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COLLUSIVE BEHAVIOUR IN FINITE REPEATED GAMES WITH BONDING

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Abstract

In finite repeated games, it is not possible to enforce collusive behaviour using deterrent strategies because of the "unravelling" of cooperative behaviour in the last period. This paper demonstrates that under certain conditions collusion among the players can be maintained if they can post a bond which they must forfeit if they defect from the cooperative mode. We show that the incentives to cooperate increase as the period of interaction grows in that the size of the bond required to deter defection becomes arbitrarily small as the number of periods in the game increases.

COLLUSIVE BEHAVIOUR IN FINITE REPEATED GAMES WITH BONDING

It is well known that it is possible (even with strictly positive discounting) to obtain collusive perfect equilibria in infinitely repeated games. However, only noncooperative perfect equilibria exist in finite games. Even though finite games may last for a long time, the cooperative behaviour of the players unravels in the final period of play: defection from the cooperative agreement is the dominanat strategy in the last period, and backward induction renders noncooperative action the dominant strategy in all earlier periods. This phenomenon of unraveling is unsatisfactory for two reasons. First, it contradicts our intuition that cooperative behaviour is more likely to occur when competitors confront each other many times. In such situations firms may voluntarily cooperate to avoid retaliation by their rivals later on. Second as T, the number of repetitions of a game grows large, the behaviour of firms in the last periods of play should have a negligible impact on their strategies in the beginning periods. This suggests that the unravelling problem should not completely destroy the incentives for cooperation as T becomes large.

In this note we demonstrate a way in which cooperative equilibrium behaviour may be maintained in finite games of sufficiently long duration. Suppose that as a show of their good faith, each firm is required to post a small performance bond at the beginning of play.[1] The bond is forfeited if the firm ever defects from the cooperative mode; if the firm does not defect it receives the value of the bond plus accrued interest after the game terminates. We show that the ability of firms to maintain collusive behaviour increases as the period of interaction between them grows, in that the size of the initial bond required to deter defection becomes arbitrarily small as the number of periods in thegame becomes large.

This work is inspired by the interesting analysis of Radner (1980). He demonstrates in a Cournot-type of model with zero discounting that if firms are content to "almost" achieve their optimal responses to other firms' strategies that a fixed number of firms will agree to collude for some set number, K, < T periods, provided T is large enough.[2] The structure of our model differs from that of Radner's in that we assume that firms discount the future at a strictly positive rate. We assume however, that the discount rate is not high enough to preclude the existence of cooperative equilibria in infinite repeated games.

Following most of the literature on repeated games, we suppose there are $N \geq 2$ symmetric players or firms who compete in a sequence of identical single period stage games. We assume that the following payoffs to the stage game are well defined and that they are identical for all players

$\pi^c \equiv$ The payoff accruing to each firm, whenever all firms cooperate

$\pi^n \equiv$ The unique (by assumption) Cournot Nash equilibrium payoff to each firm

$\pi^d \equiv$ The single period optimal payoff for a single firm defection from a collusive agreement. It is defined by

$$\pi^d = \max_{S_i} \pi(S_i | S_j = \tilde{C} \ \forall \ j \neq i)$$

where S_i is the action of the defecting firm, S_j is the action of the nondefecting firms j, and $\tilde{C}$ is the collusive action.

Define a trigger strategy for firm i by

$$S_i^t = \begin{cases} \tilde{C} \text{ if } S_j^{t'} = C \ \forall \ j \neq i \text{ and } t' < t \\ \tilde{n} \text{ otherwise} \end{cases}$$

where $\tilde{n}$ is the Cournot Nash equilibrium action for the stage game.

In what follows, we let $D = 1/(1 + r)$ be the discount factor, where r denotes the discount rate. We are only interested in those cases where it is possible to enforce cooperative behaviour in infinite repeated games using trigger strategies. While we allow for strictly positive discounting, we want to bound D from below to insure the existence of a collusive equilibrium in the infinite repeated game. Otherwise, the issue of unraveling in the finite game becomes irrelevant. This is the purpose of our first assumption, (A1) Given D, π^c, π^n, and π^d there exists a collusive equilibrium with $S_i^t = \tilde{C}$ $\forall$ i and t supported by trigger strategies for the infinite repeated game.

Our second assumption concerns bonding in the finite game case. Let T < ∞ be the number of repetitions of the stage game, let B be the size of the initial bond that each firm must put up, and let P(T) be the amount paid back to each firm after the game concludes. Then we assume,

$$\text{(A2)} \quad P(T) = \begin{cases} (1/D)^{T-1}\, B & \text{if } S_i^t = \tilde{C} \;\forall\, i, \text{ and } t \le T \\ 0 & \text{otherwise} \end{cases}$$

Imagine for example that an industry collectively obtains a performance bond from each firm. Collusion is encouraged by making the repayment of the bond and its accrued interest contingent on all firms behaving cooperatively.

Now let us determine the smallest bond required to prevent firms from defecting from the cooperative mode in a finite T period game. Let $\emptyset(t)$ represent the net gain to a single firm which defects for the first time in period t, assuming all N-1 of its rivals employ a trigger strategy. For t = T we have

$$\emptyset(T) = \pi^d - \pi_c - (1/D^{\tau-1})B \qquad (1)$$

where the last term on the right hand side of (1) is the value of the forfeited bond payment. Equation (1) defines the minimum sized bond

$\beta(T)$ that prevents unraveling of cooperative behaviour in the last period. From (1) we have

$$\beta(T) = D^{T-1}\,(\pi^d - \pi^c) \qquad (2)$$

Assumption (A1) places restrictions on the relative magnitudes of $(\pi^d - \pi^c)$, $(\pi^n - \pi^c)$ and D. According to (A1) it is not profitable for a single firm to defect when is rivals employ a trigger strategy in an infinite repeated game. This means that the net returns from defecting in the first period must be non positive or

$$\pi^d + \sum_{t=1}^{\infty} D^t \pi^n - \pi^c + \sum_{t=1}^{\infty} D^t \pi^c \qquad (3)$$

$$= (\pi^d - \pi^c) + \frac{D}{1 - D}(\pi^n - \pi^c) \le 0$$

Using (2) and (3), it is now possible to show that the bond, $\beta(T)$, is sufficiently large to prevent a firm from defecting in any period $t \le T$. Notice that if $B = \beta(T)$, then at time t, the net return from defection discounted back to time t is

$$\emptyset(t) = \pi^d + \sum_{k=t+1}^{T} D^{k-t}\,\pi^n - (\pi^c + \sum_{k=t+1}^{\tau} D^{k-t}\pi^c) \qquad (4)$$

$$- (1/D^{T-(t+1)})\;\beta(T)$$

$$= (\pi^d - \pi^c) + \frac{D(1 - D^t)}{1 - D} (\pi^n - \pi^c)$$

$$- (1/D^{T-(t+1)})\beta(T)$$

Substituting for $\beta(T)$ from (2) into (4) and using (3) we obtain

$$\phi(t) = (1 - D^t)[(\pi^d - \pi^c) + \frac{D}{1 - D} (\pi^n - \pi^c)] \leq 0 \qquad (5)$$

which is what we intended to show.

Thus according to (2) and (5) the size of the bond required to achieve collusive behaviour becomes arbitrarily small as the number of repetitions of the game grows large.[3] It seems likely that a similar kind of result can be derived for a repeated principle-agent relationship when the actions of the agent can not be observed.

NOTES

1. Alternatively, the bond may be taken out of the profits of each firm in the first periods of play.

2. Radner obtains the stronger result that K = T if one compares average profits calculated over the entire time horizon, T rather than the average profits calculated over just the remaining periods of play. This distinction is made clear in Radner (1980, Section 7). We believe it is more interesting to assume that firms only consider the remaining profit stream in making future decisions, and this is the assumption used in our analysis.

3. Furthermore, the bonding equilibrium is perfect. This is verified by showing that the trigger strategy is a best response to all conceivable histories of play. The only relevant characterization of history is whether or not defection has occurred yet.

REFERENCE

Radner, R. "Collusive Behavior in Noncooperative Epsilon-Equilibria of Oligopolies with Long but Finite Lives," _Journal of Economic Theory_ 22 (1980) pp. 136-154.