DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

INITIAL VERSUS CONTINUING PROPOSAL POWER IN LEGISLATIVE SENIORITY SYSTEMS

Richard D. McKelvey
California Institute of Technology

Raymond Riezman
University of Iowa

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Richard D. McKelvey * Raymond Riezman

ABSTRACT

We compare two different seniority systems in a legislature whose sole task is to decide on distributive issues, and which operates under a Baron-Ferejohn recognition rule, where recognition probability is based on seniority. In the first system, called "initial proposal power", recognition probability for the initial proposal is based on seniority, but once the proposal is voted on by the legislature, all members have equal recognition probabilities for any reconsideration. Under the second system, called "continuing proposal power," seniority is used to determine proposal power both in the initial consideration and in any reconsideration. We find that in the case of seniority systems embodying continuing proposal power, there does not exist an equilibrium in which incumbents are reelected, and in which legislators would endogenously choose to impose a such a seniority system on themselves. This contrasts with previous results in which we have shown that there does exist such an equilibrium for the case of initial proposal power. The reason for this result is that continuing proposal power lowers the value of senior members, since it makes them less desirable as coalition partners.

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1. INTRODUCTION

In this paper we compare two different systems of legislative seniority. In the first, senior legislators are given disproportionate power to make initial proposals, but once the proposal is brought before the legislative body, then the senior members lose their power. In the second system, senior legislators have disproportionate proposal power throughout the legislative session. While it might seem at first glance that the latter system would give more power to senior members, we show that in any equilibrium that sustains an incumbency effect and a seniority system, that senior members actually have less power under the second system than under the first. In fact under the second system, they have no more power than the junior members.

Our analysis takes place in a formal model of the legislative and electoral system which is the same as the one which is developed in McKelvey and Riezman [1990]. We model the representative process as an $\mathcal{L}+n$ player stochastic game, where $\mathcal{L}$ is the number of legislators, and n is the number of voters, partitioned into $\mathcal{L}$ distinct legislative districts. The game alternates back and forth between an election and a legislative session. The election is modeled as a game (called the Voter Game) in which all the voters in each of the $\mathcal{L}$ legislative districts vote to determine who will be their representative for the next legislative session. In the legislative session the legislators first decide whether or not to have a seniority system for the current session and then proceed to select a policy. The policy selected is a decision on a distribution of a fixed amount of money among the legislative districts. We model the policy making process in the legislative session using the approach of Baron and Ferejohn [1989], who consider the legislature as a form of a Rubinstein bargaining game: There is a random recognition rule, which depends on seniority, which determines the legislator who makes a proposal. The legislators then vote, by majority rule, whether to accept or reject the proposal. The process continues until the legislature accepts a proposal, at which time the legislature adjourns, and new elections are held (i. e., we return to the voter game) and the process begins all over again.

The difference between the two institutions we look at concerns what happens if the legislature rejects the proposal once it comes to the floor. Under continuing proposal power, in the second and subsequent considerations by the legislative body, the seniority system is the same as in the first proposal. Under initial proposal power, the seniority system is only in effect for the initial proposal. In the second and all subsequent proposals, all members have equal probability of recognition.

In our previous work, McKelvey and Riezman [1990], we show that under initial proposal power, an equilibrium exists in which the legislature always votes to impose on itself a non trivial seniority system. In the proposal stage, the proposer selects a minimum winning coalition, retaining $\frac{\mathcal{L}+1}{2\mathcal{L}}$ for its own district and allocating $\frac{1}{\mathcal{L}}$ to the districts of the remaining coalition members. Districts that are not part of the winning coalition get nothing. This proposal passes and the game proceeds to the voter game. Voters always reelect incumbents. The intuition behind the results is that voters, understanding the incentives in the legislative session, realize that their representative will be disadvantaged if it does not have seniority hence they always choose to reelect their representatives. The next three sections draw heavily on McKelvey and Riezman [1990].

2. THE GENERAL FRAMEWORK

Before introducing the model we work with, we develop some general notation for stochastic games. Our model will be a special case of such a general model.

Assume that there is a set N of ***players***, a set X of ***alternatives***, and for each player $i \in N$, a Von Neumann Morgenstern ***utility function*** u_i: $X \to \mathbb{R}$ over the set of alternatives. We assume that X contains a ***null outcome***, x_0 with $u_i(x_0) = 0$ for all $i \in N$. Let T be a finite set of ***states***. We now define a ***stochastic game***, $\Gamma = \{\Gamma^t: t \in T\}$ to be a collection of ***game elements*** $\Gamma^t = (S^t, \pi^t, \psi^t)$. Here $S^t = \prod_{i \in N} S_i^t$ is an n-tuple of ***pure strategy sets***. Next π^t: $S^t \to \mathcal{M}(\mathrm{T}) = \Delta^{|T|}$ is a ***transition function*** specifying for each $s^t \in S^t$ a probability distribution, $\pi^t(s^t)$ on T, which determines for each $s^t \in S^t$ and $y \in T$, the probability $\pi^t(s^t)(y)$ of proceeding to game element Γ^y. Finally, ψ^t: $S^t \to X$ is an ***outcome function*** which specifies for each $s^t \in S^t$ an outcome $\psi^t(s^t) \in \mathrm{X}$. We let $S = \prod_{t \in T} S^t$ be the collection of pure strategy n-tuples, one for each game element. We write $\Sigma_i^t = \mathcal{M}(S_i^t)$, where $\mathcal{M}(S_i^t)$ is the set of probability disributions over S_i^t, and then define $\Sigma_i = \prod_{t \in T} \Sigma_i^t$ to be the set of ***stationary strategies*** for player i. Elements of Σ are written in the form $\sigma = (\sigma_1, \sigma_2, \ \ldots \ , \sigma_n)$. We also use the abusive notation $\sigma^t(s^t) = \prod_{i \in N} \sigma_i^t(s_i^t)$, and $\sigma(s) = \prod_{t \in T} \sigma^t(s^t)$ to represent the probability under σ of choosing the pure strategy profile $s^t \in S^t$, and $s \in S$, respectively.

For stationary strategies, we can define the payoff function $M^t\colon \Sigma \to \mathbb{R}^n$ by

$$M_i^t(\sigma) = \sum_{\tau=1}^{\infty} \sum_{r\in T} \pi_\tau^t(\sigma)(r) \cdot u_i(\psi^r(\sigma^r)), \tag{2.1}$$

where $\pi_\tau^t(\sigma)(r)$ is defined inductively by

$$\pi_1^t(\sigma)(r) = \pi^t(\sigma^t)(r) = \sum_{s^t\in S^t} \sigma^t(s^t) \cdot \pi^t(s^t)(r),$$

$$\pi_\tau^y(\sigma)(r) = \sum_{y\in Y} \pi_{\tau-1}^y(\sigma)(y) \cdot \pi_1^y(\sigma^t)(r),$$

and $u_i(\psi^t(\sigma^t))$ is defined by

$$u_i(\psi^t(\sigma^t)) = \sum_{s^t\in S^t} \sigma^t(s^t) \cdot u_i(\psi^t(s^t)).$$

Note that the above is only well defined if the sum in (2.1) converges for all σ, t, and i.

A strategy n-tuple, $\sigma \in \Sigma$ is said to be a ***Nash equilibrium*** if $M_i(\sigma_i', \sigma_{-i}) \leq M_i(\sigma)$ for all $\sigma_i' \in \Sigma_i$. It follows from standard results of stochastic games, that any stationary Nash equilibrium can be characterized by a collection $\{v^t\}_{t\in T} \subseteq \mathbb{R}^n$ of values for each game element Γ^t, and a strategy profile, $\sigma \in \Sigma$ satisfying:

(a) For all $t \in T$, σ^t is a Nash equilibrium to the game with payoff function $\mathrm{G}^t\colon \Sigma^t \to \mathbb{R}^n$ defined by:

$$\begin{aligned}
\mathrm{G}^t(\sigma^t) &= u(\psi^t(\sigma^t)) + \sum_{y\in T} \pi^t(\sigma^t)(y) \cdot v^y \\
&= \mathrm{E}_{\sigma^t}\left[u(\psi^t(s^t)) + \sum_{y\in T} \pi^t(s^t)(y) \cdot v^y\right] \\
&= \sum_{s^t\in S^t} \sigma^t(s^t) \cdot \left[u(\psi^t(s^t)) + \sum_{y\in T} \pi^t(s^t)(y) \cdot v^y\right].
\end{aligned}$$

(b) For all $t \in T$, $v^t = \mathrm{G}^t(\sigma^t)$.

We will use the above result to characterize equilibria in the stochastic game we consider. Finally, it also follows from results in Sobel [1971] that a Nash equilibrium in the set of stationary strategies is also a Nash equilibrium in the larger class of non-stationary strategies.

3. THE GAME WITH INITIAL PROPOSAL POWER

We consider an infinitely repeated game between legislators and their constituents. The game alternates back and forth between a legislative session, in which legislators decide on a division of a fixed pie among the legislative districts, and an election, in which the voters decide whether or not to reelect their legislators.

The legislative session consists of three parts: a vote on the seniority structure, a proposal by a randomly selected member, and a vote on the proposal. The legislative session starts with a vote on the seniority structure. If a majority of the legislators vote for a seniority system, it passes, otherwise there is no seniority system. Next, a random recognition rule, like that of Baron and Ferejohn [1989] is used to select a legislator as a proposer. If no seniority system was passed, all legislators have equal probability of being selected. On the other hand, if a seniority system was passed, then the probability of recognition is an increasing function of the legislator's relative seniority. The proposer proposes a division of the pie among the $\mathcal{L}$ legislative districts. The legislature then votes on the proposal. If the proposal is defeated, a new proposer is selected and the game continues as before. Under continuing proposal power, the next proposer is selected in the same manner as in the first round. Under initial proposal power, seniority is ignored in selecting the second and all subsequent proposers. Once a proposal passes the legislature the legislative session ends.

After each legislative session there is an election. The voters can choose to re-elect their incumbent legislator, in which case the legislator has seniority in the next session and receives a salary of c, or the voters can vote not to re-elect the incumbent, in which case their legislator receives no salary and goes to the next session with no seniority. While this is not completely realistic it at least captures the idea that voters can punish their representatives if they feel that they are not acting in their best interests. Our formulation allows more limited punishments than would be the case if voters could remove the legislator from office permanently. After each election the legislative session begins again with the new seniority structure. All agents have utility functions which are the discounted present value of their lifetime stream of utility. For the legislators, in each period, payoffs consist of a salary, which depends on whether they are re-relected, and a percentage $(1-\theta)$ of what they secure for their disrict. Thus, they skim some exogenously given portion of their district's payoff. For the voters, in each period they get θ times their share of what their legislator is able to secure for the district.

We now define the legislative seniority game more formally as a special kind of stochastic game. We let the set of ***players*** be $N = L \cup V$, where L is the set of ***legislators***, with $\mathcal{L} = |L| \geq 3$ odd, and V is the set of ***voters***. We assume that there is a function ϕ: $V \to L$ identifying the ***legislative districts***, such that voter v is in legislator ℓ's district if $\phi(v) = \ell$. We assume that $n_\ell = |\phi^{-1}(\ell)|$ is odd for all $\ell \in L$. We assume that the set of ***outcomes*** is $X = X' \cup x_0$, where $X' = \Delta^{\mathcal{L}} \times \{0,1\}^{\mathcal{L}}$, and x_0 is the ***null outcome***. Elements of X' are written in the form $x = (z, q)$, where $z \in Z = \Delta^{\mathcal{L}}$, and $q \in Q = \{0,1\}^{\mathcal{L}}$. So $z = (z_1, \ldots, z_{\mathcal{L}}) \in Z = \Delta^{\mathcal{L}}$ represents a division of the resources between the $\mathcal{L}$ districts, and $q = (q_1, \ldots, q_{\mathcal{L}}) \in Q = \{0, 1\}^{\mathcal{L}}$ repesents the seniority structure of the legislature, with $q_i = 1$ indicating that legislator i has seniority, whereas $q_i = 0$ indicating it does not have seniority. We assume that utility functions over X' are of the form $u_i(x) = (1-\theta)z_i + cq_i$ for $i \in L$, and $u_i(x) = (\theta / n_{\phi(i)}) z_{\phi(i)}$ for $i \in V$. Further, for the null outcome, it is assumed that $u_i(x_0) = 0$ for all $i \in N$.

Let $0 < \delta < 1$ be a fixed discount rate, and q^* be the element of Q satisfying $q_i^* = 1$ for all i. Let p: $Q \to \Delta^{\mathcal{L}}$ be a function which indicates the proposal power of each legislator as a function of it's seniority. We assume p is strictly monotonic in each component: for all $q \in Q$, and $i \in L$, $q_i > q_i' \Rightarrow p_i(q) > p_i(q_i', q_{-i})$, and that $q_i = q_j \Rightarrow p_i(q) = p_j(q)$. Thus, more seniority means a higher probability that a legislator is selected as the proposer, and legislators with the same seniority have equal probability of being selected.

We assume that there are two basic phases of the game, called the Legislative Session and the Election, plus an ending state, called the Termination Game. The Election consists of a single component, called the Voting Game, but the Legislative Session is further divided into four stages, called the Legislative Seniority Game, the Legislative Recognition Game, the Legislative Proposal Game, the Legislative Voting Game. This yields a total of six basic game elements, represented by the set $\{LS, LR, LP, LV, V, T\}$. Each of these is further indexed by the current state variable. Let $\mathcal{T} = (\{LS\} \times Q) \cup (\{LR\} \times Q) \cup (\{LP\} \times Q \times L) \cup (\{LV\} \times Q \times Z) \cup (\{V\} \times Z) \cup (\{T\})$ be the set of states.

The strategy sets and transition functions for the game elements are defined as follows:

For $t \in \{LS\} \times Q$: $S_i^t = \begin{cases} \{0,1\} & \text{if } i \in L \\ \{0\} & \text{if } i \in N - L, \end{cases}$ LS: Legislative Seniority Game

$$\pi^t(s^t)(LR, t_1) = 1 \qquad \text{if } \Sigma_{i \in L}\, s_i^t > \frac{\ell}{2},$$
$$\pi^t(s^t)(LR, q^*) = 1 \qquad \text{if } \Sigma_{i \in L}\, s_i^t \leq \frac{\ell}{2},$$

$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

The first decision the legislature makes is whether or not to have seniority for the current session. This game is indexed by $t = (LS, t_1)$, where $t_0 = LS$ indicates that we are in the Legislative Seniority Game, and t_1 is the current seniority vector. The vote determines if seniority is used in the Legislative Recognition Game below. If a majority of the legislators vote for seniority, then the current seniority vector, t_1, is used in the Legislative Recognition Game. If there is not a strict majority voting for, then the seniority vector q^*, which assigns equal weight to all legislators, is used in the Legislative Recognition Game.

For $t \in \{LR\} \times Q$: $S_i^t = \{0\}$ if $i \in N$, LR: Legislative Recognition Game

$$\pi^t(s^t)(LP, t_1, y) = p_y(t_1) \text{ if } y \in L,$$
$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

The Legislative Recognition Game is the second stage of the legislative session. This game is indexed by $t = (LR, t_1)$, where $t_0 = LR$ indicates we are in the Legislative Recognition Game, and t_1 is the current seniority vector. If seniority passed in the Legislative Seniority Game, the seniority vector t_1 is the same as that in the Legislative Seniority Game. If seniority failed then q^* is used for the seniority vector. A legislator is selected by a random recognition rule to make a proposal for consideration by the legislature. This rule is similar to the Baron Ferejohn recognition rule, except we let the recognition rule be a function of seniority. Assumptions made above guarantee that higher seniority leads to higher probability of being selected as the proposer.

For $t \in \{LP\} \times Q \times L$: $S_i^t = \begin{cases} Z & \text{if } i = t \\ \{0\} & \text{if } i \in N - \{t\}, \end{cases}$ LP: Legislative Proposal Game

$$\pi^t(s^t)(LV, t_1, s_t^t) = 1,$$

$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

The Legislative Proposal Game is the third stage of the legislative session. This game is indexed by $t = (LP, t_1, t_2)$, where $t_0 = LP$ indicates that we are in the proposal game, t_1 is the current seniority vector, and $t_2 \in L$ is the legislator who has been selected to make a proposal. This legislator, who has been selected as the proposer in the Legislative Recognition Game, makes a proposal for a division of the dollar between the legislative districts. If the legislator proposes the division z, then we proceed to the Legislative Voting Game (LV, t_1, z).

For $t \in \{LV\} \times Q \times Z$: $S_i^t = \begin{cases} \{0,\ 1\} & \text{if } i \in L \\ \{0\} & \text{if } i \in V, \end{cases}$ L: Legislative Voting Game

$$\pi^t(s^t)(V, t_2) = 1 \text{ if } \Sigma_{i \in L}\, s_i^t > \tfrac{\ell}{2},$$

$$\pi^t(s^t)(LR, q^*) = 1 \text{ if } \Sigma_{i \in L} s_i^t \leq \tfrac{\ell}{2},$$

$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

The Legislative Voting Game is the fourth stage of the legislative session. This game is indexed by $t = (LV, t_1, t_2)$, where $t_0 = LV$ indicates that we are in the Legislative Voting Game, t_1 is the current seniority vector, and $t_2 \in Z$ indicates the proposal for division of the dollar that was selected by the proposer in the Legislative Proposal Game. In this game, the proposal t_2 is before the legislature, and the legislators must vote whether to accept it or reject it. If the legislators vote to accept the proposal, the legislative session ends, and we proceed to the Voter Game. If the legislators reject the proposal we return to the Legislative Recognition Game, however with initial proposal power seniority is ignored in selecting the proposer. Note that the Legislative Proposal and Legislative Voting Games together are similar to the closed rule version of the Baron Ferejohn model.

For $t \in \{V\} \times Z$: $S_i^t = \begin{cases} \{0, 1\} & \text{if } i \in V \\ \{0\} & \text{if } i \in L, \end{cases}$ V: Voter Game

$$\pi^t(s^t)(LS, q(s^t)) = \delta,$$

$$\pi^t(s^t)(T) = 1 - \delta,$$

$$\psi^t(s^t) = (t_1, q(s^t)),$$

where $q(s^t) = (q_1(s^t), q_2(s^t), \dots, q_L(s^t)) \in Q$ is defined by

$$q_i(s^t) = \begin{cases} 1 & \text{if } \sum_{j \in \phi^{-1}(i)} s_j^t > \frac{n_\ell}{2} \\ 0 & \text{if } \sum_{j \in \phi^{-1}(i)} s_j^t \leq \frac{n_\ell}{2}, \end{cases}$$

and where $0 < \theta < 1$ and $0 < c$ are constants.

The Voter Game consists of a set of simultaneous elections in all of the legislative districts. This game is indexed by $t = (V, t_1)$, where $t_0 = V$ indicates that we are in the Voter Game, and $t_1 \in Z$ represents the outcome of the Legislative Voting Game. In each legislative district, the voters of that district vote whether or not to reelect their legislator. In the version of the game as it is presented here, there is only one legislator in each district, and no challenger. So the effect of a negative vote in a given district is that the legislator from that district does not get a salary for the next period, and loses its seniority.

The Voter Game also determines the termination conditions of the game. With probability δ, the game proceeds to the Legislative Seniority Game. With probability $1 - \delta$ the game proceeds to the Termination Game. This is a formal way of introducing discounting into the model. It is assumed that there is a probability $1 - \delta$ of termination after each round of the game. Note that the entire game terminates when this occurs. This is equivalent to assuming that players discount future payoffs by an amount δ. The Termination game is an absorbing state with zero payoffs forever:

For $t \in \{T\}$: $S_i^t = \{0\}$ if $i \in N$, T: Termination Game

$$\pi^t(s^t)(0) = 1,$$

$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

This completes the description of the stochastic game. Note that there are no payoffs except in the Voter Game. At that point policy $x = (t_1, q(s^t))$ is implemented. Thus, the pie is divided up among the districts according to $z = t_1 \in \Delta^{\mathcal{L}}$, and $q(s^t) \in Q$ determines which legislators get reelected, and which do not. Given the utility functions we have specified, it follows that the output $t_{1\ell}$ to district ℓ is first divided up with $\theta t_{1\ell}$ actually delivered to the voters, and $(1-\theta)t_{1\ell}$ being skimmed off by legislator ℓ. The voters each get an even share of the delivered output. The legislators, in addition to their share of the output get a salary which is dependent on whether they are reelected or not.

4. EQUILIBRIUM WITH INITIAL PROPOSAL POWER

The following proposition is proven in McKelvey and Riezman [1990].

PROPOSITION 1: The following is a stationary equilibrium to the legislative seniority game defined in section 3.

For $t \in \{LS\} \times Q$, and $i \in L$: $\quad \sigma_i^t(t_{1i}) = 1$

For $t \in \{LP\} \times Q \times L$: $\quad \sigma_t^t = \frac{1}{|\Omega_t|} \sum_{w \in \Omega_t} \delta_{z_t(w)}$,

where $\Omega_t = \{\omega \in \{0,\ 1\}^{\mathcal{L}}: \sum_i \omega_i = \frac{\mathcal{L}+1}{2},\ \omega_t = 1\}$, δ_x is the Dirac delta at x, and z_t: $\Omega_t \to \mathbb{R}^{\mathcal{L}}$ is defined by:

$$z_{ti}(\omega) = \begin{cases} \frac{\mathcal{L}+1}{2\mathcal{L}} & \text{if } i = t \\ \frac{1}{\mathcal{L}} & \text{if } i \neq t,\ \omega_i = 1 \\ 0 & \text{otherwise.} \end{cases}$$

For $t \in \{LV\} \times Q \times Z$, and $i \in L$:

$$\sigma_i^t(1) = \begin{cases} 1 \text{ if } t_{1i} \geq \frac{1}{\mathcal{L}} \\ 0 \text{ if } t_{1i} < \frac{1}{\mathcal{L}}. \end{cases}$$

For $t \in \{\mathrm{V}\} \times Z$, and $i \in V$: $\sigma_i^t(1) = 1$ for all i.

The proposition gives equilibrium strategies for both the legislators and voters in the game with initial proposal power. In the Legislative Seniority Game all legislators with seniority vote in favor of the seniority system, those who do not have seniority vote against the seniority system. In this equilibrium, since all legislators get reelected the seniority system always passes.

In the Legislative Proposal Game, the proposer selects a minimal winning coalition of legislators which includes itself. The proposer retains $\frac{\mathcal{L}+1}{2\mathcal{L}}$ for its own district, leaving $\frac{1}{\mathcal{L}}$ to be allocated to the districts of each of the remaining members of the coalition. Districts whose legislators are not a part of the winning coalition are allocated zero. Thus the proposer obtains a premium of $\frac{\mathcal{L}+1}{2\mathcal{L}} - \frac{1}{\mathcal{L}} = \frac{\mathcal{L}-1}{2\mathcal{L}}$ due to its proposal power. As $\mathcal{L} \to \infty$ this premium goes to one half.

In the Legislative Voting Game, a legislator votes for a proposal if and only if it receives at least $\frac{1}{\mathcal{L}}$. Thus, in equilbrium all proposals receive $\frac{\mathcal{L}+1}{2}$ votes and pass.

Finally, in the Voter Game, the voters always vote to reelect their legislators. It should be noted that although the proof shows only that this is a Nash equilibrium for the voters, in fact the strategy of voting for the incumbent is a dominant strategy for the voters in any given legislative district.

The intuition behind Proposition 1 is straightforward: Voters know that in equilibrium the seniority system will pass, hence it is in their best interest to reelect the incumbent, since a senior legislator will be more easily able to serve the constituency than a junior legislator. Note that voters do not know that there will be a seniority system in the next session, but rather know that in the steady state equilibrium, seniority will be voted in each session. In the next section we show that the results change if legislatures use continuing proposal power rather than initial proposal power.

5. EQUILIBRIUM WITH CONTINUING PROPOSAL POWER

In the above model the seniority system works through the Legislative Proposal stage, by influencing the probability that legislators get chosen to be the proposer. We assumed that seniority only is used for selecting the proposer in the first round of any legislative session. So, if a proposal is turned down in the legislature, then seniority is no longer used to select the proposer during that legislative session. The alternative we now analyze is the case in which seniority is in effect throughout the legislative session. One might think that this system, which on its face gives more power to the senior members, would make them better off, and hence would be selected by them. However, we show that the opposite is the case. When seniority is in effect for the entire session the only equilibrium is one in which legislators with and without seniority have the same continuation values. In other words, any equilibria have the property that seniority has no benefits for legislators. So, legislators would be indifferent between having and not having such a seniority system and would hence prefer a seniority system in which seniority is used for only the first proposal in each legislative session. Thus we get the rather paradoxical result that legislators who have seniority would choose a seniority system which on its face gives more less power to senior members.

It is worth pointing out that a seniority system which gives only initial proposal power is a realistic description of the seniority system for the U.S. Congress in the sense that seniority is embodied in the committee system. The committees make proposals by sending bills to the Floor. Once the bills go to the Floor the committees lose most of their power since bills that are amended or defeated generally do not go back to committe in that session. Hence our model might explain certain features about the way in which seniority systems are set up — in particular the importance of initial proposal power.

We now turn to consideration of the Legislative Voting Game when seniority is used for selection of the proposer in every round. The rest of the stochastic game is as before. We change the Legislative Voting Game so that when a proposal is rejected the subsequent Legislative Recognition Game will use the original seniority vector. In other words, the Legislative Voting Game is

For $t \in \{\mathrm{LV}'\} \times Q \times Z$: $S_i^t = \begin{cases} \{0,\ 1\} & \text{if } i \in L \\ \{0\} & \text{if } i \in V, \end{cases}$ LV$'$:Revised Legislative Voting Game

$$\pi^t(s^t)(V, t_{\mathscr{Q}}) = 1 \ \text{ if } \ \Sigma_{i \in L}\, s_i^t > \frac{\mathscr{L}}{2},$$

$$\pi^t(s^t)(LR, t_1) = 1 \quad \text{if} \ \Sigma_{i \in L} \ s_i^t \leq \frac{\mathcal{L}}{2},$$

$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

Consider the stochastic game of section 3, substituting the above game for the previous Legislative Voting Game. Call this the Revised Game.

PROPOSITION 2: In the Revised Game there is no symmetric stationary equilibrium with the following two properties:

(1) Voters always reelect incumbents:
For $t \in \{V\} \times Z$ and $i \in V$, $\sigma_i^t(1) = 1$.

(2) The value of senior and non-senior members in the Legislative Seniority Game is different.

***Proof*:** Let $v_i^{LR,q}$ be the value to $i \in L$ in the Legislative Recognition Game, given seniority vector $q \in Q$. Assume, without loss of generality that

$$v_1^{LR,q} \leq v_2^{LR,q} \leq \ldots \leq v_{\mathcal{L}}^{LR,q}$$

Let K be the largest integer for which $v_1^{LR,q} = v_K^{LR,q}$. We will first show that either $K = \mathcal{L}$, or the legislators vote against seniority in the Legislative Seniority Game. We deal first with the case when pure strategies are adopted in the Legislative Proposal Game, and then discuss the case of mixed strategies. So assume that $K < \mathcal{L}$. There are 2 cases:

Case 1: $K \leq \frac{\mathcal{L}-1}{2}$.

First, write $v_i^* = v_i^{LR,q^*}$. By assumptions 1 and 2 of the theorem, and using symmetry, it follows that $v_i^* = v_j^*$ for all $i \neq j$. So write $v^* = v_i^*$. Now since legislators adopt an undominated Nash equilibrium in the Revised Legislative Voting Game, it follows that legislator i will vote for t_2 (i.e. $s_i^t = 1$) if $v_i^{V,t_2} \geq v_i^{LR,t_1}$. But by Assumptions 1 and 2, writing $z = t_2$,

$$v_i^{V,z} = v_i^{V,t_2} = (1-\theta)z_i + c + \delta v^*,$$

and

$$v_i^{LR,q} = v_i^{LR,t_1}.$$

Hence $s_i^t = 1$ if

$$(1-\theta)z_i + c + \delta v^* \geq v_i^{LR,q}.$$

It follows that in the Legislative Proposal game, Nash equilibrium implies $t_2 = j$ will choose z^j to maximize z_j^j subject to z^j being approved by a majority in the Legislative voting game. Hence voter j will select a coalition $C_j \subseteq L - \{j\}$ of size $\frac{\ell-1}{2}$ for which $\sum_{i \epsilon C_j} v_i^{LR,q}$ is minimized, and set

$$z_i^j = \frac{1}{1-\theta}\,(v_i^{LR,q} - c - \delta v^*) \text{ for } i \in C_j, \tag{1}$$

$$z_j^j = 1 - \sum_{i \epsilon C_j} z_i^j, \text{ and}$$

$$z_i^j = 0 \quad \text{for } i \notin C_j,\ i \neq j.$$

Since $v_1^{LR,q}$ is minimal, and $|\{i \in L{:}\, v_i^{LR,q} = v_1^{LR,q}\}| \leq \frac{\ell-1}{2}$, it follows that for all $j \neq 1$,

$$z_1^j = \frac{1}{(1-\theta)}\,(v_1^{LR,q} - c - \delta v^*) = z_1.$$

Or,

$$(1-\theta)z_1^j + c + \delta v^* = v_1^{LR,q}.$$

But

$$v_1^{LR,q} = \sum_{i \epsilon L} p_i(q) v_1^{LP,q,i},$$

where

$$v_1^{LP,q,i} = v_1^{LV,q,z^i} = (1-\theta)z_1^i + c + \delta v^*.$$

So

$$\begin{aligned}
&(1-\theta)z_1^j + c + \delta v^* = \sum_{i \epsilon L} p_i(q)[(1-\theta)z_1^i + c + \delta v^*] \\
\Rightarrow\quad & (1-\theta)z_1^j = (1-\theta)\sum_{i \epsilon L} p_i(q) z_1^i \\
\Rightarrow\quad & z_1^j = p_1(q) z_1^1 + (1 - p_1(q)) z_1^j \\
\Rightarrow\quad & z_1^1 = z_1^j
\end{aligned}$$

But we have shown that $z_1^i = z_1^1$ for all $i \in L$, hence

$$v_1^{LR,q} = \sum_{i \epsilon L} p_i(q) v_1^{LP,q,i} = \sum_{i \epsilon L} p_i(q)[(1-\theta)z_1^i + c + \delta v^*] = (1-\theta)z_1^1 + c + \delta v^*.$$

Further, from equation (1) above, it follows that for all $j \in C_1$,

$$v_j^{LR,q} = (1-\theta)z_j^1 + c + \delta v^*.$$

Hence, setting $C = C_1 \cup \{1\}$,

$$\sum_{j \epsilon C} v_j^{LR,q} = \sum_{j \epsilon C} [(1-\theta)z_j^1 + c + \delta v^*] = (1-\theta) + \frac{\mathcal{L}+1}{2}(c+\delta v^*).$$

But

$$\sum_{j \epsilon L} v_j^{LR,q} = \sum_{j \epsilon L} \sum_{i \epsilon L} p_i(q) v_j^{LP,q,i} = \sum_{j \epsilon L} \sum_{i \epsilon L} p_i(q)[(1-\theta)z_j^i + c + \delta v^*]$$

$$= \sum_{i \epsilon L} p_i(q) \sum_{j \epsilon L} [(1-\theta)z_j^i + c + \delta v^*] = \sum_{i \epsilon L} p_i(q)[(1-\theta) + \mathcal{L}(c+\delta v^*)]$$

$$= (1-\theta) + \mathcal{L}(c+\delta v^*)$$

Combining the last two equations, it follows that

$$\sum_{j \epsilon L - C} v_j^{LR,q} = [\mathcal{L} - \frac{\mathcal{L}+1}{2}](c+\delta v^*) = \frac{\mathcal{L}-1}{2}(c+\delta v^*).$$

Since we also have that for each $j \in L$, $v_j^{LR,q} \geq c + \delta v^*$, it follows from the above, that for all $j \in L - C$, $v_j^{LR,q} = c + \delta v^*$. But since $\mathcal{L} \notin C$, and for some $j \in C$ we must have $z_j^1 > 0$, it follows that $v_{\mathcal{L}}^{LR,q} < v_j^{LR,q}$, a contradiction.

Case 2: $\frac{\mathcal{L}-1}{2} < K < \mathcal{L}$.

In this case, in the Legislative Seniority Game, it follows that individual i will only vote for seniority (i. e., $s_i^t = 1$) if $v_i^{LR,q} \geq v^*$, where, as in Case 1, $v^* = v_i^* = v_i^{LR,q^*}$ for all $i \in L$. By the same argument as in Case 1,

$$\mathcal{L}v^* = \sum_{j \epsilon L} v_j^* = \sum_{j \epsilon L} v_j^{LR,q^*} = (1-\theta) + \mathcal{L}(c+\delta v^*). \tag{2}$$

So

$$v^* = \frac{1}{\mathcal{L}}(1-\theta) + (c+\delta v^*).$$

Now, it must be the case that $v_1^{LR,q} < v^*$. To see this, assume that $v_1^{LR,q} \geq v^*$. Then using $v_1^{LR,q} \leq v_j^{LR,q}$ for all $j \in L$, and $v_1^{LR,q} < v_j^{LR,q}$ for $j > K$, it follows that

$$\sum_{j \epsilon L} v_j^{LR,q^*} > \sum_{j \epsilon L} v_1^* = \mathcal{L}v^*,$$

which contradicts (2). But now since $v_j^{LR,q} = v_1^{LR,q}$ for all $j \leq K$, it follows that for all $j \leq K$, $v_j^{LR,q} < v^*$. It follows that in the Legislative Seniority Game, $s_i^t = 1$ for all

$i \leq K$. But by assumption, $\frac{\pounds - 1}{2} < K$. Or, equivalently, $\frac{\pounds + 1}{2} \leq K$. Thus a majority vote against seniority in the Legislative Seniority Game.

We have thus shown that either $K = \pounds$, or a majority vote against seniority in the Legislative Seniority Game. But now if a majority vote against seniority in the Legislative Seniority Game, it follows that $v_i^{LS,q} = v_i^{LR,q^*} = v^*$ for all $i \in L$. On the other hand, if $K = \pounds$, it follows that $v_i^{LR,q} = v_j^{LR,q}$ for all $i, j \in L$. So regardless of the vote in the Legislative Seniority Game, we have $v_i^{LS,q} = v_j^{LS,q}$ for all $i, j \in L$. Hence, in both cases, we have shown that $v_i^{LS,q} = v_j^{LS,q}$ for all $i, j \in L$, which violates assumption (2) of the proposition.

The above argument has assumed that pure strategies are adopted in the Legislative Proposal Game. However, if mixed strategies are adopted, then any mixed strategy for legislator j must mix between pure strategies each of which satisfies the condition that j will select a coalition $C_j \subseteq L - \{j\}$ of size $\frac{\pounds-1}{2}$ for which $\sum_{i \epsilon C_j} v_i^{LR,q}$ is minimized. Thus the same argument as above can be applied.

Q. E. D.

The intuition behind this result has to do with how proposers choose coalition partners. Once chosen, proposers want to include in the coalition those with the lowest continuation values because they can be given less and will still vote for the proposal. It follows then, when seniority is used throughout the legislative session, if seniority benefits senior members then they will be less likely to be included in coalitions. What Proposition 2 shows is that for senior members the effect of being included in coalitions less often swamps the advantage of being chosen as proposer more often when seniority is used throughout the session. Thus, once the proposer is chosen senior members want to look like non-senior members so they are as likely to be included in the coalition.

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