DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

AN EQUILIBRIUM MODEL OF TAX COMPLIANCE WITH A BAYESIAN AUDITOR AND SOME "HONEST" TAXPAYERS

Michael J. Graetz
Yale Law School

Jennifer F. Reinganum and Louis L. Wilde
California Institute of Technology

SOCIAL SCIENCE WORKING PAPER 506

December 1983
Revised June 1984

ABSTRACT

Empirical work on tax compliance has yielded conservative estimates of unreported taxable income in the U.S. that average 10 to 15 percent of total taxable income for recent years. Moreover, it is held by many that the rate of noncompliance has been growing dramatically. This problem is widely perceived as one of eroding ethics -- more and more people are ceasing to comply voluntarily and are instead acting "strategically" in response to the structure of the U.S. income tax laws. We propose a simple model of tax compliance in which an exogenously given fraction of taxpayers comply voluntarily, while the remainder behave strategically. We distinguish between a general decision to act strategically and a specific decision not to report honestly. This is done in an equilibrium setting where the IRS is allowed to adjust its audit policy in response to taxpayer behavior. Because the audit policy of the IRS is endogenous and thus co-determined with the reporting behavior of potential noncompliers, several non-intuitive results emerge. In particular, we find that an increase in the fraction of strategic taxpayers decreases the likelihood that a given strategic taxpayer fails to comply. In fact, the decrease in the likelihood of underreporting exactly offsets the increase in the fraction of strategic taxpayers, so that aggregate compliance (and net tax revenues) are unaffected.

AN EQUILIBRIUM MODEL OF TAX COMPLIANCE WITH A BAYESIAN AUDITOR AND SOME "HONEST" TAXPAYERS

Michael J. Graetz, Jennifer F. Reinganum and Louis L. Wilde*

1. INTRODUCTION

Empirical work on tax compliance has yielded conservative estimates of unreported taxable income in the U.S. that average 10 to 15 percent of total taxable income for recent years.[1] Moreover, many observers believe that the <u>rate</u> of noncompliance has been growing dramatically in recent years. IRS Commissioner Egger reported in 1982 that the "income tax gap" in the <u>legal</u> sector grew from $29 billion in 1973 to $87 billion in 1981, and projected a gap of $120 billion by 1985.[2] Equally shocking estimates are offered of income tax evasion in the illegal sector. These estimates of the extent of noncompliance have produced a certain sense of panic among commentators in academia, the government and the news media. One went so far as to say that "the dramatic deterioration in compliance levels witnessed thus far, if not reversed quickly and forcefully, will gain further momentum and eventually erode, beyond repair, the integrity of our present income tax system."[3]

Even assuming these estimates are roughly correct, there is still a serious problem with identifying the source of the problem. Most experts consider the collection of U.S. income taxes to be essentially automatic. The fact that taxpayers themselves provide the initial (and as a practical matter, often final) estimate of their tax liability on their tax returns has produced an almost mythological characterization of the federal income tax as "voluntary." To quote Commissioner Egger again:

> "There has always been some resistance in this country, from colonial times onward, to virtually every form of taxation. As a general rule, with some exceptions, the resistance or protest was episodic and geographically contained. The system was never seriously threatened or weakened. From early times, as de Toqueville observed, <u>most</u> Americans had an unusual willingness to engage in voluntary activity for the public good. It can be credited in part to the "frontier mentality" which required cooperation for survival. That willingness still exists in large part; <u>most</u> Americans do engage in the spirit of voluntarism and <u>most</u> Americans do subscribe voluntarily to and comply with the tax laws to which we are all subject. Unfortunately, a growing number of what are otherwise honest citizens are becoming non-persons in the tax system or are finding various ways to submerge parts of their income, so as not to have it subject to taxation."[4]

Whether one agrees with Commissioner Egger's view of history or not, it is clear that he perceives the problem to be fundamentally one of eroding ethics -- more and more people are ceasing to comply voluntarily and are instead acting "strategically" in response to the structure of the U.S. income tax laws. In other words, the proportion of taxpayers who routinely and habitually comply with income tax rules is thought to be declining -- perhaps at a rapid pace. It is of course possible that large numbers of people have always acted strategically, and we have simply become better at measuring the extent of such behavior. Finally, one might argue that the structure of enforcement policies, coupled with general increases in real income or tax rates, have increased the benefits of noncompliance relative to the costs, so that even without a change in underlying attitudes, more people now find it profitable not to comply.

Assuming that voluntary compliance is on the decline, there are at least three reasons for concern. First, at a time of substantial budget deficits, revenue losses from noncompliance become particularly significant. Second, there is an issue whether the structure of tax legislation is creating

* We would like to thank Kim Border and members of the Caltech Theory Workshop for helpful comments. The financial support of National Science Foundation Grant No. SES-8315422 is gratefully acknowledged.

a nation of criminals, the implication being that a general decrease in respect for the law stemming from noncompliance with the tax laws will "spill over" into other areas. Finally, there is the issue of equity -- compliance is desirable purely on the grounds that the tax system should be fair, with equals paying equal taxes.

We will focus mainly on the revenue issue. We will also distinguish between a general decision to act strategically and a specific decision not to comply. Unlike prior work on this subject, we treat the taxpayer's filing of a tax return as having an impact on IRS enforcement decisions and we explore the heretofore ignored impact of the existence of a group of "habitual compliers," both with respect to taxpayers who behave strategically and to IRS enforcement. This will be done in an equilibrium setting where the IRS is allowed to adjust its audit policy in response to taxpayer behavior.

While economists have long treated the decision to evade taxes as a matter of rational choice, they have only recently begun to regard it as an equilibrium phenomenon. Drawing from Becker's (1968) classic work on the economics of crime, the traditional treatment of tax evasion in a partial equilibrium framework is due to Allingham and Sandmo (1972). Their model assumes an exogenously-given penalty system consisting of a probability p of audit and a fine F which is proportional to unreported income. Srinivasan (1973) offers a similar model in which he discusses the allocation of resources for detection, but presents no analysis of equilibrium interactions. Subsequent variations on the basic partial equilibrium model can be found in Yitzhaki (1974), Christiansen (1980), and Fishburn (1981).

There have also been multi-period analyses of the choice of a penalty system to minimize tax evasion subject to a budget constraint (Greenberg, 1983), to maximize net revenue (Landsberger and Meilijson, 1982), and to

maximize an exogenously given social utility function (Rubinstein, 1979).

These analyses have essentially treated the probability of audit p as independent of a taxpayer's reported income, although the extent of noncompliance depends on detection probabilities. Although detection probabilities are treated as uniform across taxpayers, some analysts have explored the optimal (uniform) probability of detection. In some cases, the probability of auddit is made contingent upon other past data such as whether the taxpayer had been caught underreporting in the past (e.g., Landsberger and Meilijson, 1982, Greenberg, 1983 and Rubinstein, 1979).[5]

In a previous paper (Reinganum and Wilde, forthcoming), two of us have analyzed a principal/agent model of income tax evasion in which the IRS is designated the principal and the taxpayer the agent. This formulation permits the IRS to take account of the information contained in a taxpayer's report, and treats the IRS audit and enforcement strategy as an endogenous policy dependent on tax return information. Below we analyze a somewhat simpler model with this same feature; that is, the probability that a taxpayer is audited depends upon his report. In contrast to our previous model, we take a Nash equilibrium (rather than a principal/agent) approach to this problem. With such a model, each agent's strategies must maximize his respective payoff, given the other agent's strategy choice. The principal/agent approach requires the IRS to commit itself to an audit policy which will typically not be a "best response" to the agent's reporting strategy. The Nash equilibrium framework neither permits nor requires such commitment; instead, it follows the natural temporal order of play: first the taxpayer reports his income; subsequently the IRS decides (on the basis of his reported income) whether to perform an investigative audit. If the taxpayer is not audited, the tax liability is computed on the basis of reported income; if the taxpayer is audited, then the tax liability is computed on the basis of true income (which is discovered in the audit process), plus any applicable fines.

The next section of this paper will present a simple model of this "tax compliance game" given that some fraction of all taxpayers are voluntarily "honest" in paying their taxes. Income levels, tax rates and fines will all be taken as exogenous. To keep things simple, we assume income takes one of two values, high or low. The IRS does not observe true income, only a report made by the taxpayer. Equilibrium involves a probability of audit, chosen by the IRS, and a probability of noncompliance, chosen by those taxpayers who act strategically. Thus the model assumes the IRS can audit as many taxpayers as it wants; no budget constraint is imposed. Because the audit policy of the IRS is endogenous and thus co-determined with the reporting behavior of potential noncompliers, several non-intuitive results emerge. For example, an increase in the proportion of strategic taxpayers has precisely one effect; it <u>decreases</u> the likelihood of under-reporting by strategic taxpayers. In fact, the decrease in the likelihood of under-reporting exactly offsets the increase in the percentage of strategic taxpayers, so neither aggregate noncompliance nor aggregate revenue is affected.

Another initially surprising result is that an increase in audit costs results in an <u>increase</u> in both individual and aggregate noncompliance and an <u>increase</u> in the aggregate number of audits. Again, this is an equilibrium phenomenon -- when audit costs rise, strategic taxpayers are less likely to comply. Hence it also pays to audit them more often. Finally, an increase in the fine for under-reporting leads to less noncompliance, as expected, but it also leads to less auditing. This happens in spite of the fact that an increase in the fine makes auditing more profitable -- since noncompliance

falls, the equilibrium level of auditing can fall as well. It is worth emphasizing that some (but not all) of these results depend upon the absence of a budget constraint on the IRS' ability to audit. Budget-constrained auditing is considered in Graetz, Reinganum and Wilde (1984).

The next section of this paper will present our formal model and characterize the equilibrium.[6] Section 3 will discuss various comparative statics results and Section 4 will analyze several interesting extensions. A final section will summarize our results, discuss weaknesses of the model and suggest possible avenues for future research.

2. THE MODEL

Suppose that some taxpayers are "habitual compliers;" that is, they report their income truthfully regardless of their pecuniary self-interest. The remaining taxpayers examine their incentives carefully and act so as to maximize expected utility, taking as given the probability of audit associated with the income they choose to report. Denote the proportion of potential noncompliers by ρ and the proportion of habitual compliers by $1 - \rho$, where $0 < \rho < 1$.

For simplicity, we assume there are only two income classes -- high and low, denoted I_H and I_L, respectively. Since the IRS does not directly observe income, the taxpayer may report either high or low income. Let ϕ_H denote a report of high income and ϕ_L a report of low income. A <u>strategy</u> for the taxpayer is a function $\alpha : \{I_H, I_L\} \rightarrow [0,1]$, where

$$\alpha_H \equiv \alpha(I_H) = \Pr\{\text{potential noncomplier reports } \phi_L \mid I_H\}$$

and

$$\alpha_L \equiv \alpha(I_L) = \Pr\{\text{potential noncomplier reports } \phi_L \mid I_L\}.$$

Similarly, a <u>strategy</u> for the IRS is a function $\beta : \{\phi_H, \phi_L\} \rightarrow [0,1]$, where

$$\beta_H \equiv \beta(\phi_H) = \Pr\{\text{IRS audits} \mid \text{taxpayer reported } \phi_H\}$$

and

$$\beta_L \equiv \beta(\phi_L) = \Pr\{\text{IRS audits} \mid \text{taxpayer reported } \phi_L\}.$$

Let T_H and T_L represent the tax payments owed by high and low income taxpayers, respectively. We assume that $T_H \geq T_L$. Taxpayers who are discovered to be underreporting income are fined F in addition, but the IRS suffers an audit cost c per audit to discover underreporting. We suppose that $T_H + F - T_L > c$. That is, the increment to revenue associated with uncovering a noncomplier exceeds the audit cost. If this were not true, then even if noncompliers could be identified a priori, it would not pay to audit them and collect the taxes owed plus the fines.[7] We assume that both the taxes T_H, T_L and the fine F are taken as fixed by the IRS.[8] Let q represent the fraction of high income taxpayers in the populace; q is also the probability that a randomly chosen taxpayer has high income.

A simple game tree describes the information and actions available to each player. With probability ρ, the taxpayer is a potential noncomplier in which case the taxpayer and the IRS play the game described by Figure 1. With probability $1 - \rho$, the taxpayer is a habitual complier and simply reports his or her income truthfully. In this case, the game tree is truncated, and is displayed in Figure 2. The taxpayer knows which tree is relevant to his or her decision-making, but the IRS does not. Thus the actual game tree is a hybrid of the trees in Figures 1 and 2, and is shown in Figure 3. Since the IRS cannot distinguish between habitual compliers and potential noncompliers a priori, it must compute the conditional distribution of income, given the

taxpayer's report.

[Figures 1, 2 and 3 approximately here]

Let $\mu_H = \Pr\{I_H \mid \phi_H\}$. By Bayes' Rule,

$$\mu_H = \Pr\{\phi_H \mid I_H\}\Pr\{I_H\}/[\Pr\{\phi_H \mid I_H\}\Pr\{I_H\} + \Pr\{\phi_H \mid I_L\}\Pr\{I_L\}].$$

For any given strategy α of the potential noncompliers, this reduces to

$$\mu_H = (\rho(1-\alpha_H)+1-\rho)q/[(\rho(1-\alpha_H)+1-\rho)q+\rho(1-\alpha_L)(1-q)].$$

This expression accounts for the fact that $1-\rho$ percent of the populace always reports truthfully while the other ρ percent use the strategy α. Similarly, if $\mu_L = \Pr\{I_H \mid \phi_L\}$,

$$\mu_L = \rho q\alpha_H/[\rho q\alpha_H+(\rho\alpha_L+1-\rho)(1-q)]. \quad (1)$$

We assume the IRS is risk-neutral; it deals with a large population of taxpayers and thus achieves (nearly) its expected net revenue. Expected net revenue to the IRS when it observes a report of high income, and the strategies are (α,β), is

$$\Pi(\phi_H;\alpha,\beta) = \beta_H[\mu_H(T_H-c) + (1-\mu_H)(T_L-c)] + (1-\beta_H)T_H.$$

Similarly, the IRS' expected net revenue when a report of ϕ_L is received is

$$\Pi(\phi_L;\alpha,\beta) = \beta_L[\mu_L(T_H+F-c) + (1-\mu_L)(T_L-c)] + (1-\beta_L)T_L.$$

The payoff to a potential noncomplier who has high income, in expected utility terms, is

$$U(I_H;\alpha,\beta) = \alpha_H[\beta_L u(I_H-T_H-F) + (1-\beta_L)u(I_H-T_L)] + (1-\alpha_H)u(I_H-T_H).$$

We assume that $u'(.) > 0$ and $u''(.) < 0$. For a potential noncomplier with low income,

$$U(I_L;\alpha,\beta) = \alpha_L u(I_L-T_L) + (1-\alpha_L)[\beta_H u(I_L-T_L)+(1-\beta_H)u(I_L-T_H)].$$

A <u>best response</u> for the IRS to a given strategy α for potential noncompliers is a strategy $\hat{\beta}(\alpha)$ such that $\Pi(.;\alpha,\hat{\beta}(\alpha)) \geq \Pi(.;\alpha,\beta)$ for all other strategies β. Similarly, a <u>best response</u> for potential noncompliers to any given auditing policy β is a strategy $\hat{\alpha}(\beta)$ such that $U(.;\hat{\alpha}(\beta),\beta) \geq U(.;\alpha,\beta)$ for all strategies α. Finally, a <u>Nash equilibrium</u> is a pair of strategies (α^*,β^*) such that $\alpha^* = \hat{\alpha}(\beta^*)$ and $\beta^* = \hat{\beta}(\alpha^*)$.

For any given strategy for potential noncompliers, α, the IRS wants to choose $\beta = (\beta_H,\beta_L)$ to maximize $\Pi(.;\alpha,\beta)$. The marginal benefit of auditing a taxpayer who reports high income is

$$\partial\Pi(\sigma_H;\alpha,\beta)/\partial\beta_H = (1-\mu_H)(T_L-T_H) - c < 0. \qquad (2)$$

Thus there is a dominant choice of $\beta_H^* = 0$; audit no one who reports high income. The marginal benefit of auditing a taxpayer who reports low income is

$$\partial\Pi(\sigma_L;\alpha,\beta)/\partial\beta_L = \mu_L(T_H+F-T_L) - c. \qquad (3)$$

This gain is increasing with μ_L, the conditional probablity that the taxpayer has high income given that he or she reported low income, with T_H, the tax owed by high-income taxpayers, and with F, the fine; since μ_L is an increasing function of α_L, ρ and q, the marginal benefit of auditing a taxpayer who reports low income also increases with α_L, the conditional probability of noncompliance, with ρ, the fraction of potential noncompliers in the population, and with q, the proportion of high-income taxpayers. This gain is decreasing with T_L, the tax owed by low-income taxpayers, and with c, the

audit cost. Consequently, the IRS' best response to a strategy α is

$$\hat{\beta}_L(\alpha) \begin{cases} = 1 & \text{if } \mu_L(\alpha) > \bar{\mu}_L \\ \varepsilon\ [0,1] & \text{if } \mu_L(\alpha) = \bar{\mu}_L. \\ = 0 & \text{if } \mu_L(\alpha) < \bar{\mu}_L \end{cases}$$

where $\mu_L(\alpha)$ is as described in equation (1), and

$$\bar{\mu}_L = c/(T_H+F-T_L).$$

For a given auditing policy β, the strategic taxpayer or potential noncomplier wishes to choose a policy $\alpha = (\alpha_H,\alpha_L)$ so as to maximize expected utility. The marginal gain to reporting low income when one actually has low income is

$$\partial U(I_L;\alpha,\beta)/\partial\alpha_L = (1-\beta_H)[u(I_L-T_L) - u(I_L-T_H)] \geq 0 \qquad (4)$$

for all $\beta_H \geq 0$ (with strict inequality when $\beta_H > 0$). Thus it is a dominant strategy for a low-income individual to report low income; $\alpha_L^* = 1$. The marginal gain to reporting low income when one actually has high income -- that is, the marginal benefit of noncompliance -- is

$$\partial U(I_H;\alpha,\beta)/\partial\alpha_H = (1-\beta_L)[u(I_H-T_L) - u(I_H-T_H)]$$
$$+ \beta_L[u(I_H-T_H-F) - u(I_H-T_H)]. \qquad (5)$$

This gain is decreasing with β_L, the probability of audit, with F, the fine, and with T_L, the tax owed by low-income taxpayers. Its dependence upon T_H, the tax owed by high-income taxpayers, is ambiguous in general. This is because the expression above consists of two parts; the coefficient of $(1-\beta_L)$ represents the gain due to noncompliance when one is not audited; this gain increases with T_H. The coefficient of β_L represents the loss due to noncompliance when one is apprehended and punished; this loss is increased (in absolute value) by an increase in T_H. For the case of risk-neutral taxpayers, the marginal benefit of noncompliance increases with T_H. Equation (4) implies that a best response for the taxpayer to the strategy β is

$$\hat{\alpha}_H(\beta) \begin{cases} = 1 & \text{if } \beta_L < \bar{\beta}_L \\ \varepsilon\ [0,1] & \text{if } \beta_L = \bar{\beta}_L, \\ = 0 & \text{if } \beta_L > \bar{\beta}_L \end{cases}$$

where

$$\bar{\beta}_L = [u(I_H-T_L) - u(I_H-T_H)]/[u(I_H-T_L) - u(I_H-T_H-F)]. \tag{6}$$

Clearly $\bar{\beta}_L \ \varepsilon\ (0,1)$. Substituting $\alpha_L* = 1$ in the definition of μ_L implies that

$$\hat{\beta}_L(\alpha) \begin{cases} = 1 & \text{if } \alpha_H > \bar{\alpha}_H \\ \varepsilon\ [0,1] & \text{if } \alpha_H = \bar{\alpha}_H, \\ = 0 & \text{if } \alpha_H < \bar{\alpha}_H \end{cases}$$

where

$$\bar{\alpha}_H = (1-q)c/\rho q(T_H+F-T_L-c). \tag{7}$$

Here $\bar{\alpha}_H > 0$ unless F is literally infinite (assuming that $1 < q < 1$ and $c > 0$). But we assume that $F \leq I_H-T_H$; that is, the IRS cannot take more than a person's income. Hence there is always some chance that strategic taxpayers

will underreport. However, it might be that $\bar{\alpha}_H \geq 1$. We can graph these best reply functions in [0,1] X [0,1]. This gives two possible Nash equilibrium configurations.

[Figures 4 and 5 approximately here]

In the first equilibrium (see Figure 4), $(\alpha_H*,\beta_L*) = (1,0)$. That is, all strategic taxpayers underreport but there is no auditing. This occurs if $\bar{\alpha}_H > 1$.[9] In this case, it is not worth auditing any individual who reports low income (given that one cannot tell whether the individual is reporting truthfully or is failing to comply; the assumption that $T_H + F - T_L > c$ implies that it is always worth auditing someone who is known to be evading). Thus the interesting case is that illustrated in Figure 5, in which the unique equilibrium is $(\alpha_H*,\beta_L*) = (\bar{\alpha}_H,\bar{\beta}_L)$. In this case, a fraction $\bar{\alpha}_H$ of high-income potential noncompliers actually do underreport, and a fraction $\bar{\beta}_L$ of taxpayers who report low income are audited.

3. COMPARATIVE STATICS

There are four equilibrium expressions which are of interest:

$\bar{\alpha}_H$ -- the conditional probability of noncompliance given that the individual is a potential noncomplier and has high income;

$P_N = q\rho\bar{\alpha}_H$ -- the unconditional probability of noncompliance;

$\bar{\beta}_L$ -- the conditional probability of audit given a low report; and

$P_A = (q\rho\bar{\alpha}_H + 1-q)\bar{\beta}_L$ -- the unconditional probability of audit.

Recall that $\bar{\alpha}_H$ and $\bar{\beta}_L$ are given by equations (8) and (9) below, respectively.

$$\bar{\alpha}_H = (1-q)c/\rho q(T_H+F-T_L-c). \tag{8}$$

$$\bar{\beta}_L = [u(I_H-T_L) - u(I_H-T_H)]/[u(I_H-T_L) - u(I_H-T_H-F)]. \tag{9}$$

There are many parameters of potential interest, and comparative statics results are summarized in Table 1 below.

Table 1

	T_H	T_L	F	c	q	ρ
$\bar{\alpha}_H$	–	+	–	+	–	–
P_N	–	+	–	+	–	0
$\bar{\beta}_L$	*	–	–	0	0	0
P_A	**	**	–	+	–	0

* ambiguous in general; when taxpayers are risk-neutral, this entry is +.

** ambiguous in general; when taxpayers are risk-neutral, $\partial P_A/\partial T_H = -\partial P_A/\partial T_L \geq 0$ (≤ 0) as $F \geq c$ ($\leq c$).

Some of these results are quite intuitive; for instance, consider the result that $\partial\bar{\alpha}_H/\partial F < 0$. Recall that $\bar{\alpha}_H$ is the equilibrium probability with which a high-income potential noncomplier actually fails to comply. An increase in the fine for evasion reduces the likelihood that a potential noncomplier actually fails to comply. This is a standard result in the economics of crime and the tax evasion literature. But, with our model, we also see the equilibrium effect that $\partial\bar{\beta}_L/\partial F < 0$; an increase in the fine F results in less noncompliance and less auditing, both at the level of the individual taxpayer and at the aggregate level. This is an equilibrium effect

because the direct impact of an increase in the fine is (from equation (3)) to increase the marginal benefit of auditing; however, since taxpayers respond by increasing their compliance rate, the IRS can actually reduce its equilibrium number of audits. As we have already pointed out, driving equilibrium noncompliance (and the equilibrium probability of audit) to zero would require a literally infinite fine.

Indeed, several of these results are counterintuitive until one remembers that equilibrium effects play a crucial role in this analysis. For instance, an increase in the audit cost actually results in an <u>increase</u> in individual and aggregate noncompliance, and an <u>increase</u> in the aggregate number of audits. To understand why this must be so, suppose there is an exogenous increase in audit costs; if potential noncompliers made no adjustment, then it would no longer pay to audit anyone (because the IRS is just indifferent in equilibrium between auditing and not auditing taxpayers who report low income). But then potential noncompliers should underreport with probability 1, which in turn implies that the IRS should audit with probability 1, and so on. Thus there must be an adjustment by potential noncompliers, and they must adjust their probability of noncompliance upwards, so that audits will in general be more successful (i.e., catch more noncompliers and collect more fines). The probability of audit for a taxpayer who reports low income is unaffected in equilibrium. Because the aggregate number of low-income reports increases, and because each of these taxpayers is audited with an unchanged probability, the aggregate number of audits will also increase.

Recall that q is the fraction of taxpayers with high income. As q increases, it becomes less likely that a given taxpayer who reports low income actually is a low-income individual. Thus in equilibrium, each potential noncomplier must respond to an increase in q by decreasing his probability of noncompliance (alternatively, fewer potential noncompliers can actually fail to comply). Again the probability of audit given a report of low income is unaffected, so aggregate noncompliance and the aggregate probability of audit decrease.

The effects of ρ, the proportion of potential noncompliers in the population, are perhaps the most interesting. As remarked earlier, in the literature on tax evasion there is frequent reference to the claim that people are becoming more strategic: "Increasingly, the way taxpayers seem to view our present tax system is as a game to be won or lost each year. While it is fair to say that there are taxpayers still who treat the tax laws as rules to be obeyed because they are normative legal rules, . . . many taxpayers have ceased to accept the normativity of our tax structure and instead have begun to view tax laws as outcome determinative rules to be considered when develping a strategy for action (Hoeflich, 1982, pp. 31-32.)" This increase in strategic behavior is often taken to be synonymous with a decrease in compliance. The alleged deterioration in compliance is also cited as a cause of declining tax revenues. The question is often posed as one of policy -- what should the IRS do in the face of increased strategic behavior on the part of taxpayers?

Policy recommendations to curb this decline in law-abidingness include both harsher penalty systems (more audits, higher fines) and normative campaigns. "Field-experimental research undertaken in the United States suggests that taxpayer norms are an important factor underlying taxpayer behavior and that normative appeals may be more effective than sanctions in inducing compliance . . . The apparently strong impact of norms . . . suggests that tax authorities stand much to gain in compliance terms from

normative appeals. Such appeals could take the form of education programmes aimed both at existing taxpayers and children as potential taxpayers (Spicer and Lundstedt 1976, pp. 295,302)."

In contrast, the answer provided by this paper is striking. Here there is a clear distinction between the percentage of strategic taxpayers, ρ, and the likelihood of noncompliance $\bar{\alpha}_H$. From Table 1 we see that an increase in ρ has precisely one effect; it reduces the number of potential noncompliers who actually fail to comply (alternatively, each potential noncomplier evades with a lower probability). Again, this is necessary because with more potential noncompliers (i.e., more taxpayers who may falsely report low income), a report of low income is correspondingly more likely to have come from a noncomplier than a habitual complier with low income. Thus although there are more potential noncompliers, each is more likely to comply, and these effects exactly cancel each other out. <u>The aggregate number of noncompliant taxpayers is unaffected.</u> Similarly, both the conditional probability of audit for an individual who reports low income, and the unconditional or aggregate probability of audit, are unchanged. Expected revenue net of audit costs is

$$\text{Revenue} = (1-q)[\bar{\beta}_L(T_L-c)+(1-\bar{\beta}_L)T_L] + q[(1-\rho)T_H+\rho(1-\bar{\alpha}_H)T_H]$$

$$+ q\rho\bar{\alpha}_H[\bar{\beta}_L(T_H+F-c)+(1-\bar{\beta}_L)T_L].$$

Simplifying this expression and keeping in mind that both $P_N = q\rho\bar{\alpha}_H$ and $P_A = (P_N+1-q)\bar{\beta}_L$ are independent of ρ, we see that equilibrium expected revenues are also independent of ρ. That is, an exogenous increase in the fraction of strategic taxpayers has no impact on aggregate expected revenues or aggregate compliance, and should consequently have no affect on aggregate auditing policy. Not only is no change in audit policy warranted to correct for the increase in strategic behavior, but the problem itself seems unrelated to its hypothetical cause. As long as a strictly positive fraction of taxpayers behaves strategically, increases in this fraction do not account for declining compliance and tax revenues (at least in this world with no budget constraint on the IRS' audit capability). Of course, if <u>all</u> taxpayers were habitual compliers, then no audits would be required. Thus there is a discontinuity in the equilibrium at $\rho = 0$, but the existence of the policy debate presumes $\rho > 0$.

4. EXTENSIONS

An obvious and relevant extension of our basic model is to treat taxation as proportional at rate t, so that $T_H = tI_H$ and $T_L = tI_L$. In the United States, penalties for underreporting are proportional to evaded tax, so that $F = \pi t(I_H - I_L)$, where π is the penalty rate on evaded tax. These substitutions can be made directly into the equilibrium expressions $\bar{\alpha}_H$, and $\bar{\beta}_L$ to yield

$$\bar{\alpha}_H = (1-q)c/\rho q[t(I_H-I_L) + \pi t(I_H-I_L) - c] \tag{10}$$

and

$$\bar{\beta}_L = [u(I_H-tI_L) - u(I_H(1-t))]/[u(I_H-tI_L) - u(I_H(1-t)-\pi t(I_H-I_L))]. \tag{11}$$

Comparative statics of $\bar{\alpha}_H$, P_N, $\bar{\beta}_L$ and P_A in the tax rate t, the penalty rate π and the income dispersion I_H-I_L are summarized in Table 2 below.

Table 2

	π	t	I_H-I_L
$\bar{\alpha}_H$	-	-	-
P_N	-	-	-
$\bar{\beta}_L$	-	*	*
P_A	-	**	**

* ambiguous in general; when taxpayers are risk-neutral, these entries are 0.

** ambiguous in general; when taxpayers are risk-neutral, these entries are -.

An increase in the penalty rate π decreases equilibrium noncompliance and equilibrium auditing, both at the individual and the aggregate levels. It is straightforward to show that individual and aggregate noncompliance decrease with increases in the tax rate t. The popular press often assumes the opposite, and partial equilibrium models are generally ambiguous on this matter; Allingham and Sandmo (1972) found that when the fine is proportional to unreported income (e.g., $F = \pi(I_H-I_L)$), an increase in the tax rate t has a both an income and a substitution effect. Since the substitution effect is negative, while the income effect is positive (negative) if absolute risk aversion is decreasing (increasing), the net effect of an increase in t is ambiguous in the (presumed most likely) case of decreasing absolute risk aversion. Yitzhaki (1974) has noted, however, that penalties for evasion are in fact proportional to evaded tax, not unreported income, so that the income effect is spurious. Instead he finds that if absolute risk aversion is a decreasing function of income, then an increase in the tax rate t unambiguously enhances compliance. If absolute risk aversion is increasing with income, then Yitzhaki's result too is ambiguous. We find that equilibrium compliance is enhanced by an increase in the tax rate t, irrespective of any restrictions on the coefficient of absolute risk aversion, and irrespective of whether the fine is based on unreported income or evaded tax. The intuition behind this result is as follows: an increase in t has the (partial equilibrium) effect of making auditing a more attractive prospect for the IRS. In order that the IRS remain indifferent about auditing a taxpayer who reports low income, potential noncompliers must comply with a greater probability. A similar argument explains why equilibrium compliance increases with income inequality, as measured by $I_H - I_L$. In general, the dependence of both $\bar{\beta}_L$ and P_A upon t and $I_H - I_L$ are ambiguous; in the case of risk-neutral taxpayers, $\bar{\beta}_L = 1/(1+\pi)$, which is independent of t and $I_H - I_L$, while P_A decreases with t and $I_H - I_L$.

Fishburn (1981) has analyzed the impact of inflation upon the extent of tax evasion in the standard "portfolio" model of tax evasion. Holding nominal income constant, he shows that evasion increases (decreases) with the price level if relative risk aversion is increasing (decreasing). It is obvious from equations (10)-(11) that, in our model, scaling all monetary parameters (including nominal income) up by a constant λ has no effect on noncompliance, although the number of audits may be affected. If we hold nominal incomes fixed (as did Fishburn), but scale up other monetary parameters, then inflation is equivalent to an increase in the audit cost c. From Table 1 we know that this has the effect of increasing both noncompliance and the number of audits. Indexation of incomes to the rate of inflation would restore neutrality. Another way of modeling the impact of inflation is to assume that the tax rate t is a function of the price level with $t'(\lambda) > 0$

(i.e., bracket creep). Then if all monetary variables are scaled up by λ, the only remaining impact of inflation is to raise the tax rate t. From Table 2, we know that an increase in t increases compliance. Finally, if taxation is proportional to income and audit costs and income are subject to inflation, but fines are fixed in nominal terms, then an increase in the rate of inflation will result in decreased compliance. To see this, scale up audit costs and incomes by λ. The equilibrium number of noncompliers is

$$P_A(\lambda) = \lambda c(1-q)/\rho q[\lambda t(I_H - I_L) - \lambda c + F].$$

It is straightforward to show that $\partial P_A(\lambda)/\partial\lambda > 0$. That is, inflation <u>unaccompanied</u> by an adjustment in the fine F results in greater noncompliance. However, if the fine is proportional to underreported income or evaded taxes, then inflation will have no effect on compliance.

We have heretofore assumed that audits themselves were costless to the taxpayer; in fact, an audit can be a costly and time-consuming process, even if one can demonstrate the accuracy of one's report. Suppose that the taxpayer suffers a cost of \$k when audited. The net revenue to the IRS is unaffected since k is a deadweight loss rather than a transfer. Thus it is clear that again $\beta_H^* = 0$; it never pays to audit individuals who report high income. Moreover, the same function $\hat{\beta}_L(\alpha)$ governs the IRS' best response to a report of low income when the strategy α is used by taxpayers. Using the fact that $\beta_H^* = 0$, the expected utility for high- and low-income taxpayers, respectively, are:

$$U(I_H;\alpha,\beta) = \alpha_H[\beta_L u(I_H - T_H - F - k) + (1-\beta_L)u(I_H - T_L)] + (1-\alpha_H)u(I_H - T_H).$$

and

$$U(I_L;\alpha,\beta) = \alpha_L[\beta_L u(I_L - T_L - k) + (1-\beta_L)u(I_L - T_L)] + (1-\alpha_L)u(I_L - T_H).$$

The taxpayer wishes to choose (α_H, α_L) so as to maximize $U(.;\alpha,\beta)$, given β_L. The fact that $\beta_H^* = 0$ leaves open the possibility that, if $I_L \geq T_H$, a low-income taxpayer may over-report simply to avoid an audit.

$$\partial U(I_H;\alpha,\beta)/\partial\alpha_H = \beta_L u(I_H-T_H-F-k) + (1-\beta_L)u(I_H-T_L) - u(I_H-T_H) \quad (12)$$

$$\partial U(I_L;\alpha,\beta)/\partial\alpha_L = \beta_L u(I_L-T_L-k) + (1-\beta_L)u(I_L-T_L) - u(I_L-T_H). \quad (13)$$

Let $f_1(\beta_L)$ denote the right-hand side of equation (12), and $f_2(\beta_L)$ the right-hand side of equation (13). Then the best response function for the strategic taxpayer is

$$\hat{\alpha}_H(\beta) \begin{cases} = 1 & \text{if } f_1(\beta_L) > 0 \\ \varepsilon\ [0,1] & \text{if } f_1(\beta_L) = 0 \\ = 0 & \text{if } f_1(\beta_L) < 0 \end{cases}$$

and

$$\hat{\alpha}_L(\beta) \begin{cases} = 1 & \text{if } f_2(\beta_L) > 0 \\ \varepsilon\ [0,1] & \text{if } f_2(\beta_L) = 0. \\ = 0 & \text{if } f_2(\beta_L) < 0 \end{cases}$$

Note that $f_1(0) < f_2(0)$ by the strict concavity of $u(.)$.[10] The functions f_1 and f_2 are linear functions with

$$f_1'(\beta_L) = u(I_H-T_H-F-k) - u(I_H-T_L)$$

and

$$f_2'(\beta_L) = u(I_L-T_L-k) - u(I_L-T_L).$$

A sufficient condition for $f_1'(\beta_L) < f_2'(\beta_L)$ is that $I_H-T_H-F \leq I_L-T_L$. That is, the net income of a truthful low income taxpayer is no less than that of a discovered high-income noncomplier. This assumption is plausible, but it is not implied by previous ones. We proceed under this assumption since it greatly simplifies the subsequent analysis.[11] It follows that $f_1(\beta_L) < f_2(\beta_L)$ for all β_L. This leaves five mutually exclusive and exhaustive possibilities:

1. $f_1(\beta_L) > 0$, $f_2(\beta_L) > 0$
2. $f_1(\beta_L) = 0$, $f_2(\beta_L) > 0$
3. $f_2(\beta_L) = 0$, $f_1(\beta_L) < 0$
4. $f_2(\beta_L) < 0$, $f_1(\beta_L) < 0$
5. $f_2(\beta_L) > 0$, $f_1(\beta_L) < 0$.

Possibilities (3)-(5) can be ruled out as possible equilibrium configurations as follows. Suppose there exists an equilibrium (α^*,β^*) such that $f_1(\beta_L^*) < 0$. Then $\alpha_H^* = 0$. Consequently $\mu_L(\alpha^*) = 0$, implying that $\beta_L^* = 0$. But then $f_1(\beta_L^*) = f_1(0) > 0$. This is a contradiction. Hence only cases (1) and (2) are possible in equilibrium.

Case 1 implies that $\alpha_L^* = \alpha_H^* = 1$. Then $\beta_L^* = 1$ is impossible since $f_1(1) < 0$. But $\beta_L^* = 0$ is possible. Thus case 1 corresponds to Figure 4. Case 2 corresponds to Figure 5, the interior equilibrium, with $\alpha_L^* = 1$,

$$\alpha_H^* = c(1-q)/(T_H-T_L-F-c)$$

and

$$\beta_L^* = [u(I_H-T_L) - u(I_H-T_H)]/[u(I_H-T_L) - u(I_H-T_H-F-k)].$$

Note that the cost k has no effect on equilibrium noncompliance; no low-income taxpayers elect to overreport and the same fraction of high-income taxpayers elect to underreport. The taxpayer audit cost k affects only the equilibrium audit probability, which is reduced. Thus the same level of noncompliance is sustained with a lower level of auditing. The taxpayer's audit cost k is something of a policy variable for the IRS; unfortunately, an increase in k is likely to be accompanied by an increase in c, the IRS' audit cost. Thus an increase in the complexity of the audit process results in both more noncompliance (due to the increase in c) and a lower probability of audit for each taxpayer who reports low income (due to the increase in k). However, since the number of low income reports is increased, the net effect on the aggregate number of audits is ambiguous.

5. CONCLUSIONS AND QUALIFICATIONS

The results of this paper demonstrate the value of regarding tax evasion as an equilibrium phenomenon; most (but not all) of the comparative static results generated by the usual analysis of the taxpayer's decision problem in isolation are contradicted in a simple equilibrium model. In particular, we find that increases in the magnitude of sanctions have the conjectured effect; that is, an increase in the magnitude of sanctions results in more compliance and less auditing in equilibrium. However, an increase in the cost of auditing results in both more noncompliance and more auditing, while increases in the tax rate and in the degree of income inequality actually enhance equilibrium compliance.

We have also shown that, at least in this simple model, a decline in law-abidingness does not account for purported declines in tax compliance and tax revenue when the IRS is free to adjust its audit policy to taxpayer behavior (i.e., when the IRS does not face a binding budget constraint). Alternative explanations which account for both decreasing compliance and

declining tax revenues include an effective decrease in the penalty for evasion F, an exogenous increase in the cost of audit c, or an exogenous decrease in the percentage of high-income individuals q.

This said, we should mention some of the model's limitations. It is evident that we have made assumptions which dramatically simplify an extremely complex problem. In addition, the use of the Nash equilibrium concept requires both the IRS and the taxpayer to possess a great deal of information. It is often argued that were taxpayers really aware of the true probabilities of audit and levels of fines, we would observe much more noncompliance. In fact, survey research suggests that the probability of noncompliance increases after exposure to IRS review. The model, as formulated, does not incorporate imperfect information of this sort.

This model is based on only two categories of taxpayers: high and low income. A more realistic model would allow for a continuous distribution of income possibilities. Such a model, in which all taxpayers are assumed to be strategic, has been developed in Reinganum and Wilde (1984). In addition, the model presupposes that the IRS can audit as many taxpayers as it desires. Budget constraints, however, are a very real problem for the IRS. Introducing a budget constraint into this model is a nontrivial task, and we will not attempt it here. See Graetz, Reinganum and Wilde (1984) for some initial results when total audits are constrained.

FOOTNOTES

1. Henry (1983) provides a good summary and critique of this work.

2. Compliance Gap: Hearings Before the Subcommittee on Oversight of the Committee on Finance, 97th Congress, 2d Session (1982).

3. Vitez (1983), p. 191.

4. Egger (1983), p. 12.

5. There have been a number of papers in the "economics of crime" literature which analyze the optimal penalty system (p,F) using a utilitarian criterion (Becker, 1968; Stigler, 1970; Brown and Reynolds, 1973; Stern, 1975; Polinsky and Shavell, 1979). While these papers incorporate a kind of equilibrium approach, they are not directly relevant to the tax evasion problem, since the probability of detection is not sensitive to the actions of the agents.

6. Our model of the interaction between taxpayers and the IRS is a very standard two-state, two-action game. P'ng (1983) and Salant and Rest (1982) have used this type of model to analyze the litigation of settlement demands in civil torts cases. Subsequently, Salant (1983) has generalized their analysis to include an interval of possible settlement demands.

7. We also implicitly assume that $T_i + F \leq I_i$, for i = L,H.

8. Both taxes and the general structure of penalties are fixed by the legislative branch, although the IRS has some control over the choice of penalty (e.g., civil versus criminal). We ignore the latter in this analysis.

9. The knife-edge case of $\bar{\alpha}_H = 1$ has a continuum of equilibria corresponding to the heavily outlined portion of the right-hand boundary of Figure 4. In this case, all strategic taxpayers underreport, and the IRS is indifferent regarding the probability with which it audits taxpayers who report low income.

10. $f_1(0) - f_2(0) = u(I_H - T_L) - u(I_L - T_L) - [u(I_H - T_H) - u(I_L - T_H)] < 0$ by the strict concavity of u(.).

11. If taxpayers are risk neutral, then it follows that $f_1'(\beta_L) < f_2'(\beta_L)$ without any additional parametric restrictions.

REFERENCES

Allingham, Michael G., and Agnar Sandmo, "Income Tax Evasion: A Theoretical Analysis," Journal of Public Economics 1 (1972), 323-338.

Becker, Gary S., "Crime and Punishment: An Economic Approach," Journal of Political Economy 76 (1968), 169-217.

Brown, William W., and Morgan O. Reynolds, "Crime and Punishment: Risk Implications," Journal of Economic Theory 6 (1973), 508-514.

Christiansen, Vidar, "Two Comments on Tax Evasion," Journal of Public Economics 13 (1980), 389-393.

Egger, Roscoe L., Jr., "Taxpayer Compliance -- The Keynote Address," in Sawicki (ed.), Income Tax Compliance, American Bar Association (1983), 11-14.

Fishburn, Geoffrey, "Tax Evasion and Inflation," Australian Economic Papers 20 (1981), 325-332.

Graetz, Michael J., Jennifer F. Reinganum and Louis L. Wilde, "A Simple Model of Tax Compliance Under Budget-Constrained Audits," manuscript, Yale Law School (June 1984).

Greenberg, Joseph, "Avoiding Tax Avoidance: A (Repeated) Game-Theoretic Approach," Journal of Economic Theory, forthcoming.

Henry, James S., "Noncompliance with U.S. Tax Law -- Evidence on Size, Growth, and Composition," in Sawicki (ed.), Income Tax Compliance (1983), 15-112.

Hoeflich, M. H., "Of Reason, Gamesmanship, and Taxes: A Jurisprudential and Games Theoretical Approach to the Problem of Voluntary Compliance," The American Journal of Tax Policy 2 (Spring 1983), 9-88.

Landsberger, Michael, and Isaac Meilijson, "Incentive Generating State Dependent Penalty System," Journal of Public Economics 19 (1982), 333-352.

Lewis, Alan, "An Empirical Assessment of Tax Mentality," Public Finance/Finances Publiques 34 (1979), 245-257.

P'ng, I. P. L., "Strategic Behavior in Suit, Settlement and Trial," Bell Journal (Autumn 1983), forthcoming.

Polinsky, A. Mitchell, and Steven Shavell, "The Optimal Tradeoff Between the Probability and Magnitude of Fines," American Economic Review 69 (1979), 880-891.

Reinganum, Jennifer F. and Louis L. Wilde, "Income Tax Compliance in a Principal-Agent Framework," Journal of Public Economics, forthcoming.

Reinganum, Jennifer F. and Louis L. Wilde, "Sequential Equilibrium Verification and Reporting Policies in a Model of Tax Compliance," Social Science Working Paper No. 525, California Institute of Technology, April 1984.

Rubinstein, Ariel, "An Optimal Conviction Policy for Offenses that May Have Been Committed By Accident," Applied Game Theory (1979), 406-413.

Salant, S. and G. Rest, "Litigation of Questioned Settlement Claims: A Bayesian Nash-Equilibrium Approach," The Rand Corporation, P-6809, September 1982.

Salant, S., "Litigation of Settlement Demands Questioned by Bayesian Defendants," Social Science Working Paper No. 516, California Institute of Technology, March 1984.

Spicer, M. W. and S. B. Lundstedt, "Understanding Tax Evasion," Public Finance/Finances Publiques 31 (1976), 295-305.

Srinivasan, T. N., "Tax Evasion: A Model," Journal of Public Economics 2 (1973), 339-346.

Stern, Nicholas, "On the Economic Theory of Policy Towards Crime," in Economic Models of Criminal Behavior, J. M. Heinke, editor, North-Holland Publishing Company, 1978.

Stigler, George, "The Optimum Enforcement of Laws," Journal of Political Economy 78 (1970), 526-536.

Vitez, Thomas G., "Information Reporting and Witholding as Stimulants of Voluntary Compliance," in Sawicki (ed.), Income Tax Compliance, American Bar Association (1983), 191-216.

Witte, Ann D., and Diane F. Woodbury, "What We Know About Factors Affecting Compliance With the Tax Laws," in Sawicki (ed.), Income Tax Compliance, American Bar Association (1983), 133-148.

Yitzhaki, Shlomo, "A Note on 'Income Tax Evasion: A Theoretical Analysis,'" Journal of Public Economics 3 (1974), 201-202.

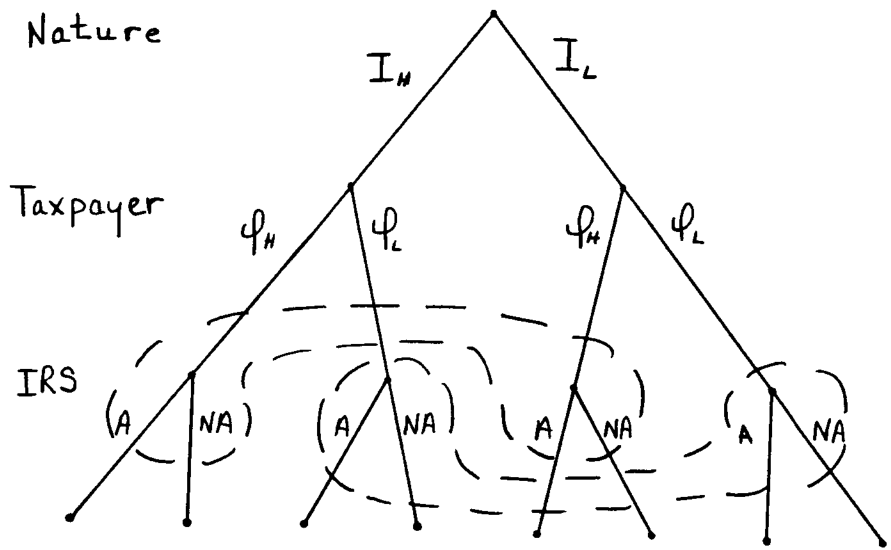


Figure 1

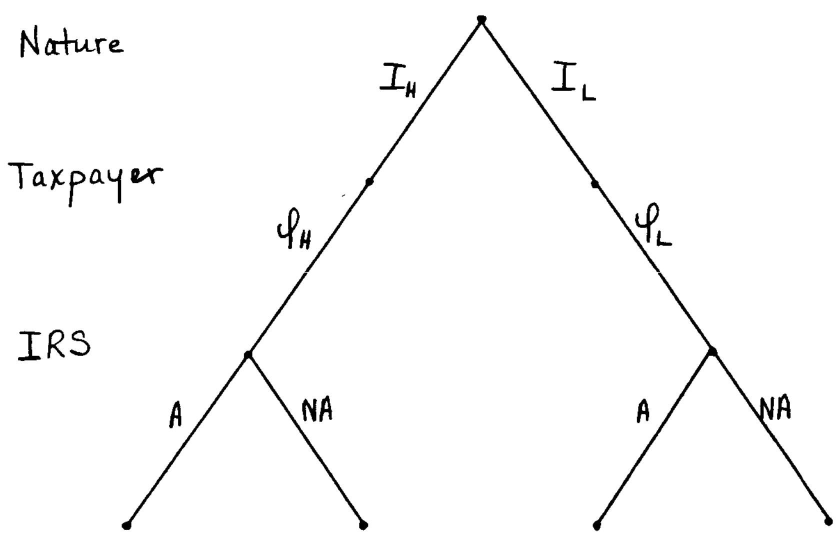
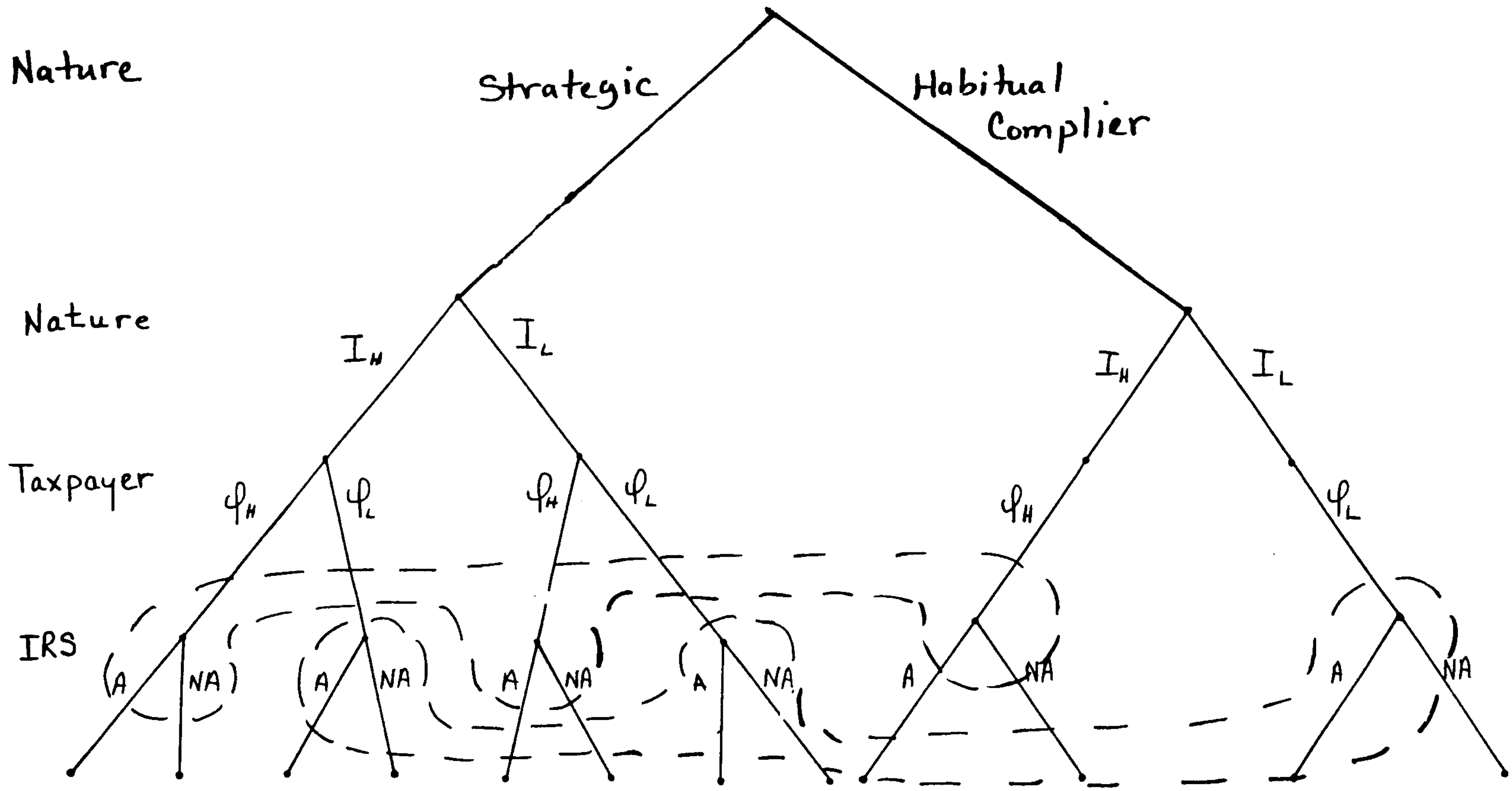
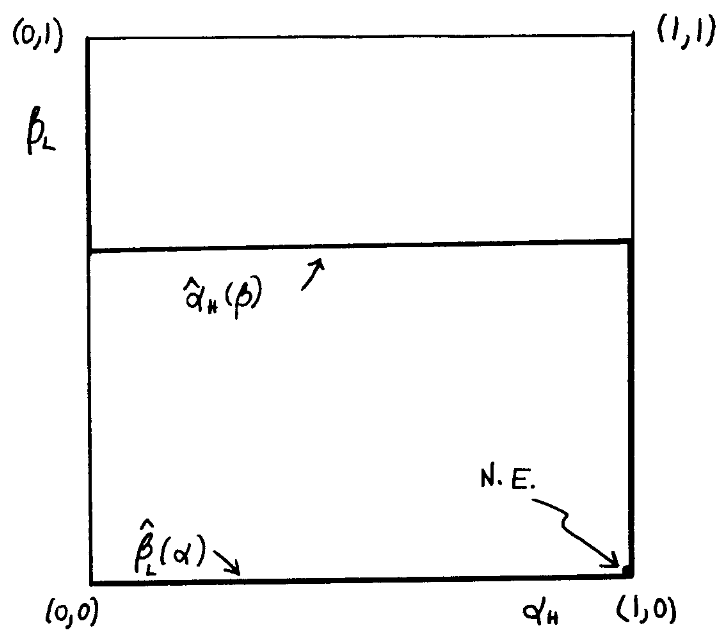


Figure 2



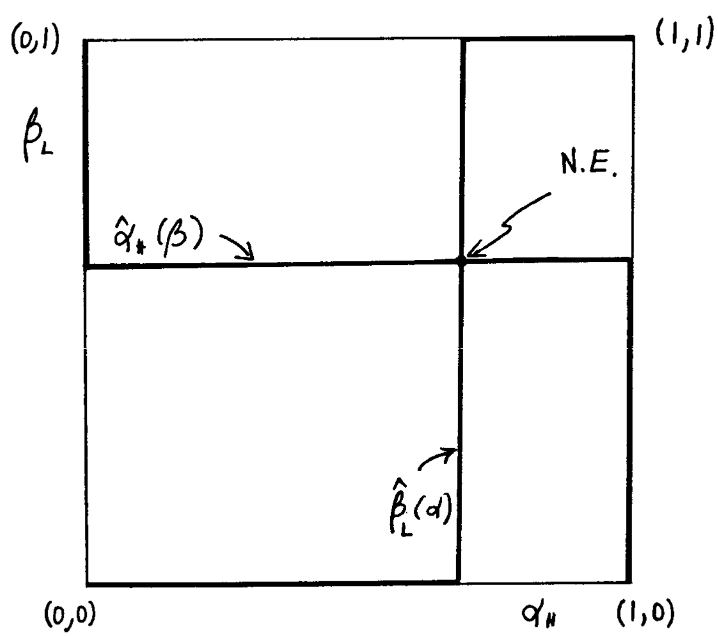
Nature
Strategic
Habitual Complier
Nature
I_H
I_L
I_H
I_L
Taxpayer
φ_H
φ_L
φ_H
φ_L
φ_H
φ_L
IRS
A
NA
A
NA
A
NA
A
NA
A
NA
A
NA

Figure 3



$\bar{\alpha}_H \geq 1$

Figure 4



$\bar{\alpha}_H < 1$

Figure 5