risk parameters r are distributed according to some publicly known distribution $G(r)$. In the special case when the variance of $G(r)$ is zero, the CRRAM becomes the RASNE.

Figures 4A through 4F illustrate different features of the CRRAM bidding function. The CRRAM individual bidding function, which will be described in Section 4, is a function of individual risk parameter r, the mean, $E(r)$, of the distribution $G(r)$ and the price variance, S_r^2, of $G(r)$. In previous theoretical studies, bids were found to be increasing with increasing risk aversion in first price auctions. The effects of $E(r)$ and S_r^2 on the bidding function are more subtle and less intuitive. Both variables control the slope and the curvature of the bidding function in the interval [0.5,1]. As one can see from Figures 4B and 4C, the bidding function is relatively constant to varying $E(r)$ and S_r^2 in the interval [0,0.5]. Figures 4D through 4F show the effects of varying r, $E(r)$ and S_r^2, respectively, when $a = 0.2$ as opposed to the $a = 0.8$ used in Figures 4A through 4C.

The CRRAM is the most supported theory in the CSW study. However, as was mentioned in the introduction, there are still some inconsistencies between the data and CRRAM:

(a) In some cases, the subject has a bidding function with a significant nonzero intercept, while CRRAM predicts a zero intercept (CSW).

(b) In other experimental data, bidding is observed to be consistently above the dominant strategy in single unit second price auction (KHL, p. 33).

(c) In third price auctions, 60 percent of all bids lie above the RNNE line with ten subjects, while risk aversion requires bids to be below the RNNE line. This may be an indication of failure of the theory, or that subjects are risk-loving (Kagel and Levin, 1991).

7. The Belief Free Constant Relative Risk Aversion Model (BFCRRAM)

The BFCRRAM is identical to the CRRAM except that the rational expectation hypothesis is relaxed. The rational expectation hypothesis is imposed on the estimation procedure by requiring the subject to believe the mean and variance of the distribution of the risk parameters r_i are equal to the estimated ones.

In the BFCRRAM, the beliefs about the mean and variance of the distribution of risk attitudes (which are used to derive the optimal individual behavior) are estimated as free parameters. In contrast, when the CRRAM is estimated, these two parameters are restricted to the average and the variance estimate of the estimated individual risk parameters.

4 Formal Theoretical Development

This section develops the CRRAM model in our special nonlinear case and shows how it can be solved by numerical methods.

Consider a first price auction where there are $N \geq 2$ bidders. Each bidder's monetary value v_i, $i = 1,...N$, for the auctioned object is independently drawn from the probability distribution with c.d.f. $H(\cdot)$ on [0,1]. It is assumed that each bidder knows his own v_i but knows only the distribution from which his rivals' values are drawn. Each bidder is also assumed to know N, the number of bidders.

The one parameter utility of bidder i is $u(x;r_i) = x^{r_i}$. Assume that each bidder knows his/her own risk parameter r_i. Also, assume that r_i is distributed independently with distribution $G(\cdot)$. Each bidder knows $G(\cdot)$ and he/she is an expected utility maximizer.

Assume each bidder i believes everyone is using bidding function $b(\cdot;r)$, his expected utility is $Eu=(v-b)^{r_i}\{E_rH(\pi(b,r))\}^{N-1}$ where v = i's private value and $\pi(\cdot;r)$ = inverse function of $b(\cdot;r)$. The notation E_r denotes an expectation taken on r.

To maximize Eu, the first order condition is

$$-\frac{r_i}{v-b}+(N-1)E_r\left\{H'(\pi(b,r))\frac{\partial\pi(b,r)}{\partial b}\right\}/E_rH(\pi(b,r))=0$$

$$\Rightarrow E_rH(\pi(b,r))=\frac{N-1}{r_i}(\pi(b,r_i)-b)E_r\left\{H'(\pi(b,r))\frac{\partial\pi(b,r)}{\partial b}\right\}$$

with the substitution $v = \pi(b,r_i)$.

Let

$$f(b)=E_rH(\pi(b,r))/E_r\left\{H'(\pi(b,r))\frac{\partial\pi(b,r)}{\partial b}\right\}$$

$$\Rightarrow f(b)=\frac{N-1}{r_i}(\pi(b,r_i)-b)$$

$$\Rightarrow \pi(b,r_i)=\frac{r_i}{N-1}f(b)+b \tag{1}$$

$$\Rightarrow \frac{\partial \pi(b,r_i)}{\partial b} = \frac{r_i}{N-1} f'(b) + 1 \tag{2}$$

As mentioned above, the form of distribution $H(x)$ studied in our investigation is

$$H(x) = \begin{cases} 2ax & 0 \le x \le \frac{1}{2} \\ 2(1-a)x + 2a - 1 & \frac{1}{2} \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

with density

$$H'(x) = \begin{cases} 2a & 0 \le x < \frac{1}{2} \\ 2(1-a) & \frac{1}{2} \le x < 1 \\ 0 & x \ge 1 \end{cases}$$

This distribution is uniform in the intervals [0,½] and [½,1]. But the probability of being in either interval is different, where $P(x \in [0,½]) = a$ and $P(x \in [½,1]) = 1 - a$.

Substituting (1) into $E_r H(\pi(b,r))$, and writing E_r in an integral form, we have

$$E_r H(\pi(b,r)) = \int_o^{u_1} 2a\left(\frac{r}{N-1} f(b) + b\right) dG(r) + \int_{u_1}^{u_2} \left[2(1-a)\left(\frac{r}{N-1} f(b) + b\right) + 2a - 1 \right] dG(v) + \int_{u_2}^{\infty} dG(r)$$

$$= f(b)\left\{\frac{2a}{N-1}\int_0^{u_1} r dG(r) + \frac{2(1-a)}{N-1}\int_{u_1}^{u_2} r dG(r)\right\} + b\left\{ 2aG(u_1) \right.$$

$$\left. + 2(1-a)[G(u_2) - G(u_1)] \right\} + 1 - G(u_2) + (2a-1)[G(u_2) - G(u_1)]$$

where $u_1 = \frac{N-1}{f(b)}(\frac{1}{2} - b)$ and $u_2 = \frac{N-1}{f(b)}(1-b)$.

Similarly, substituting (2) into $E_r\left\{H'(\pi(b,r))\frac{\partial \pi}{\partial b}\right\}$ we have

$$E_r\left\{H'(\pi(b,r))\frac{\partial \pi(b,r)}{\partial b}\right\} = \int_o^{u_1} 2a\left(\frac{r}{N-1} f'(b) + 1\right) dG(r) + \int_{u_1}^{u_2} 2(1-a)\left(\frac{r}{N-1} f'(b) + 1\right) dG(r)$$

$$= f'(b)\left[\frac{2a}{N-1}\int_0^{u_1} r dG(r) + \frac{2(1-a)}{N-1}\int_{u_1}^{u_2} r dG(r)\right]$$

$$+ 2aG(U_1) + 2(1-a)[G(u_2) - G(u_1)]$$

Let

$$W_1 = \frac{2a}{N-1}\int_0^{u_1} rdG(r) + \frac{2(1-a)}{N-1}\int_{u_1}^{u_2} rdG(r)$$

$$W_2 = 2aG(u_1) + 2(1-a)[G(u_2) - G(u_1)]$$

$$W_3 = (2a-1)[G(u_2) - G(u_1)] + 1 - G(u_2)$$

Substituting

$$W_1 f(b) + W_2 b + W_3 = f(b)[(W_1 f'(b) + W_2])$$

$$\Rightarrow f'(b) = 1 - \frac{W_2}{W_1} + \left(\frac{W_2 b + W_3}{W_1}\right)\frac{1}{f(b)} \qquad (3)$$

From (1)

$$\Rightarrow f(b) = \frac{N-1}{r_i}(v-b)$$

$$\Rightarrow f'(b) = \frac{N-1}{r_i}\left(\frac{dv}{db} - 1\right)$$

and $u_1 = \dfrac{r_i(\frac{1}{2} - b)}{v-b}$, $u_2 = \dfrac{r_i(1-b)}{v-b}$

And (3) becomes

$$\frac{db}{dv} = \frac{(N-1)W_1}{(N-1+r_i)W_1 - r_i W_2 + \dfrac{r_i^2(W_2 b + W_3)}{(v-b)(N-1)}} \qquad (4)$$

Equation (4) is a differential equation that allows one to solve for the optimal bidding function $b(v;r)$ numerically. The following properties of the solution are useful:

Lemma 1) $b(0;r_i) = 0$. This gives us the initial condition for the differential equation (4)

Lemma 2) $\lim_{v \to 0} \dfrac{db(v,r_i)}{dv} = \dfrac{n-1}{n-1+r_i}$ (see Appendix B for proof)

Lemma 3) if $b(v;r_i)$ is a solution to $\max_x (v-x) r_i \{E_r H(\pi(x;r_i))\}^{n-1}$, then $\tilde{b}(\tilde{v};r_i)$ is a solution to $\max_x (\tilde{v} - x)^{r_i}\{E_r \tilde{H}(\pi(x;r_i))\}^{n-1}$, where for all $\lambda \in \Re, \beta \in \Re+$

$$\tilde{v} = \lambda + \beta v$$
$$\tilde{b}(\tilde{v};r) = \lambda + \beta\, b(v\,;r)$$
$$\tilde{H}(\lambda + \beta\, y) = H(y) \qquad \text{for all } y \text{ (see Appendix C for proof).}$$

Lemma 3 shows that the maximization problem is invariant to linear transformation. In our experiments, a number of different intervals were chosen as support of v. Lemma 3 legitimizes the use of (4) as the base solution for all of our parameter choices.

We have initial condition $b(0;r_i) = 0$, so $b(v;r_i)$ can be integrated numerically from (4).

Notice that equation (4) depends upon the distribution G of r. We assume r has a log normal distribution. There are two intuitive reasons to make this choice. First, we believe that people tend to behave similarly. Second, the structure of the utility function constrains r on the open interval $(0, \infty)$. A log normal distribution has support on $(0, \infty)$. Let u_0 and σ^2 be the mean and variance of $\log(r)$.

$$G(r) = \Phi\left(\frac{\log r - u_0}{\sigma}\right)$$

The density is

$$g(r) = \frac{1}{\sigma\, r}\Phi\left(\frac{\log r - u_0}{\sigma}\right)$$

It is advantageous to parameterize G in terms of $E(r)$ and σ_r^2 instead of u_0, σ^2.

$$\begin{aligned} E(r) &= \int_0^\infty r g(r) dr \\ &= \int_0^\infty \frac{1}{\sigma}\phi\left(\frac{\log r - Uo}{\sigma}\right) dr \\ &= e^{u_0 + \frac{\sigma^2}{2}} \end{aligned}$$

$$\begin{aligned} E(r^2) &= \int_0^\infty \frac{r}{\sigma}\phi\left(\frac{\log\ r - u_0}{\sigma}\right) dr \\ &= e^{2u_0 + 2\sigma^2} \end{aligned}$$

$$\begin{aligned} \sigma_r^2 &= E(r^2) - (E(r))^2 \\ &= e^{2u_0 + 2\sigma^2}(1 - \bar{e}^{\sigma^2}) \end{aligned}$$

and

$$u_0 = \log E(r) - \frac{1}{2}\log\left(1+\frac{\sigma_r^2}{E(r)^2}\right)$$

$$\sigma^2 = \log\left(1+\frac{\sigma_r^2}{E(r)^2}\right)$$

5 Data Analysis/Econometrics

The data is analyzed with the following goals in mind:

1. To compare CRRAM to the Markdown Model (MM), the simple and sophisticated ad hoc model (SIMAM and SUPAM).

2. To show (in a pseudo R^2 sense) how well CRRAM explains the bids.

3. To test whether risk parameters of subjects in different experiments are drawn from the same distribution.

4. To test whether the bidding functions go through the point (offset, offset) = (T,T).

5. To test the rational expectation hypothesis.

Both the RNNE and RASNE models were also analyzed. Recall that RNNE and RASNE are special cases of CRRAM and both models were rejected in CSW's paper. In separate (unreported) tests, both RNNE and RASNE models can be rejected using the likelihood ratio test.

Using the framework developed in Section 3, for given risk parameter r and risk distribution G_r, one can calculate the bidding function $b(v;r,E_r(r),\sigma_r^2$ where $E_r(r) = \int r dGr$ and $\sigma_r^2 = E_r((r-E(r))^2)$.

The following econometric model is used:

$$B_{it} = B(V_{it};r_i,E(r),\sigma_r^2)+\xi_{it} \qquad (5)$$

where

$B(\cdot)$	= CRRAM bidding function in franc units
B_{it}	= bid submitted by subject i at time t
V_{it}	= private value of subject i at time t
ξ_{it}	is distributed *i.i.d.* $n(0, \sigma_i^2)$
r_i	= risk parameter of subject i

The following assumptions are made:

- The variance of σ_r^2 of ξ_{it} is constant across periods but can vary from subject to subject.
- Since $b(v_{it}, r_i, E(r), \sigma_r^2)$ depends also on $E(r)$ and σ_r^2, we use $\bar{r}=\frac{1}{n}\sum_i r_i$ to estimate $E(r)$ and $\frac{1}{n-1}\sum_i (r_i - \bar{r})^2$ to estimate σ_r^2.

The method of maximum likelihood is used to estimate r_i for all subjects. The results will be presented in the next section. The maximum likelihood procedures were carried out on a CRAY X-MP/18 supercomputer at the Jet Propulsion Laboratories and a CRAY Y/MP supercomputer at the NASA Goddard Space Flight Center.

Each evaluation of the bidding function typically included a 100-step integration. Since the bidding functions of the subjects are all interrelated under CRRAM, the risk aversion parameters of all subjects in an experiment have to be estimated simultaneously. Each experiment consists of 12 subjects and typically 100 bids per subject. To calculate the maximum likelihood, we have to maximize a likelihood function with 1200 data points in a 12 dimensional space. With the level of precision used in the analysis, the calculation typically takes about six to twelve hours on a CRAY. Better precision of the integration routine may be desirable since some divergence of bidding functions were encountered upon integration. None of the numbers reported below involve the divergence problem. The problem itself can be overcome by computer techniques or by a willingness to devote much more time to computations.

The pseudo R^2 was also calculated for each subject. The pseudo R^2 is a measure of how much explaining power the model has.

$$\text{pseudo } R_i^2 = 1 - \frac{SSR_i}{\overline{SSR_i}} \tag{6}$$

where

$$SSR_i = \sum_t (B_{it} - B(V_{it}, \hat{r}_i, \hat{E}(r), \hat{\sigma}_r^2))^2$$

$$\overline{SSR_i} = \sum_t (B_{it} - \overline{B_{it}})^2$$

The pseudo R^2 is between 0 and 1 since $SSR_i \leq \overline{SSR_i}$. The closer the pseudo R^2 is to 1, the better explanatory power the model has. A model that explains all the variation of the bids will have a pseudo R^2 of 1. The result is presented in the next section.

Maximum likelihood estimation is also carried out for the following two ad hoc models:

The Simple Ad hoc Model (SIMAM)

$$B_{it} = \alpha_i + \beta_i V_{it} + \xi_{it} \tag{7}$$

And the Sophisticated Ad hoc Model (SOPAM)

$$B_{it} = \begin{cases} \alpha_i + \beta_i (V_{it} - T) + \xi_{it} \\ \alpha_i + \beta_i (V_{it} - T) + \gamma_i (V_{it} - T - 500) + \xi_{it} \end{cases}$$

if $T \le V_{it} < T + 500$

if $T + 500 \le V_{it} \le T + 1000$

(8)

where ξ_{it} is *i.i.d.* with constant variance σ_i^2. These two models have been outlined in previous sections. Both models are rewritten in a statistical fashion here.

The Vuong's model Selection Test was used to compare the SIMAM and the SOPAM to the CRRAM. Let us consider the general case where one wants to select between two strictly non-nested models *f* and *g*.

Let

Y	=	$\{y_t\}_{t=1}^{n}=1$ be a data set with n points.
θ	=	parameter of model f
γ	=	parameter of model g
$\hat{\theta}_n$	=	maximum likelihood estimate of θ under Y
$\hat{\gamma}_n$	=	maximum likelihood estimate of γ under Y
l_f	=	likelihood function of f
l_g	=	likelihood function of g

Define the estimated log likelihood ratio of f to g to be

$$LR_n\left(\hat{\theta}_n, \hat{\gamma}_n\right) \equiv \sum_{t=1}^{n} \log \frac{l_f(y_t|\hat{\theta}_n)}{l_g(y_t|\hat{\gamma}_n)}$$

Now, consider the hypothesis H_o : f and g are equivalent,

$$\hat{W}_n^2 \equiv \frac{1}{n}\sum_{t=1}^{n}\left[\log\frac{l_f(y_t|\hat{\theta}_n)}{l_g(y_t|\hat{\gamma}_n)}\right]^2 - \left[\frac{1}{n}\sum_{t=1}^{n}\log\frac{l_f(y_t|\hat{\theta}_n)}{l_g(y_t|\hat{\gamma}_n)}\right]$$

Consider the following three hypotheses:

- H_0: *f* and *g* are equivalent,
- H_f: *f* is better than *g*, and

- H_g: g is better than f.

Vuong's theorem 5.2 states that

(i) under H_0 : $n^{-1/2} LR_n(\hat{\theta}_n, \hat{\gamma}_n) / \hat{W}_n \xrightarrow{a.s.} n(0,1)$

(ii) under H_f: $n^{-1/2} LR_n(\hat{\theta}_n, \hat{\gamma}_n) / \hat{W}_n \xrightarrow{a.s.} +\infty$

(iii) under H_g: $n^{-1/2} LR_n(\hat{\theta}_n, \hat{\gamma}_n) / \hat{W}_n \xrightarrow{a.s.} -\infty$ (9)

Equation (9) provides a very simple directional test for model selection. If the value of the statistics $n^{-1/2} LR_n(\hat{\theta}_n, \hat{\gamma}_n) / \hat{W}_n$ is higher than some critical value c, which is decided by the significance level, then one rejects the null hypothesis that the models are equivalent in favor of f being better than g. If $n^{-1/2} LR_n(\hat{\theta}_n, \hat{\gamma}_n) / \hat{W}_n$ is smaller than $-c$ then one rejects the null hypothesis in favor of g being better than f. Finally, if $| n^{-1/2} LR_n(\hat{\theta}_n, \hat{\gamma}_n) / \hat{W}_n | \leq c$ then one cannot discriminate between f and g given the data.

In our case, let

L_{CRRAM}	=	estimated maximum log likelihood of $CRRAM$
L_{MM}	=	estimated maximum log likelihood of MM
L_{SIMAM}	=	estimated maximum log likelihood of $SIMAM$
L_{SOPAM}	=	estimated maximum log likelihood of $SOPAM$
LR_{MM}^{CRRAM}	=	estimated maximum log likelihood ratio of $CRRAM$ to MM
LR_{SIMAM}^{CRRAM}	=	estimated maximum log likelihood ratio of $CRRAM$ to $SIMAM$
LR_{SOPAM}^{CRRAM}	=	estimated maximum log likelihood ratio of $CRRAM$ to $SOPAM$
$\hat{W}^{CRRAM}$	=	estimated variance of LR_{MM}^{LCRRAM}
$\hat{W}_{SIMAN}^{CRRAM}$	=	estimated variance of LR_{SIMAM}^{CRRAM}
$\hat{W}_{SOPAM}^{CRRAM}$	=	estimated variance of LR_{SOPAM}^{CRRAM}
n	=	number of data points in an experiment

The critical value of a normal distribution at 0.05 level of significance is 1.65. Therefore, we select CRRAM over SIMAM if $n^{-1/2} LR_{SIMAM}^{CRRAM} / W_{SIMAM}^{CRRAM} > 1.65$ and $SIMAM$ over $CRRAM$ if $n^{-1/2} LR_{SIMAM}^{CRRAM} / W_{SIMAM}^{CRRAM} < 1.65$ and we cannot select one over the other if $| n^{-1/2} LR_{SIMAM}^{CRRAM} / W_{SIMAM}^{CRRAM} | \leq 1.65$. Similarly, we can compare $CRRAM$ to $SOPAM$ and MM.

Vuong noted the existence of other model selection criteria that, when appropriately normalized, are asymptotically equivalent to the LR-statistics (9).

More generally, let

$$\hat{LR}_n(\hat{\theta}_n,\hat{\gamma}_n) \equiv LR_n(\hat{\theta}_n,\hat{\gamma}_n) - K_n(f,g)$$

where $K_n(f, g)$ is a correction factor depending on the characteristics of models f and g. The statements in (9) hold true when LR_m is replaced by $\tilde{LR}_n$.
It is found that some of our results depend on whether a correction factor

$K_n(f,g) = p - q$ or $K_n(f,g) = \frac{p}{2} \log n - \frac{q}{2} \log n$ is applied. (Where p= number of parameters of f, g = number of parameters of g).

We have also considered using other procedures like the bounded-size likelihood ratio (BLR) test. We decided that the Vuong's Model Selection Test is most suited to our purpose.

In the maximum likelihood procedures, we obtained estimates of the risk parameters of the subjects. In each experiment there are 12 estimates. The Kolomogorov-Smirnov test was used to test whether the sets of 12 estimates from different experiments are drawn from the same distribution or not. The results are presented in the next section.

To test whether the bidding functions go through the point (offset, offset), the following econometric model was used:

$$B_{it} = \alpha_i + \sum_{j=i}^{5} B_{ji}(V_{it} - \text{offset})^j + \xi_{it} \qquad (10)$$

If the bidding function for subject i goes through (offset, offset), then α_i = offset. After α_i is estimated, a simple t-test can tell whether α_i is significantly different from the offset. The results are reported in the next section.

The reason a polynomial model was chosen is that any well-behaved function can be closely approximated by a finite polynomial.

6 Results

The CRRAM is estimated for the six experiments conducted. The eight figures, 6A through 6H, are examples of individual bids and estimated bidding functions . As can be seen in the figures, the CRRAM can yield highly nonlinear predictions. Among these figures there are "good fits" (Figure 6A), "worst fits" (Figure 6B) and "typical fits" from each experiment (Figures 6C-6H). Notice that in Figures 6D and 6F the bidding functions have slightly different features than the others probably due to the fact that in 6D and 6F (experiment 2 and 4), the parameter a = 0.2 while in the others a = 0.8.

Together these figures suggest how the CRRAM respond to the single (risk aversion) parameter. These visual supports for the CRRAM suggest that if people are using rules of thumb, then these rules of thumb highly resemble the CRRAM.

Result 0 is included for completeness. It deals with the special cases of risk neutrality and with the risk averse symmetric cases with homogenous bidders. As was mentioned, the RNNE and RASNE are both special cases of the CRRAM which have been rejected by CSW. RASNE is the special case where the constraint $r_1 = r_2 = \cdots = r_{12}$ is put on the CRRAM. RNNE is the special case where the constraint $r_1 = r_2 = \cdots = r_{12} = 1$ is put on the CRRAM. Result 0 simply states that neither of these models can be accepted and so the result sets the stage for an investigation of a more elaborate class of models.

Result 0: Both the RNNE and the RASNE can be rejected.

Support: Both constraints are tested using the likelihood ratio test. The statistics of the test of constraints are listed in Table 2B. In separate tests, both RNNE and RASNE models can be rejected at the 0.05 level of significance in all of the six experiments and also in the pooled data.□

The Result 0 is consistent with the CSW conclusion that subjects behave as if they are risk averse and heterogeneous. It is therefore necessary to explore in detail the more general model that they propose.

The results of the estimation of the CRRAM and other models together with relevant summary statistics are contained in Tables 2 through 5. Table 2 contains the likelihood estimates of the models, the results of Vuong's Model Selection Test discussed in Section 5 above and the results of testing the rational expectation hypothesis. For each of the models considered Table 3 gives the means and the standard deviation of the pseudo R^2 of all of the subjects in all experiments. Table 4 contains the means and the standard deviations of the estimates of the CRRAM risk aversion parameters of subjects. Table 5 contains statistics to determine if the estimates of the risk aversion parameters from different experiments are drawn from the same distribution.

The first result addresses one of the central issues directly. When separated within a fixed parametric environment, the CRRAM captures the subtleties of individual behavior that the Markdown Model does not capture.

Result 1: The CRRAM is a better model than the Markdown Model (MM).

Support: The support of this result comes at two levels of data analysis. The first is the data set for each whole experiments using the Vuong's Model Selection Test. In five of the six experiments, the MM can be rejected at five percent significance in favor of the CRRAM. In experiment 5, one cannot discriminate between the two models given the data. The statistics are in Table 2A. The analysis of the pooled data also favors the CRRAM.

The second level of support for Result 1 is provided by the pseudo R^2 analysis. In 72 percent of the subjects, the CRRAM gives a higher pseudo R^2 than MM reflecting a greater explanatory power of the CRRAM. These higher proportions are reflected in the tendency for higher means of the pseudo R^2 contained in Table 3 which are reported here rather than the pseudo R^2 for each of the individuals.□

Result 1 indicates that people respond to the strategic consideration of the environment (CRRAM) instead of following some rule of thumb (MM) blindly. Result 2 is that even when we increase the sophistication of the rule of thumb to the two parameter SIMAM, the CRRAM is still a better model than the rule of thumb. This result suggests that the full power of the rationality postulates does a better job of describing choice behavior than a model that suggests that people follow a linear rule of thumb with a free slope and free intercept. Notice that SIMAM has double the number of parameters than CRRAM has.

Result 2: The CRRAM is a better statistical model than the Simple Ad hoc Model (SIMAM).

Support: Using the Vuong's Selection Test, in three out of six experiments and in the pooled data set, the SIMAM can be rejected in favor of the CRRAM at five percent significance level independent of whether the correction factor K_n is applied. In the remaining three experiments, if the correction factor K_n is applied, all are indistinguishable from the CRRAM. However, if the correction factor K_n is not applied, in two experiments (experiments 2 and 4) the CRRAM can be rejected at the five percent level in favor of SIMAM. Notice that in the experiments that the CRRAM performed worst (experiments 2 and 4), the parameter a = 0.2. The statistics are listed in Table 2A.

In 56 percent of the subjects, the CRRAM gives a higher pseudo R^2 than the SIMAM. Again, this phenomenon is reflected in the means reported in Table 3.□

The support for Result 2 contains the perplexing fact that serves as a warning not to be over confident about the rationality postulates. When the probabilities governing individual values changed across experiments, support for rationality was reversed. Thus, Result 2 is sensitive to the underlying environment.

The next result, Result 3, gives the limit of the explanatory power of the CRRAM in comparison to rules of thumb. The result is that the SOPAM explains the data better than CRRAM. The SOPAM can be viewed as a natural extension of the SIMAM. It also captures aspects of the strategic situations since the bid function is allowed to change slope when the distribution of private values changes slope. Formally stated the result is:

Result 3: The Sophisticated Ad hoc Model (SOPAM) is a better statistical model than the CRRAM.

Support: When Vuong's Model Selection Test is used without the correction factor K_n, in all six experiments and the pooled data the CRRAM can be rejected at five percent significance in favor of the SOPAM. When the correction factor is applied, the CRRAM can be rejected in four of the six experiments. The statistics are listed in Table 2A. In 92 percent of the subjects, the SOPAM gives a higher pseudo R^2 than the CRRAM. This phenomenon is reflected in the means reported in Table 3.□

The SOPAM is a piece wise linear decision rule. The pieces cover what might be considered to be the prominent parts of the individual values space. Thus, SOPAM in conjunction with some rule of thumb regarding where the breaks of the decision rule might fall is in a sense the limits to which rationality is exhibited.

The results do not appear to be due to statistical techniques. Result 1 is independent of the correction factor $K_n = \frac{p}{2}\log(n) - \frac{q}{2}\log(n)$. Results 2 and 3 depend only slightly on this correction factor. As mentioned in Section 5, since all the ad hoc models have at least as many parameters as CRRAM, applying K_n will only tilt the Vuong's Selection Test in favor of the CRRAM.

Notice that the CRRAM performs better in experiments 1,3,5 and 6 in which the experimental parameter a = 0.8 than in experiments 2 and 4 in which a = 0.2. It seems that the performance of the CRRAM depends on the experimental environment.

Results 1 through 3 evaluate the rationality postulates relative to linear rules of thumb behaviors. There is no absolute measure to judge how well a model explains data. However, there are facts that suggests that the CRRAM is explaining the data reasonably well. The following facts should help the reader to form his/her own judgment about the absolute explanatory power of the CRRAM.

Result 4: The CRRAM has good explanatory power.

Support: The pseudo R^2s are generally high. Ninety percent of the subjects have pseudo R^2s greater than 0.8 and 67 percent of the subjects have pseudo R^2s greater than 0.9. Figure 7 shows the distribution of pseudo R^2 of all subjects for all experiments.

The CRRAM predicts that the bidding functions go through the point (offset, offset) = (T,T). Equation (11) was estimated for each of the 72 subjects. For each a t-test was performed on the hypothesis that $\partial_i = T$ (i.e., the bidding function goes through the point (T,T)). For 86 percent of the subjects the hypothesis $\partial_i = T$ cannot be rejected at the five percent level of significance.□

The support of the above result suggests an important aspect of model evaluation that can be easily overlooked. The CRRAM has a property of translational invariance.

That is, a translation of the support of probabilities of values leaves the prediction of the theory the same relative to the translation. This can be seen in Figures 6E, 6F and 6H in which the predictions of CRRAM with the offset parameters are displayed against the data. This is actually a translation of the predicted bidding function of the case with offset equal to zero by adding (500,500) to each point. The support of Result 4 suggests that the data has this property also. In particular, if the SOPAM is modified to, say, a sophisticated two piece linear bidding rule such that value = 0 implies bid = 0, then the CRRAM is a better model. If a rule of thumb does not have this translational invariant property, it is most likely that it will be inconsistent with the data. Formally the observation is as follows:

Observation: The data are consistent with a model with the translational invariant property. That is, rules of thumb that are not translation invariant will generally be rejected in favor of the CRRAM.

Until now, the analysis has focused on the full set of rationality postulates in comparison with the ad hoc rules. The natural questions to pose concern the ability of some reduced set of rationality postulates to explain the data. The next result addresses the issue of the rational expectation hypothesis and identifies it as a possible source of error of the CRRAM. As stated in the result, in all the experiments, the rational expectation hypothesis is not consistent with the data. This result suggests that subjects have beliefs about the distribution of risk behavior that are neither true nor consistent with each other.

Result 5: The rational expectation hypothesis is rejected. Hence, the parameters $E(r)$ and σ_r^2 from individual bidding functions are not consistent with the hypothesis that they are the same across individuals.

Support: Using the likelihood ratio test, we reject the hypothesis that all the individual $E(r)$ and σ_r^2 are equal at five percent significance in all experiments. The statistics are listed in Table 2B. □

Since the rational expectation hypothesis is rejected in Result 5 a natural extension of the investigation is to investigate the CRRAM without the rational expectation hypothesis. We will call the relaxed model the Belief Free CRRAM (BFCRRAM). The strategy is to test BFCRRAM against the three linear decision rules (MM, SIMAM and SOPAM). Using the set of three linear rules as a benchmark, we are able to measure the "degree" of rationality exhibited by the subjects. Although the BFCRRAM gained substantial accuracy over CRRAM as demonstrated by Result 5, its relative accuracy compared to the three linear rules did not change. The BFCRRAM is more accurate than MM and SIMAM but not quite as accurate as SOPAM. This fact is stated formally in the next three results.

Result 6: The BFCRRAM is a better statistical model than the Markdown Model (MM).

Support: Using the Vuong's Selection Test, in five out of six experiments and in the pooled data set, the MM can be rejected in favor of the BFCRRAM at a five percent significance level. The statistics are listed in Table 2C. □

Result 7: The BFCRRAM is a better statistical model than the Simple Ad hoc Model (SIMAM).

Support: Using the Vuong's Selection Test, in five out of six experiments and in the pooled data set, the MM can be rejected in favor of the CRRAM at a five percent significance level. The statistics are listed in Table 2C.□

Result 8: The Sophisticated Ad hoc Model is a better statistical model than the BFCRRAM.

Support: Using the Vuong's Selection Test, the BFCRRAM is rejected in favor of the SOPAM in experiment 1 and the SOPAM is rejected in favor of the BFCRRAM in experiment 2 - both at the five percent significance level. In all the other experiments, they are indistinguishable from each other. In the pooled data set, the BFCRRAM is rejected in favor of the SOPAM. The statistics are listed in Table 2C.□

Notice that the support for SOPAM over BFCRRAM is based only on the pooled data thereby suggesting that the results of statistical tests of BFCRRAM against rules like the SOPAM might be sensitive to econometric specification. In addition, the great improvement of the CRRAM when modified to become the BFCRRAM demonstrates the constraining power of the theory of beliefs in the game theoretic model and suggests that generalization of the theory should focus on beliefs. Such an approach is precisely the same as that of GPV who identified the consistency conditions of game theory as its major source of error.

The next result addresses the validity of the assumption employed in the application of the CRRAM that the risk parameters of the subjects are distributed with a log-normal distribution. The result is that the estimated risk parameters $\{r_i\}$ are consistent with a log-normal distribution with mean $\bar{r} = \frac{1}{n}\sum_i r_i$ and variance $\frac{1}{n-1}\sum_i (r_i - \bar{r})^2$.

Result 9: The risk parameters estimated from the data in all the experiments are consistent with the hypothesis that they are log-normally distributed.

Support: The Kolomogorov-Smirnov test statistics are listed in Table 5. For risk parameters estimated from all experiments, we fail to reject the hypothesis that the risk parameters are drawn from a log-normal distribution at a five percent level of significance.□

The next result addresses a very interesting question. Is the distribution of risk parameters estimated from the data the same for all experiments? The subjects in these experiments are drawn independently from the same general population. It is not unreasonable to expect the measured risk attitudes in different experiments to be statistically similar. If they were not the same then, the possibility exists that either the sampling of subjects does not result from independent draws from the population or there is an error in CRRAM in a broad sense. Randomly drawn risk attitudes should show up as a random distribution of risk parameter estimates when measured by the application of CRRAM. If the subjects' risk parameters are drawn from the same distribution, it is not necessary for them to have the same beliefs of this distribution. Table 4 shows the means and the standard deviations of the estimated risk parameter r in all of the experiments. The result is that all of the subjects in most of the experiments behaved as if their risk parameters were drawn from the same distribution.

Result 10: The risk parameters estimated from the data in most of the six different experiments are consistent with the hypothesis that they are drawn from the same distribution.

Support: Kolomogorov-Smirnov test statistics are listed in Table 5. For 12 out of 15 possible pairings of different experiments, we fail to reject the hypothesis that risk parameter estimates are drawn from the same distribution at five percent significance.□

7 Summary and Conclusions

This research was motivated by the possibility that people may not exhibit the full degree of rationality that game theory assumes. In particular, two sets of principles of rationality are of interest. The first set consists of the principles of maximizing behavior. This set of principles is tested against the alternatives that people are following simple rules of thumb. We use a set of three linear decision rules (the MM, the SIMAM and the SOPAM, which is a series of increasingly sophisticated linear rules) as a benchmark of how much maximizing behavior the subjects are exhibiting. The second set of principles of rationality consists of those which generate the rational expectation hypothesis which states that, in equilibrium, all of the beliefs of all of the agents are consistent with experience/reality. We estimated the subjects' beliefs about the aggregate risk behavior in the unconstrained CRRAM and thus we were able to test whether the beliefs are consistent with the true aggregate risk behavior.

The first two results suggests that the principles of maximizing behavior should not be abandoned in favor of simple ad hoc decision rules in a given economic environment. The increasingly sophisticated linear rules of thumb are MM, SIMAM, and SOPAM. The CRRAM is better than the first two. The SOPAM was introduced to answer the question of how complicated can one get in the class of piece wise linear models before a rule is found that outperforms the CRRAM? Result 3 shows that the CRRAM does not perform as well as the SOPAM. This by no means suggests that people are using non-optimal rules of thumb disregarding the strategic situation. In fact, if the SOPAM is constrained to assume some constant characteristic parameters regardless of the environment (e.g., if the SOPAM is constrained to go through (0,0) regardless of what the offset is) then there is no doubt that the CRRAM will perform better than all three models. Thus, we know that game theory has limitations on its ability to predict but if it is to be improved upon by ad hoc models then those models must exhibit some sophistication and leave parameters free to vary from environment to environment.

Result 5 suggests that the relaxation of the rational expectation hypothesis is supported by the data. Subjects do not appear to have fully consistent beliefs about the true aggregate risk behavior of their fellow subjects. However, given beliefs, a subject's behavior is quite consistent with game theory.

Results 6, 7, and 8 show that with no constraint that requires subjects' beliefs to be the truth, the CRRAM performs better than the two simpler linear models (MM and SIMAM) while CRRAM still does not perform as well as the sophisticated piece wise linear model (SOPAM). The accuracy gap between the two models is very small and a real possibility exists that Result 8 might be sensitive to the statistical specifications of the model. That is, game theory without the full constraints of rational expectations is sufficiently accurate that the measurement technology might be a real factor in comparisons with the most sophisticated rules of thumb.

In summary, the results in this paper suggest that the fundamental conclusions of CSW, KHL and GPV are all correct. The simple markdown models do not account for behavior as well as does the CRRAM which is based on game theory. In spite of a long history of application in the social and economic sciences as alternatives to models based on strategic and rational behavior, neither the simple markdown model nor its natural generalization (SIMAM) are as good as the CRRAM. The CRRAM is reasonably accurate in a certain absolute sense (Result 4) and the estimates of the parameters have a type of internal consistency that might be expected (Result 9 and 10). However, while the support for optimal strategic behavior is strong, it appears that people do not exhibit the full extent of the kind of rationality that game theory assumes. The CRRAM is not as accurate as the SOPAM nor is the rational expectation hypothesis supported by the data (Result 3 and 5).

It stands as an challenge for theorists to improve game theory to account for the limited form of rationality that is observed in these experiments. The results reported here suggest that the theory of beliefs and belief formation might be the most productive place to work. The power of the theory is increased dramatically when the constraints that the theory places on beliefs is relaxed. In this sense, the conclusion of this study are precisely the same as GPV who concluded that the "consistency conditions" of game theory were the primary source of the theory's error.

Appendix A

General Information:

This is an experiment in the economics of market decision making. The instructions are simple and if you follow them carefully and make good decisions you might earn a considerable amount of money, which will be paid to you in cash after the experiment.

In this experiment, you are going to participate in a market in which you will be buying units in a sequence of independent market days or trading periods. You will each receive a sequence of numbers from the computer, one for each period, which describe the value to you of any decisions you might make. These numbers may differ among individuals. *You are not to reveal this information to anyone.* It is your own private information.

Redemption Values and Earnings:

During each market period you are free to purchase a unit if you want. If you purchase a unit, you will receive the redemption value indicated on the computer for that period. Your earnings from a unit purchased is the difference between your redemption value for that unit and the price you paid for the unit. The formula is:

Your earnings = (redemption value) - (purchase price).

Suppose, for example, that you buy a unit and that your redemption value is 200. If you pay 150 for the unit, then your earnings are

Earnings from unit = 200 - 150 = 50.

Notice that if the price paid is above the redemption value, the buyer experiences a loss. Anyone with a net loss at the end of the experiment is allowed to work to pay the loss at a rate of $6 per hour. The earnings will be calculated for you by the computer in each period. The currency used in the market is *francs*. Each *franc* will be worth _____ dollars to you.

Market Organization:

In each period, one or more markets will be open and you will be participating in one of the markets. The computer will determine which market you are participating in randomly. There will be 3 participants in each market. In each market, buyers submit bids by entering their bid into the computer when prompted. The bids will be arranged from the highest bid to the lowest. A single unit will be sold to the highest bidder. The highest bid and the bidder of each market will be announced by the computer. The buyer will pay a price equal to the bid and as a result will earn the difference between his/her redemption value for the unit and the bid. Ties are resolved randomly by the computer. The bids of all other bidders are nullified. They receive no redemption value and pay nothing, and so have earning of zero for that period.

Determination of Redemption Values:

For each buyer, the redemption value each period is determined randomly from the following distribution. The chance of having a value between 0 and 499 is 80% and each number from 0 to 499 has equal chance of appearing. The chance of having a value between 500 and 999 is 20% and each number from 500 to 999 has equal chance of appearing. It is as if each number between 0 and 499 is stamped on 4 balls and placed in an urn. And each number between 500 and 999 is stamped on 1 ball and placed in the same urn. A draw from the urn determines the redemption value for an individual. The ball is replaced and a second draw determines the redemption value for another player. The redemption value each period is determined the same way. The following is a table of which the probability of getting a value in a certain range is listed: (It is for your reference)

Range of Redemption value	Probability of a value in this range
0-49	8%
0-99	16%
0-149	24%
0-199	32%
0-249	40%
0-299	48%
0-349	56%
0-399	64%
0-449	72%
0-499	80%
0-549	82%
0-599	84%
0-649	86%
0-699	88%
0-749	90%
0-799	92%
0-849	94%
0-899	96%
0-949	98%
0-999	100%

Appendix B

To Prove:

$$\frac{db(v;r_i)}{dv}\Big|_{v\to 0} = \frac{N-1}{N-1+r}$$

Proof:

$$b(0;r_i) = 0$$

This implies

$$\lim_{v\to 0} b(0;r_i) = 0$$

since $b(v;r)$ is a continuous function

$$\lim_{v\to 0} u_1 = \lim_{v\to 0} \frac{r_i\left(\frac{1}{2}-b\right)}{v-b} = \infty$$

$$\lim_{v\to 0} u_2 = \lim_{v\to 0} \frac{r_i(1-b)}{v-b} = \infty$$

substitute into W_2 and W_3 we have

$$\begin{aligned}
\lim_{v\to 0} W_2 &= \lim_{v\to 0}\ 2aG(u_1) + 2(1-a)\left[G(u_2) - G(u_1)\right] \\
&= 2a + 2(1-a)(1-1) \\
&= 2a \qquad\qquad \text{since } \lim_{x\to\infty} G(\chi) = 1 \\
\lim_{v\to 0} W_3 &= \lim_{v\to 0}\ (2a-1)\left[G(u_2) - G(u_1)\right] + 1 - G(u_2) \\
&= (2a-1)(1-1) + 1 - 1 \\
&= 0
\end{aligned}$$

put into equation (4) and let $\lim_{v\to 0} \frac{db(v;r_i)}{dv} = x$, we have

$$\begin{aligned}
x &= \frac{(N-1)W_1}{(N-1+r)W_1 - 2ar + \frac{2ar^2 b}{(v-b)(N-1)}} \\
&= \frac{(N-1)W_1}{(N-1+r)W_1 - 2ar + \frac{2ar^2}{\left(\frac{1}{x}-1\right)(N-1)}}
\end{aligned}$$

since

$$\lim_{v\to 0}\frac{b}{v}=\frac{db}{dv}\Big|_{v=0}$$

solving for x, we have

$$x=\frac{N-1}{N-1+r}$$

therefore

$$\lim_{v\to 0}\frac{db(v;r_i)}{dv}=\frac{N-1}{N-1+r} \qquad \text{QE.D.}$$

Appendix C

To prove If $b(v;r)$ is a solution to $\max_x (v-x)^r\{E_r H(\pi(x;r))\}^{n-1}$

then $\tilde{b}(\tilde{v},r)$ is a solution to $\max_x (\tilde{v}-x)^r\{E_r \tilde{H}(\pi(x;r))\}^{n-1}$

where for all $\alpha \in \Re, \beta \in \Re^+$

$$\tilde{v} = \alpha + \beta v$$

$$\tilde{b}(\tilde{v};r) = \alpha + \beta b(v;r)$$

$$\tilde{H}(\alpha + \beta y) = H(y) \quad \text{for all } y \in \Re^+$$

<u>Proof</u>

Let

$$Eu(x) = (v-x)^r\left\{E_r H(\pi(x;r))\right\}^{n-1}$$

$$E\tilde{u}(x) = (\tilde{v}-x)^r\left\{Er\tilde{H}(\pi(x;r))\right\}^{n-1}$$

since b is solution to $\max_x Eu(x)$ for fix v

$$Eu(b) \geq Eu(x) \text{ for all } x \in \Re \tag{1}$$

now

$$E\tilde{u}(\tilde{b}) = (\tilde{v}-\tilde{b})^r\left\{E_r\tilde{H}(\tilde{\pi}(\tilde{b};r))\right\}^{n-1}$$

$$= \beta^r(v-b)^r\left\{E_r\tilde{H}(\alpha+\beta\tilde{\pi}(b;r))\right\}^{n-1} \text{ since } \tilde{v}-\tilde{b} = \alpha+\beta v-(\alpha+\beta b) = \beta(v-b)$$

$$= \beta^r(v-b)^r\left\{E_r H(\pi(b;r))\right\}^{n-1} \qquad \text{and let } \tilde{\pi}(\tilde{b};r) = \alpha+\beta\pi(b;r)$$

$$= \beta^r Eu(b)$$

similarly $E\tilde{u}(\alpha+\beta x) = \beta^r Eu(x)$ for all $x \in \Re$ (1)implies

$$\beta^r E\tilde{u}(\tilde{b}) \geq \beta^r E\tilde{u}(\alpha+\beta x)$$

$$\geq \beta^r E\tilde{u}(y) \text{ for all } y$$

$$\Rightarrow E\tilde{u}(\tilde{b}) \geq E\tilde{u}(y) \textit{ since } \beta > 0$$

$$\Rightarrow \tilde{b} \textit{ is a solution to } \max_x Eu(x)$$

Table 1: Experimental Parameters

Experiment	No. of Subjects	No. in Each Auction	No. of Periods	a	T = Offset	Subject Pool*	Franc/Dollar Conversion Rate	Date
1	12	3	60	0.8	0	Caltech	0.01	11/6/88
2	12	3	120	0.2	0	Caltech	0.01	1/30/89
3	12	3	70	0.8	500	Caltech	0.01	5/01/89
4	12	3	100	0.2	500	Caltech	0.01	5/02/89
5	12	3	100	0.8	0	PCC	0.01	5/03/89
6	12	3	100	0.8	500	PCC/Caltech	0.01	5/05/89

*The subjects are either Caltech or PCC (Pasadena City College) students.

Table 2A: Maximum Log Likelihood Statistics

	Experiment						
	1	2	3	4	5	6	pooled
Model Estimates							
L_{CRRAM}	-3392	-7736	-4599	-6818	-6414	-6598	-35557
L_{MM}	-3757	-7780	-4979	-6880	-6428	-6982	-36806
L_{SIMAM}	-3641	-7682	-4731	-6792	-6380	-6677	-35903
L_{SOPAM}	-3190	-7512	-4504	-6706	-6289	-6516	-34717
Log Likelihood Ratio Estimates							
LR_{MM}^{CRRAM}	365	44.2	380	62.3	13.9	384	1143
LR_{SIMAM}^{CRRAM}	248	-54.0	132	-25.9	-34	79.0	345
LR_{SOPAM}^{CRRAM}	-203	-224	-95.2	-112	-125	-81.6	-840.8
Log Likelihood Ratio Standard Deviation							
$n^{\frac{1}{2}}\hat{W}_{MM}^{CRRAM}$	26.5	12.6	37.5	12.2	28.2	38.4	27.0
$n^{\frac{1}{2}}\hat{W}_{SIMAM}^{CRRAM}$	23.8	16.6	20.7	13.6	25.0	24.3	20.0
$n^{\frac{1}{2}}\hat{W}_{SOPAM}^{CRRAM}$	32.5	22.5	16.2	20.1	17.8	14.6	20.7
Test Statistics CRRAM vs. others							
$\frac{LR_{MM}^{CRRAM}}{n^{\frac{1}{2}}\hat{W}_{MM}^{CRRAM}}$	13.8[a]	3.49[a]	10.1[a]	5.08[a]	0.494	10.0[a]	46.3[a]
$\frac{LR_{SIMAM}^{CRRAM}}{n^{\frac{1}{2}}\hat{W}_{SIMAM}^{CRRAM}}$	10.4[a]	-3.26[b]	6.39[a]	-1.91[b]	-1.37	3.24[a]	17.3[a]
$\frac{LR_{SOPAM}^{CRRAM}}{n^{\frac{1}{2}}\hat{W}_{SOPAM}^{CRRAM}}$	-6.16[b]	-10.3[b]	-5.87[b]	-5.56[b]	-7.03[b]	-5.58[b]	-40.6[b]
Test Statistics CRRAM vs. others with correction* $K_n = \frac{p}{2}\log(n) - \frac{q}{2}\log(n)$							
$\frac{LR_{SIMAM}^{CRRAM}}{n^{\frac{1}{2}}\hat{W}_{SIMAM}^{CRRAM}}$	12.2[a]	-0.63	8.34[a]	-1.65	0.35	4.99[a]	19.9[a]
$\frac{LR_{SOPAM}^{CRRAM}}{n^{\frac{1}{2}}\hat{W}_{SOPAM}^{CRRAM}}$	-3.72[b]	-6.28[b]	-0.86	-5.21[b]	-2.22[b]	0.24	-18.7[b]

Table 2B: Maximum Log Likelihood Statistics

	Experiment						
	1	2	3	4	5	6	pooled
Model Estimates							
L_{CRRAM}	-3392	-7736	-4599	-6818	-6414	-6598	-35557
L_{RNNE}	-5343	-8165	-6236	-7052	-8966	-8925	-44687
L_{RASNE}	-3975	-7807	-5034	-6926	-6848	-7023	-37613
Log likelihood of CRRAM w/o rational expectation hypothesis $L_{CRRAM\ w/o\ R.E.H.}$	-3254	-7390	-4563	-6717	-6383	-6542	-34849
Likelihood Ratio Test Statistics							
$2(L_{CRRAM} - L_{RNNE})$	3902	858	3214	468	5104	4654	18200
$2(L_{CRRAM} - L_{RASNE})$	1170	142	870	216	868	850	4116
$2(L_{CRRAM\ w/o\ R.E.H} - L_{CRRAM})$	1442	834	942	418	930	962	5228
p-value							
CRRAM vs. RNNE	1.00+	1.00+	1.00+	1.00+	1.00+	1.00+	1.00+
CRRAM vs. RASNE	1.00+	1.00+	1.00+	1.00+	1.00+	1.00+	1.00+
CRRAM w/o R.E.H. vs. CRRAM	1.00+	1.00+	1.00+	1.00+	1.00+	1.00+	1.00+

[a] Denotes the CRRAM is a better model than the other.

[b] Denotes the CRRAM is a worse model than the other.

* Since the MM and the CRRAM have the same number of parameters, no correction is needed. Correction for other models are: for the SIMAM, q=24; for the SOPAM, q=36; for the CRRAM, p=12.

+ Indicates the model can be rejected at 5 percent significance.

Table 2C: Maximum Log Likelihood Statistics

	Experiment						
	1	2	3	4	5	6	pooled
Model Estimates							
$L_{BFCRRAM}$	-3254	-7390	-4563	-6717	-6383	-6542	-34849
L_{MM}	-3757	-7780	-4979	-6880	-6428	-6982	-36806
L_{SIMAM}	-3641	-7682	-4731	-6792	-6380	-6677	-35903
L_{SOPAM}	-3190	-7512	-4504	-6706	-6289	-6516	-34717
Log Likelihood Ratio Estimates							
$LR^{BFCRRAM}_{MM}$	503	390	416	163	45	440	1957
$LR^{BFCRRAM}_{SIMAM}$	387	292	168	75	3	135	1054
$LR^{BFCRRAM}_{SOPAM}$	-64	122	-59	-11	-94	-26	-132
Log Likelihood Ratio Standard Deviation							
$n^{\frac{1}{2}}\hat{W}^{BFCRRAM}_{MM}$	34.8	34.3	50.7	23.8	82.5	76.4	56.0
$n^{\frac{1}{2}}\hat{W}^{BFCRRAM}_{SIMAM}$	31.6	32.1	43.1	21.8	81.2	67.8	51.9
$n^{\frac{1}{2}}\hat{W}^{BFCRRAM}_{SOPAM}$	30.1	24.0	41.0	21.9	78.3	63.3	48.7
Test Statistics CRRAM vs. others							
$\frac{LR^{BFCRRAM}_{MM}}{n^{\frac{1}{2}}\hat{W}^{BFCRRAM}_{MM}}$	14.5[a]	11.4[a]	8.21[a]	6.85[a]	0.55	5.76[a]	34.9[a]
$\frac{LR^{BFCRRAM}_{SIMAM}}{n^{\frac{1}{2}}\hat{W}^{BFCRRAM}_{SIMAM}}$	12.2[a]	9.10[a]	3.90[a]	3.44[a]	0.04	1.99[a]	20.3[a]
$\frac{LR^{BFCRRAM}_{SOPAM}}{n^{\frac{1}{2}}\hat{W}^{BFCRRAM}_{SOPAM}}$	-2.13[b]	5.08[a]	-1.44	-0.50	-1.20	-0.41	-2.71[b]

[a] Denotes the CRRAM is a better model than the other.
[b] Denotes the CRRAM is a worse model than the other.

Table 3: Pseudo R^2

	Experiment					
	1	2	3	4	5	6
CRRAM						
Mean of Pseudo R^2	0.97	0.90	0.83	0.89	0.91	0.96
Standard deviation of Pseudo R^2	0.02	0.10	0.02	0.05	0.09	0.12
MM						
Mean of Pseudo R^2	0.92	0.91	0.66	0.89	0.90	0.74
Standard deviation of Pseudo R^2	0.05	0.09	0.24	0.05	0.08	0.22
SIMAM						
Mean of Pseudo R^2	0.94	0.92	0.80	0.90	0.91	0.85
Standard deviation of Pseudo R^2	0.08	0.07	0.19	0.05	0.03	0.11
SOPAM						
Mean of Pseudo R^2	0.97	0.93	0.84	0.91	0.92	0.87
Standard deviation of Pseudo R^2	0.02	0.07	0.21	0.05	0.07	0.11

Table 4: Estimation of Risk Aversion Parameter (r)

	Experiment					
	1	2	3	4	5	6
Mean of r	0.476	0.528	0.713	0.654	0.350	0.423
Standard deviation of r	0.187	0.119	0.596	0.291	0.182	0.251

Only the means and standard deviation across subjects are reported.

Table 5: Kolomogorov-Smirnov (K-S) Test Statistics of Risk Parameter (*r*) Estimates

Experiment		Experiment 1	2	3	4	5	6
1	K-S Stats	-	0.17	0.25	0.42	0.50	0.42
	p-value		1.00*	0.85*	0.25*	0.10*	0.25*
2	K-S Stats	0.17	-	0.25	0.33	0.58	0.50
	p-value	1.00*		0.85*	0.52*	0.03	0.10*
3	K-S Stats	0.25	0.25	-	0.33	0.58	0.42
	p-value	0.85*	0.85*	-	0.52*	0.03	0.25*
4	K-S Stats	0.42	0.33	0.33	-	0.58	0.50
	p-value	0.25*	0.52*	0.52*		0.03	0.10*
5	K-S Stats	0.50	0.58	0.58	0.58	-	0.33
	p-value	0.10*	0.03	0.03	0.03		0.52*
6	K-S Stats	0.42	0.50	0.42	0.50	0.33	-
	p-value	0.25*	0.10*	0.25*	0.10*	0.52*	

	K-S Statistics	p-value
All experiments vs. log-normal distribution	0.14	0.13+

* Indicates failure to reject the hypothesis that the risk parameters estimates are drawn from the same distribution in both experiments at 5 percent significance.

+ Indicates failure to reject the hypothesis that the risk parameters estimates are drawn from a log-normal distribution at 5 percent significance.

Figure 1 : MM and SIMAM

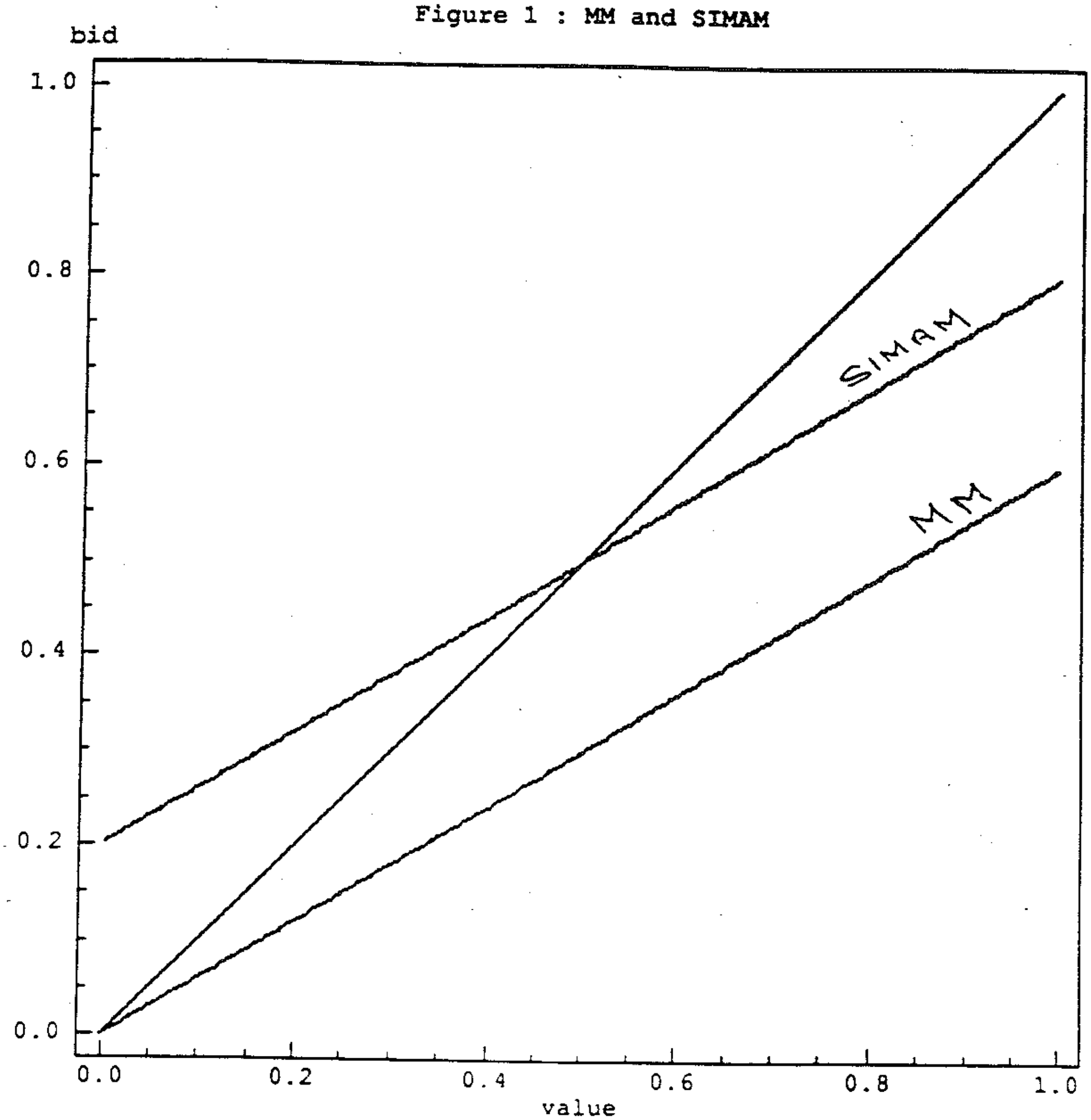


Figure 2 : Sophisticated Ad hoc Model (SOPAM)

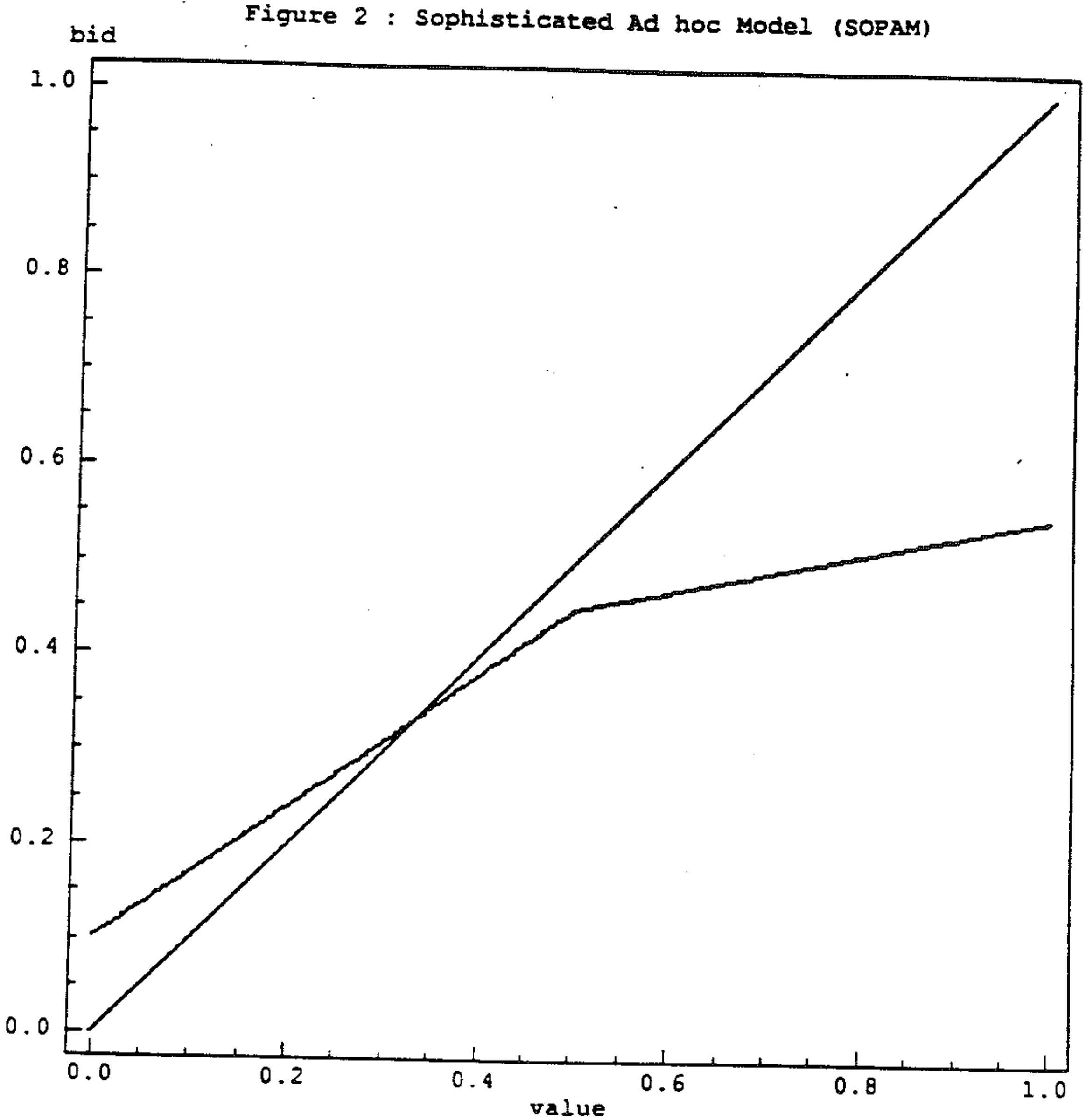


Figure 3A : RANE & RNNE model with Nonuniform Distribution a=0.8

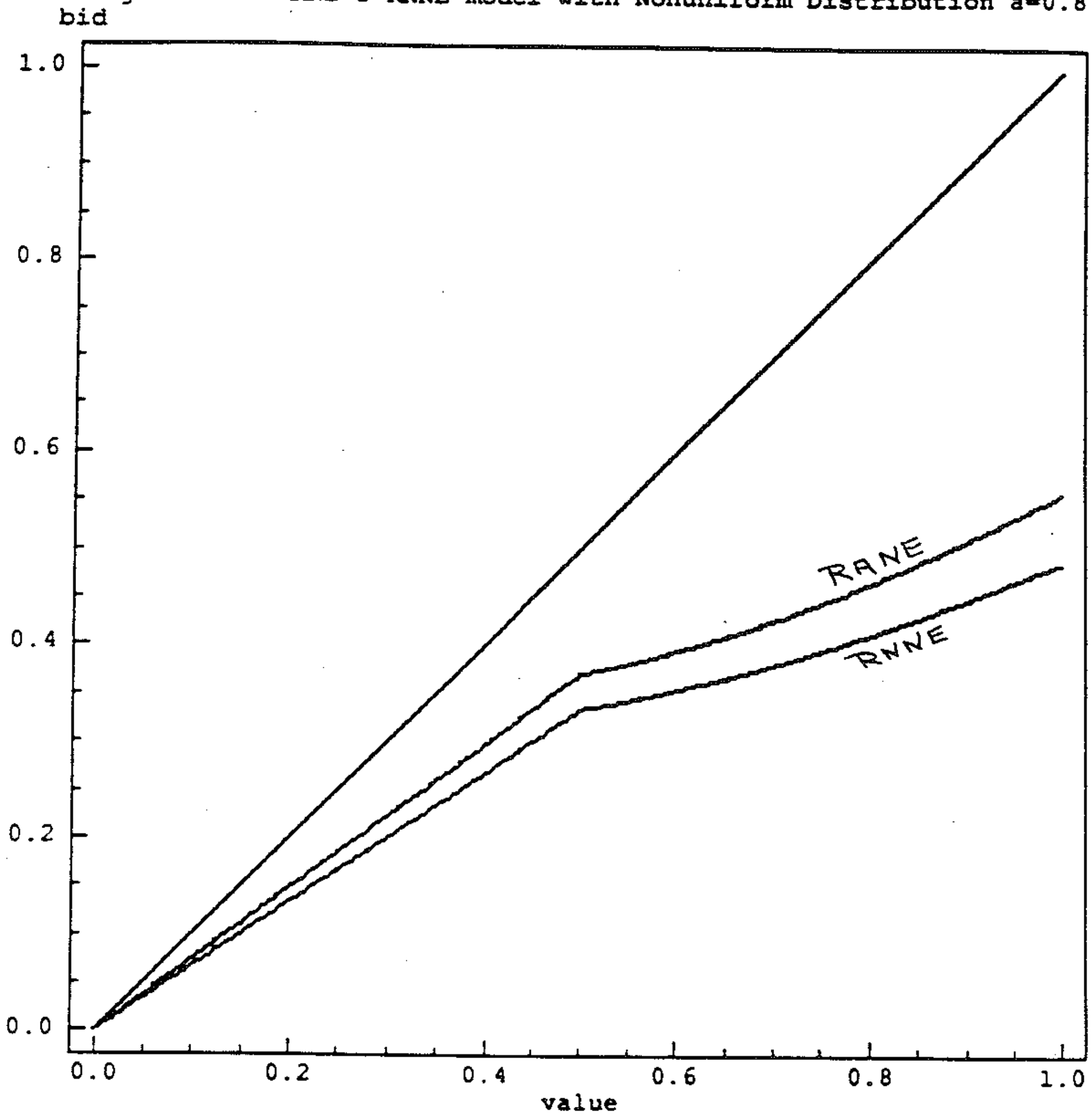


Figure 3B : RANE & RNNE model with Nonuniform Distribution a=0.2

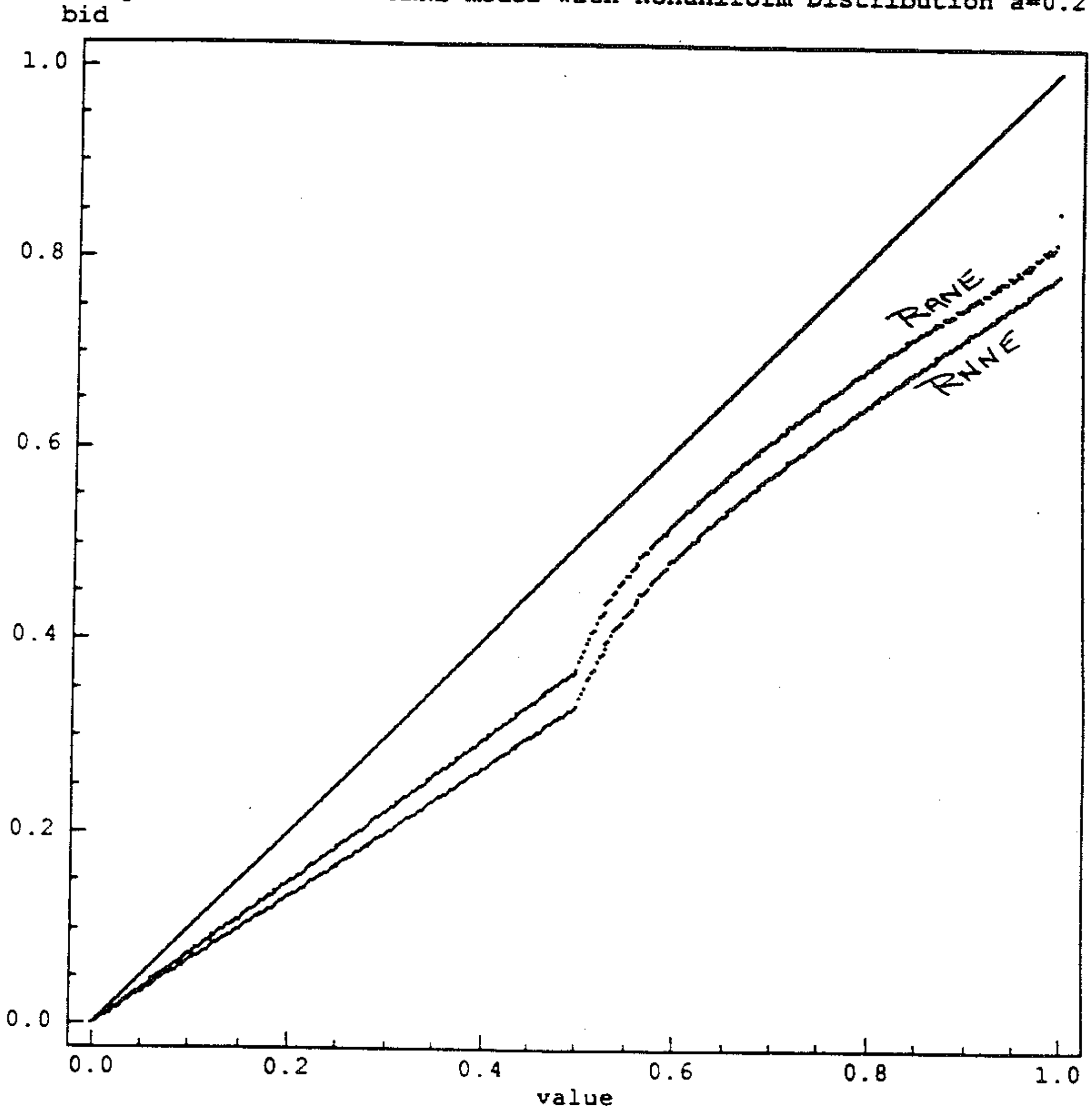


Figure 4A : CRRAM with a=0.8, E(r)=0.7 sr2=0.1

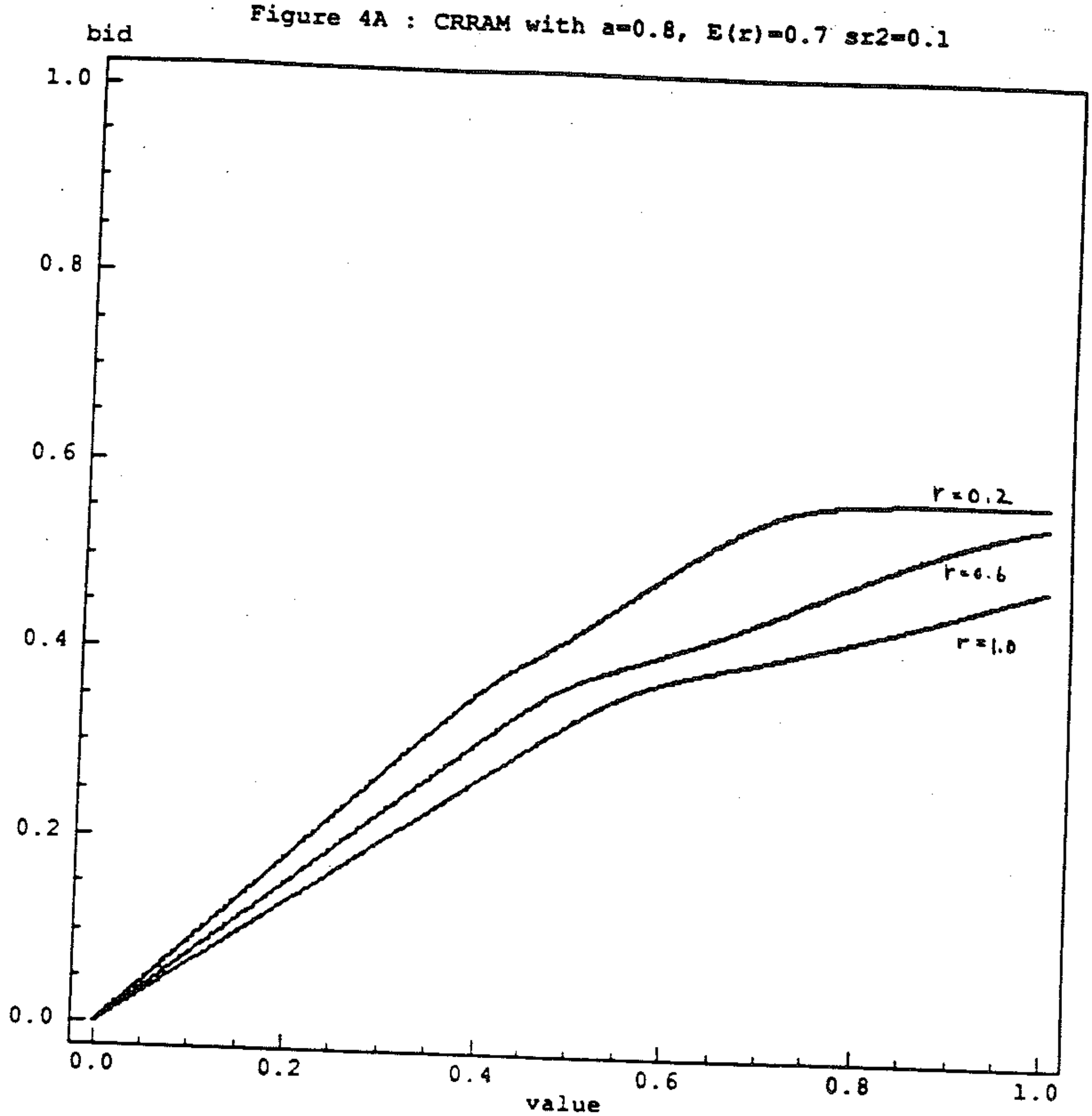


Figure 4B : CRRAM with a=0.8, r=0.7 sr2=0.1

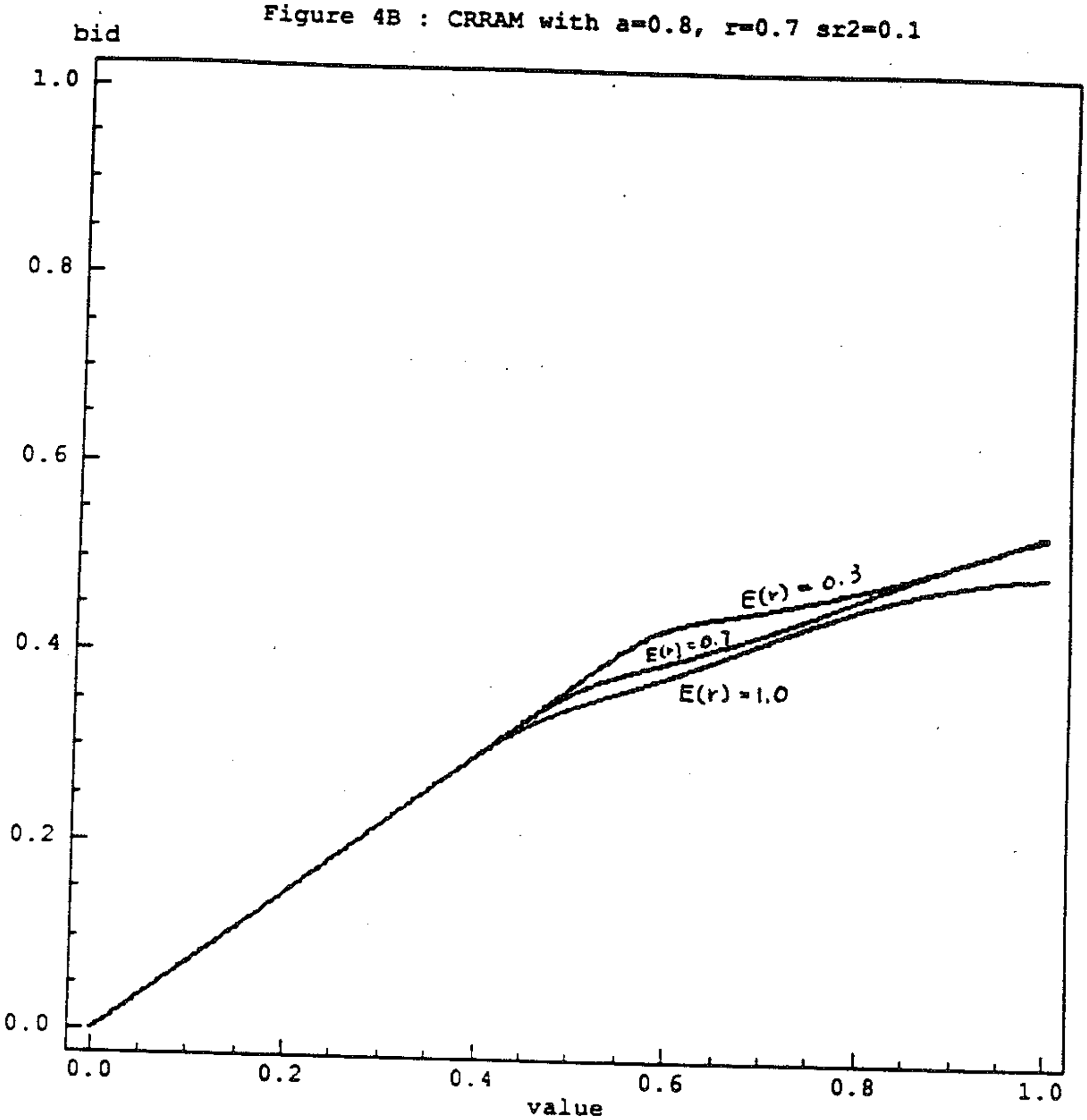


Figure 4C : CRRAM with a=0.8, r=0.7, E(r)=0.7

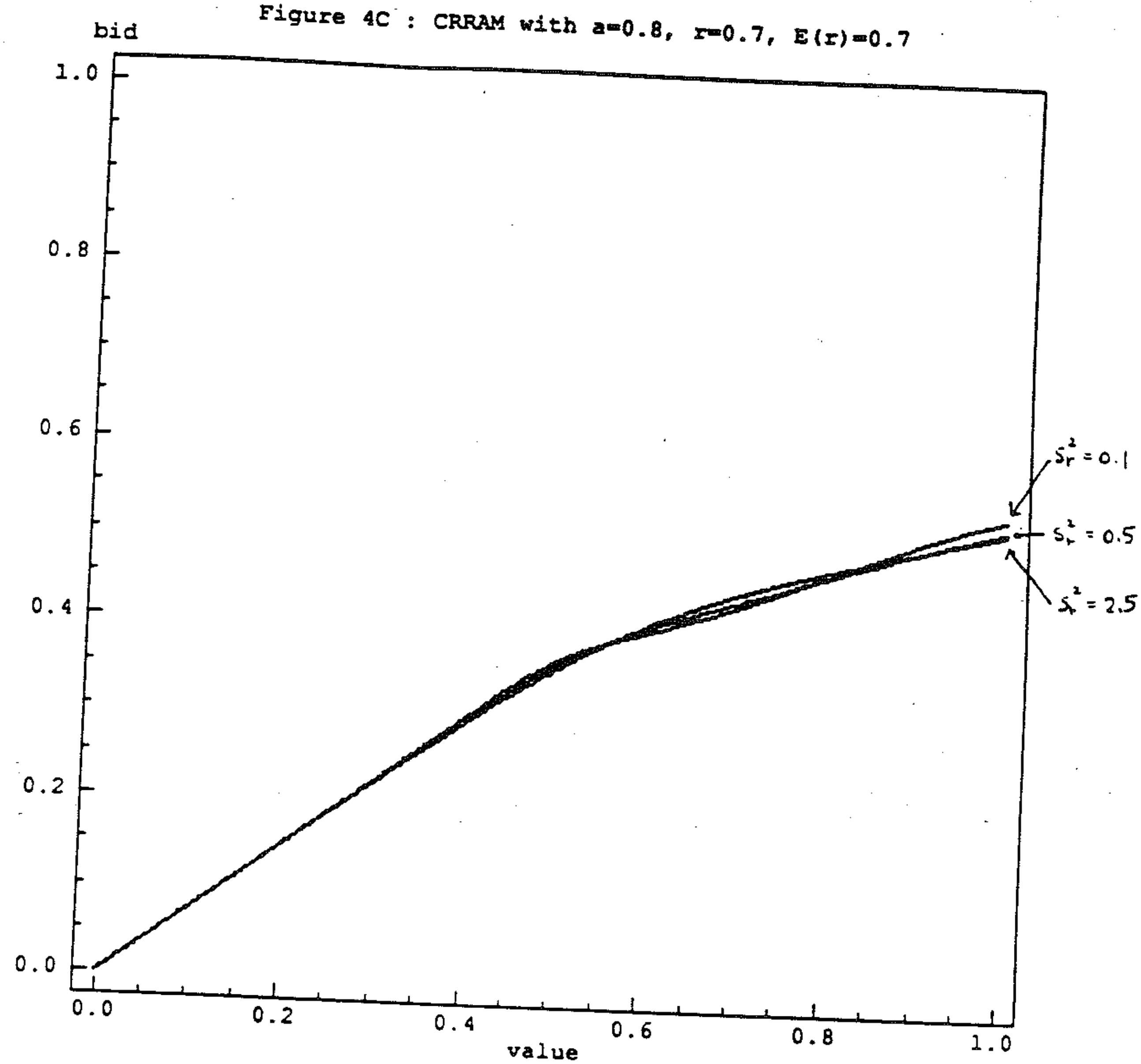


Figure 4D : CRRAM with a=0.2, E(r)=0.7 sr2=0.1

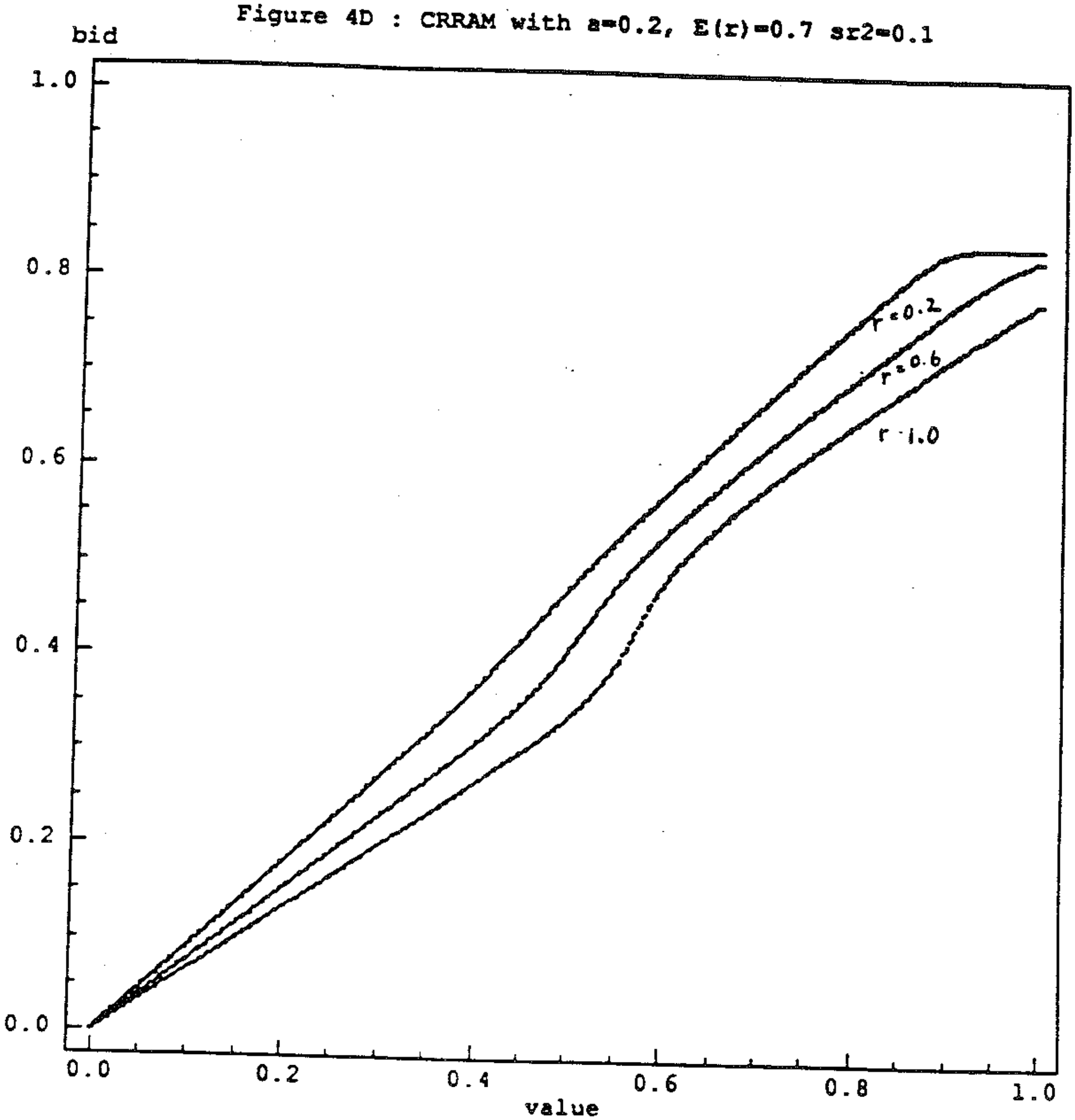


Figure 4E : CRRAM with a=0.2, r=0.7 sr2=0.1

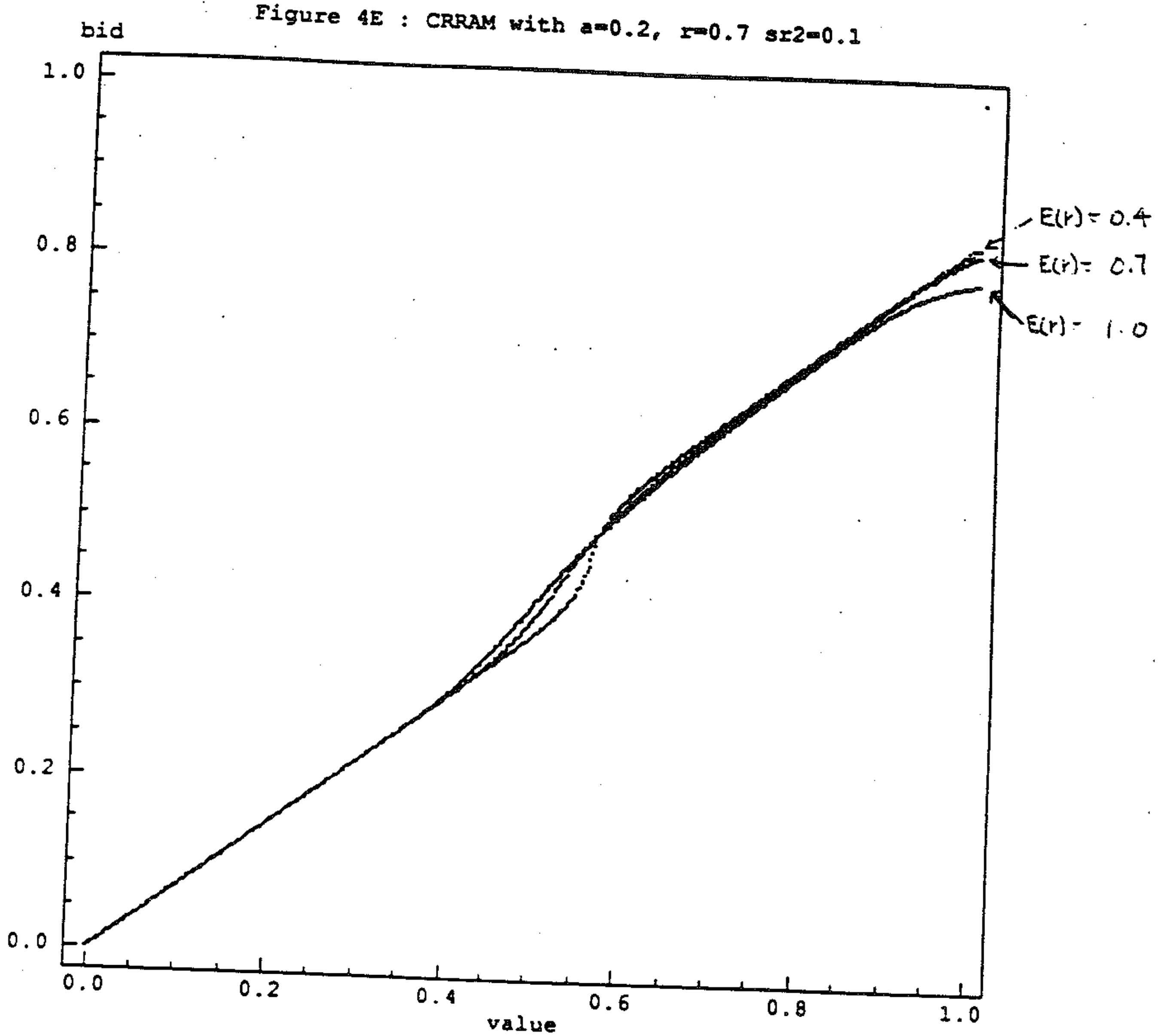
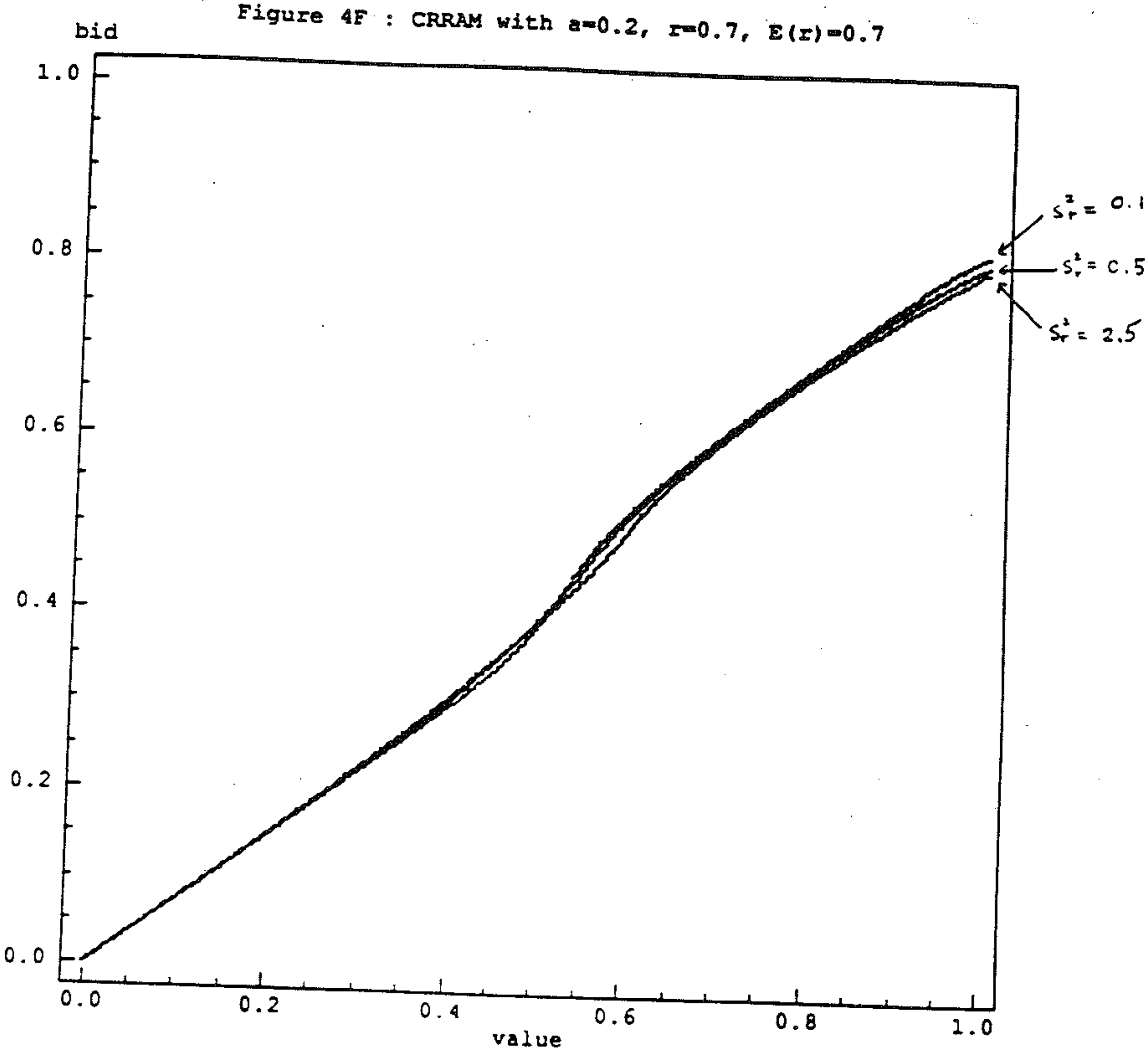
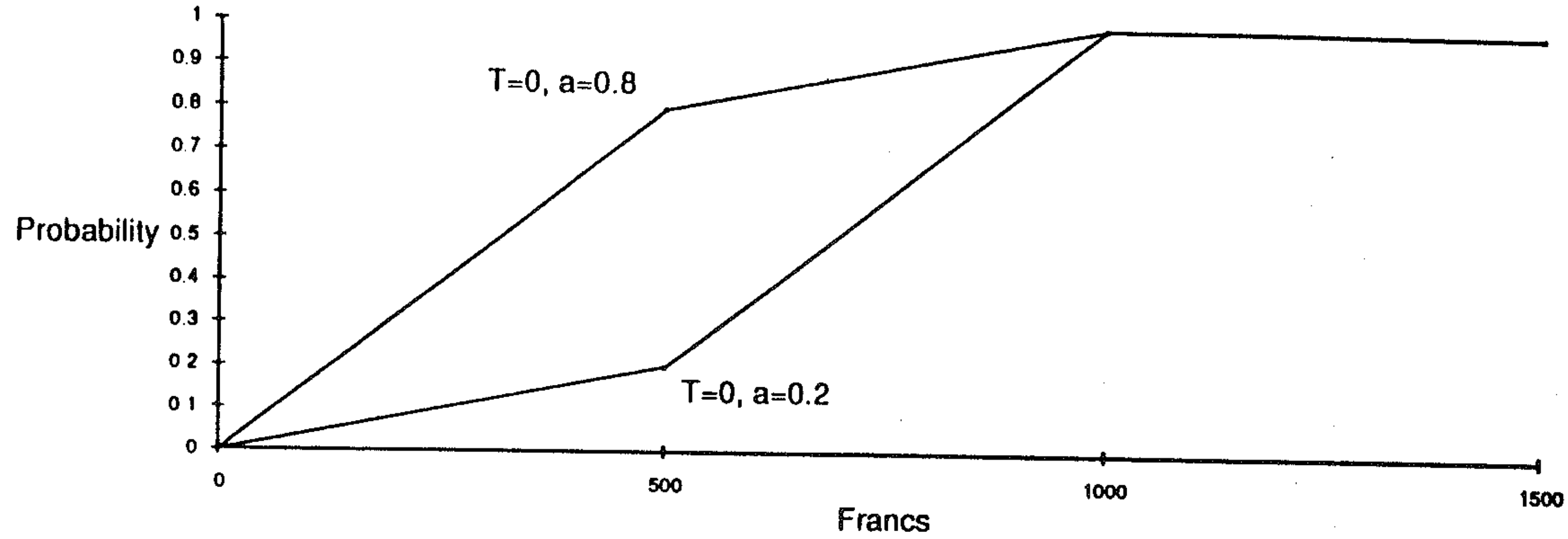
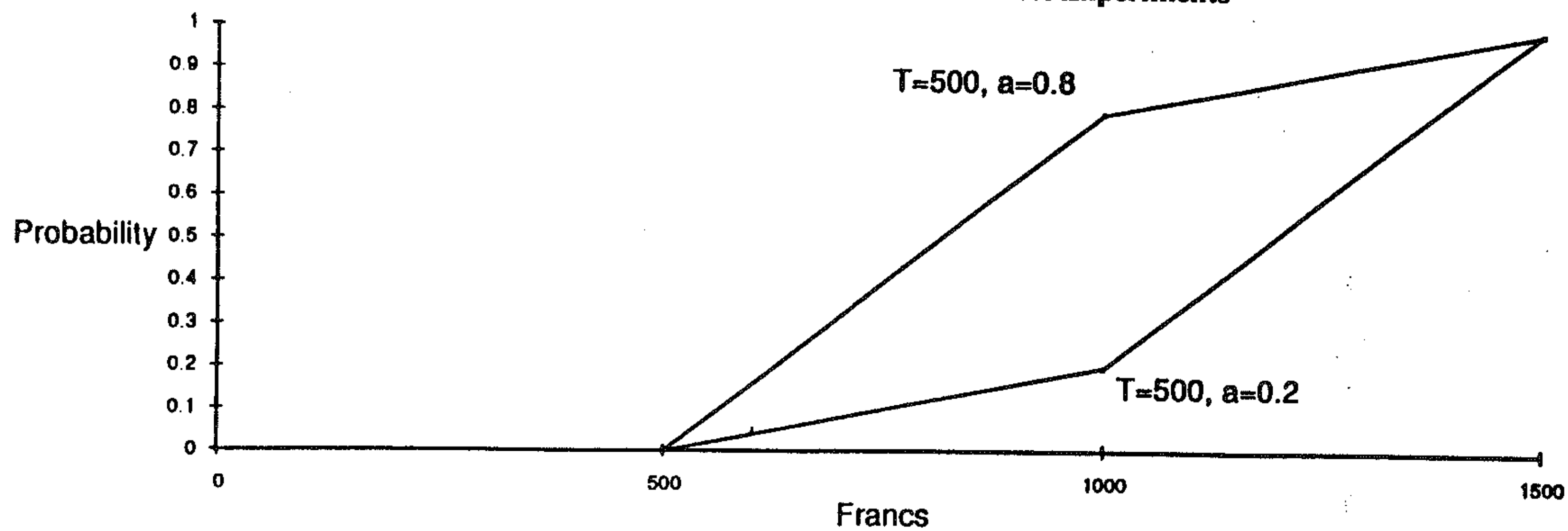


Figure 4F : CRRAM with a=0.2, r=0.7, E(r)=0.7





5.A Distribution Function for 0 Offset Experiments



5.B Distribution Function for 500 Offset Experiments

Figure 6A : Estimation Of Bidding Function (Exp 3 Subject 4)

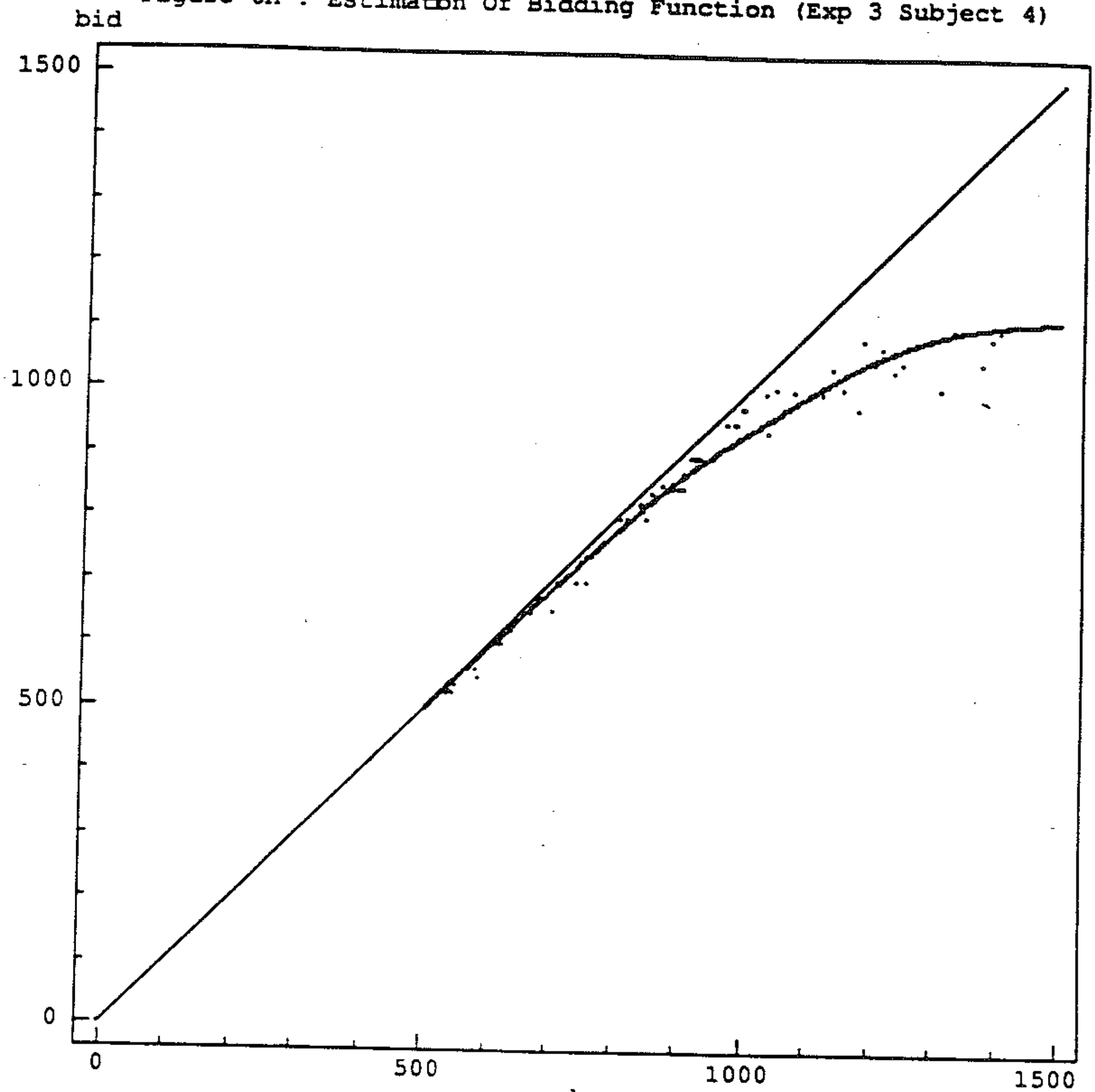


Figure 6B : Estimation Of Bidding Function (Exp 5 Subject 4)

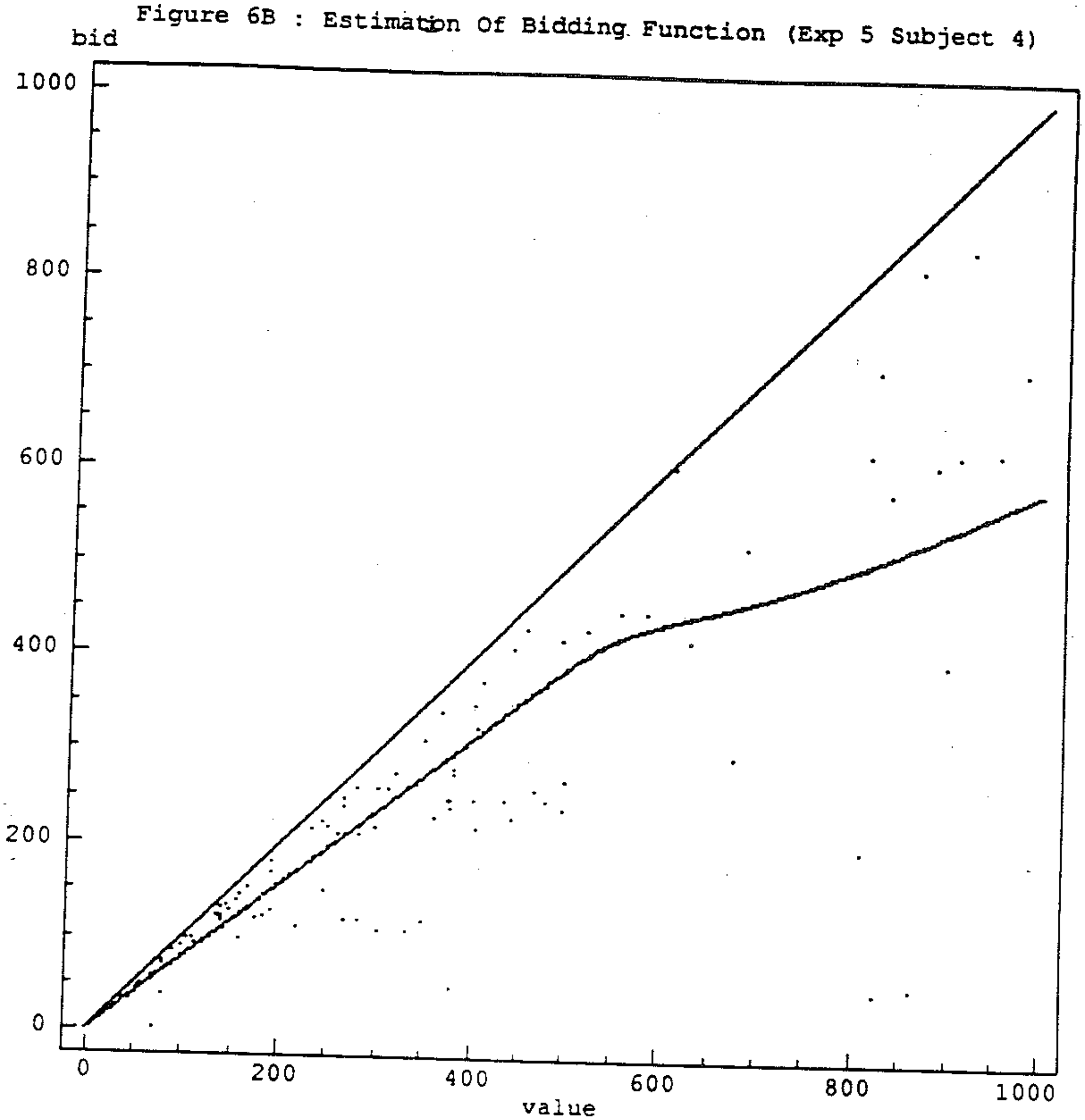
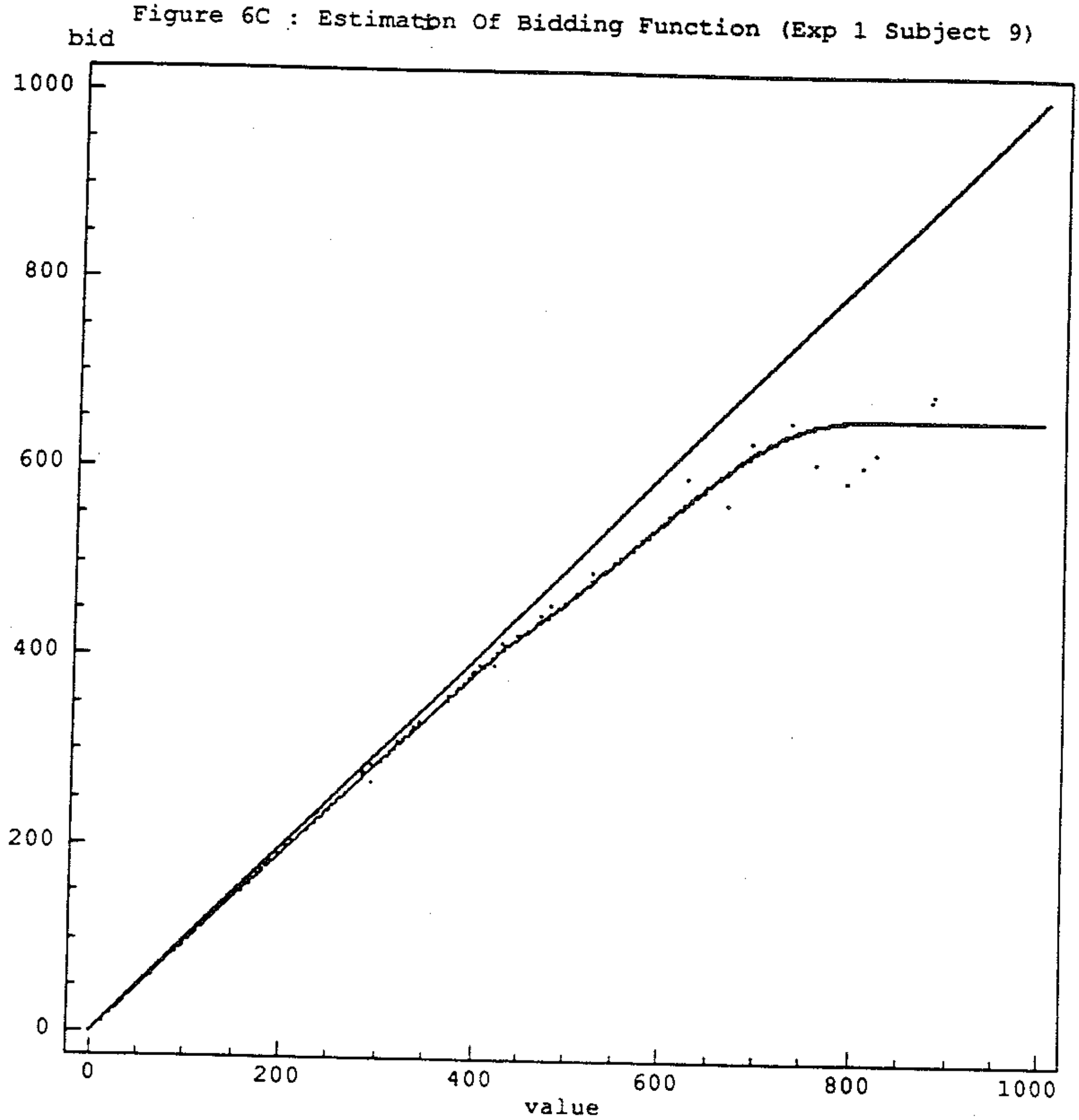


Figure 6C : Estimation Of Bidding Function (Exp 1 Subject 9)



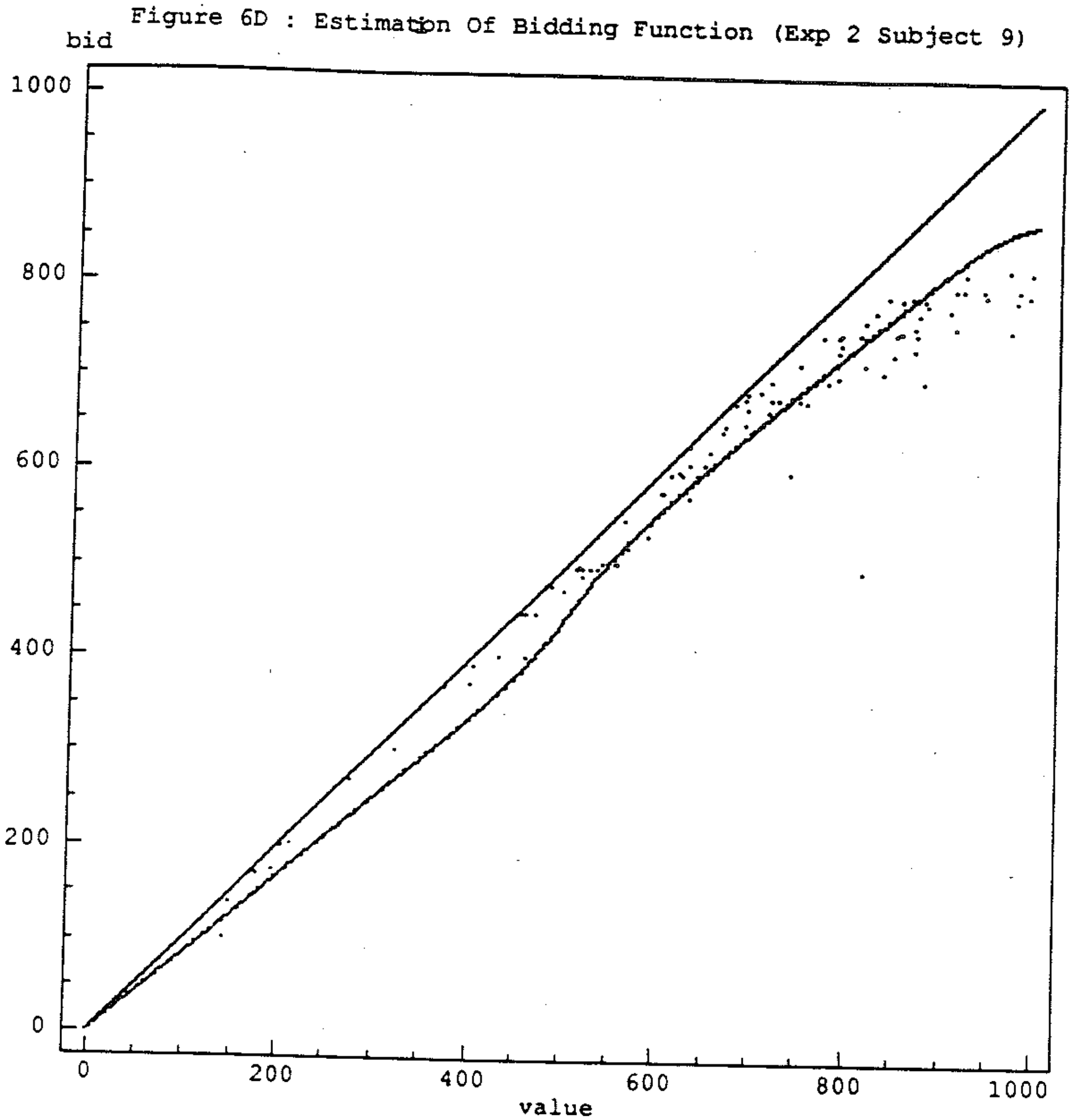


Figure 6D : Estimation Of Bidding Function (Exp 2 Subject 9)
bid
1000
800
600
400
200
0
0
200
400
600
800
1000
value

Figure 6E : Estimation Of Bidding Function (Exp 3 Subject 3)

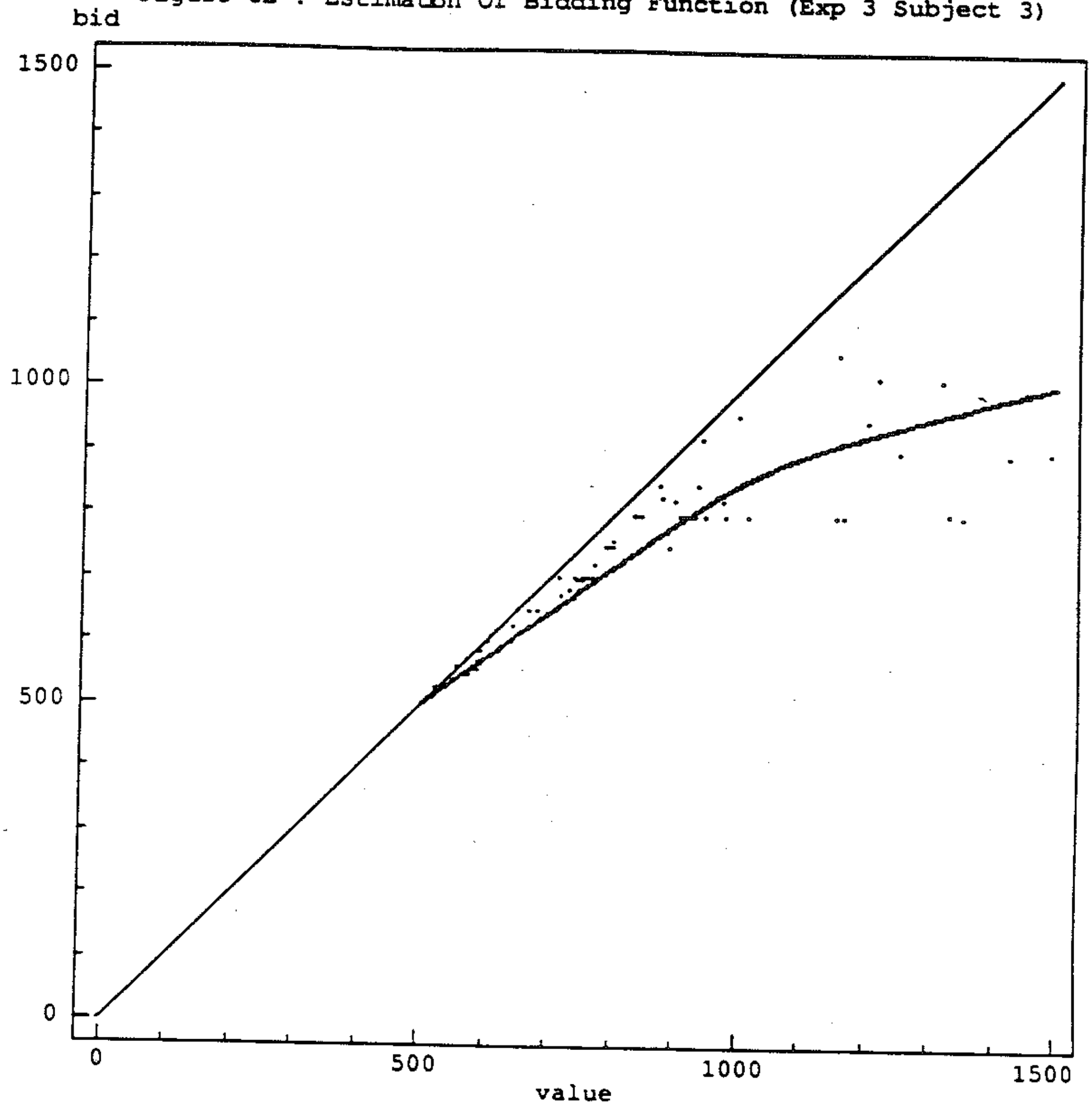


Figure 6F : Estimation Of Bidding Function (Exp 4 Subject 4)

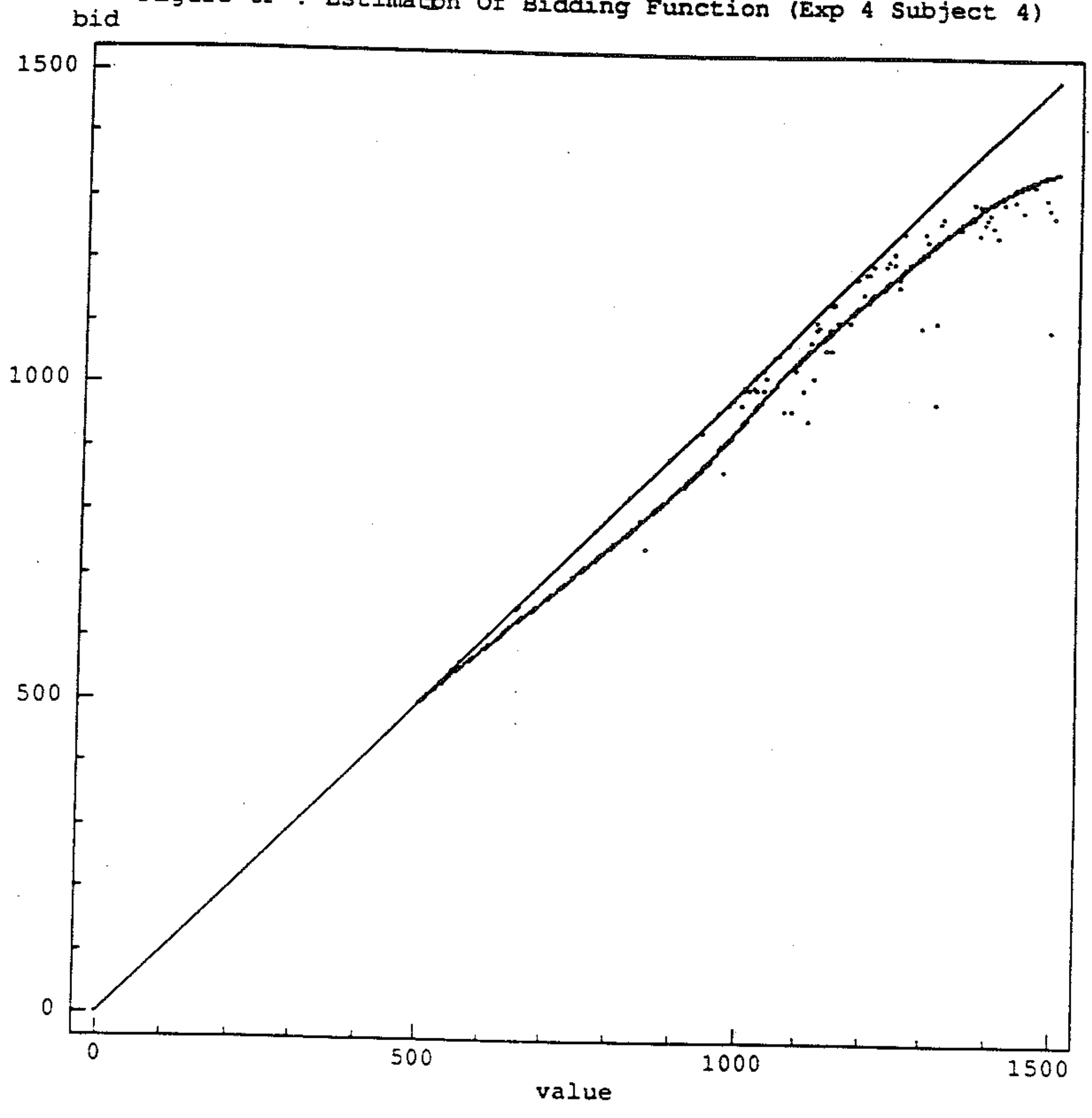
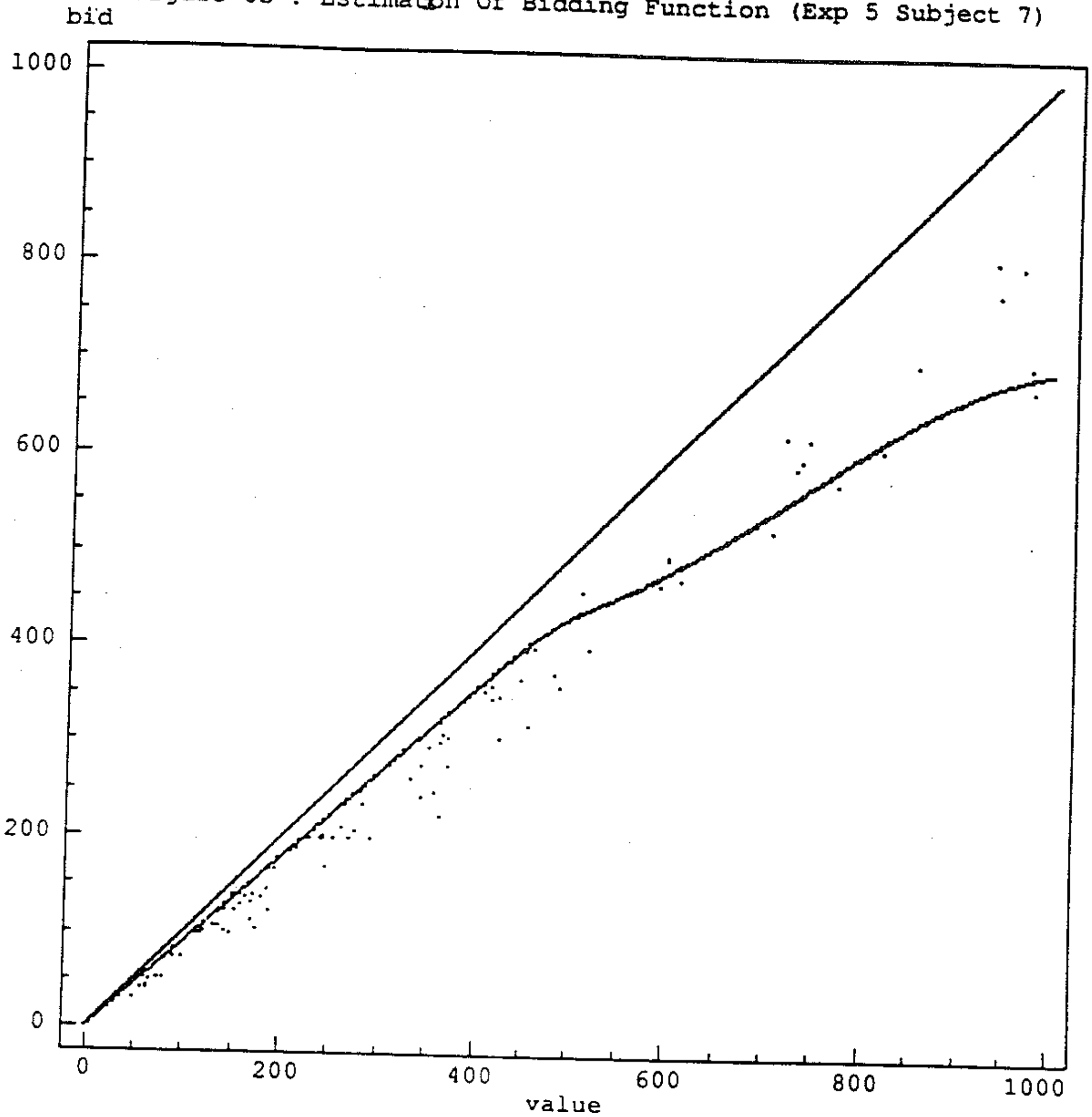


Figure 6G : Estimation Of Bidding Function (Exp 5 Subject 7)



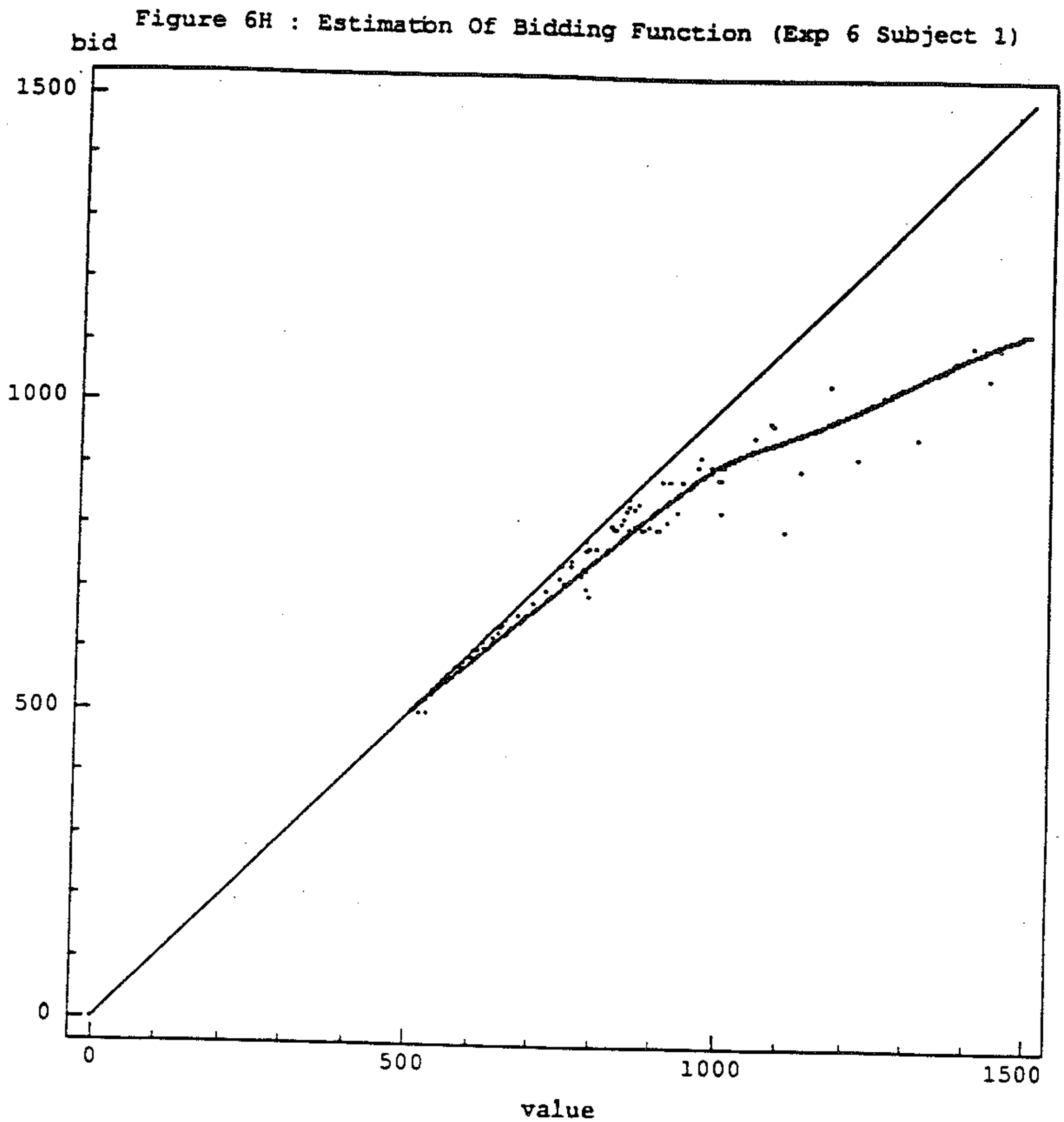
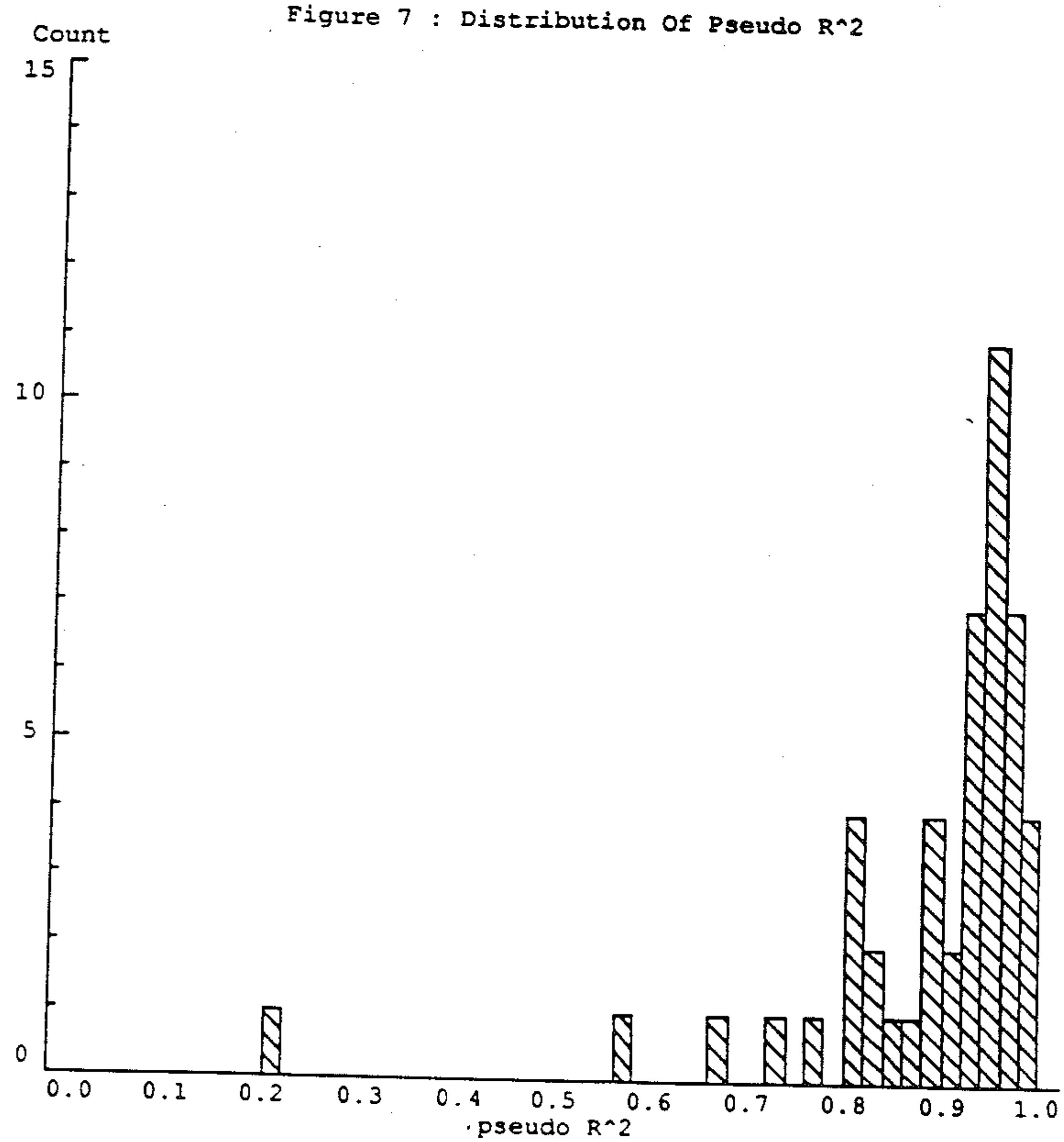


Figure 6H : Estimation Of Bidding Function (Exp 6 Subject 1)
bid
1500
1000
500
0
0
500
1000
1500
value

Figure 7 : Distribution Of Pseudo R^2



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