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TOWARD A THEORY OF PROFESSIONAL DIAGNOSIS AND SERVICE: CONSUMER BEHAVIOR

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ABSTRACT

A model is developed for situations in which consumers depend upon producers of a good or service for information which has an impact on their demand. Nonsupply sources of information do not exist and consumers are forced to rely on comparisons between suppliers as their only check on potential fraud. The optimal search strategies are characterized and some of the implications for the resulting patterns of advice are analyzed.

TOWARDS A THEORY OF PROFESSIONAL DIAGNOSIS AND SERVICE: CONSUMER BEHAVIOR*

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1. Introduction

Very little is understood about markets in which consumers depend upon producers of a good or service for information which has an impact on their demand for that good or service. The current "state of the art" in economic thought on the subject is summarized in the statement that "[t]he provision of joint [information and product] implies that some fraud can be successful because of the high, if not prohibitive, costs of discovery of the fraud."[1] Unfortunately the analysis based on this observation yields limited insights since it focuses on the relationship between non-supplier sources of information (e.g. independent experts, knowledgable friends, and repeated personal experience) and individual supplier responses (e.g. the "client relationship"). It thus ignores one of the most natural sources of information which consumers can tap to check potential fraud, comparisons <u>between</u> suppliers. In this paper we assume that nonsupplier sources of information do not exist, and that consumers are therefore forced to rely on comparisons between suppliers as their only check on potential fraud. The question we ultimately seek to answer is whether, under these conditions, competitive pressure will force (uniform) fraudulent behavior. As such, the focus of our research is on the market as a whole, not on individual agents.

The model developed in the next section differs from traditional models, so an explanation of our motivation for using it will be useful. Our initial investigation of "seller induced demand" began with a series of laboratory experiments, reported in Plott and Wilde [1980]. In these experiments buyers were given the opportunity to purchase one of two products, product "a" or product "b". The value of purchasing a was known with certainty but the value of purchasing b was random, depending on which of two personalized, underlying states of nature was realized. Additional information designed to provide a clue as to which state of nature had actually been realized for each individual was also provided. In one set of experiments this information was given directly to buyers. In another set, identical in all other respects to the first, this information was only given to sellers, but buyers were allowed to shop sellers for recommendations as well as low prices. Sellers were not constrained in any way regarding the nature of their recommendations to buyers.

One of the crucial features of these experiments was that no additional information was provided to help buyers learn how well they or the sellers assessed clues. In the case where only sellers were given clues this forced buyers to rely on comparisons between sellers as their only means of obtaining a check on accuracy or veracity. Since, in our laboratory experiments, "search costs" were relatively low, this generated substantial shopping

activity. We start the present paper with a theoretical model of consumer behavior under these circumstances.

The type of real-world markets we had in mind when we designed the experiments described above includes medical services, auto repairs, and the like. Typically one observes very little shopping in these markets. Instead consumers often rely heavily on the opinions of friends or other indirect sources of information. Given these observations, one might well question the usefulness of a model characterizing optimal buyer behavior based on the assumption that sellers are their only source of information. There are two reasons why such a theoretical exercise is of interest. The first reason is that a formal model can help us understand why buyers might not wish to engage in much shopping in these markets. The most immediate explanation for this behavior is that search costs are high, but the model developed in the next section will uncover other factors which might be important. The second reason why the exercise is of interest is that it will help us understand ways in which sellers might respond to buyer behavior in these markets.

It is this last issue with which we are most concerned. While this paper will not present a full equilibrium model it will construct a reasonable argument, based on our model of buyer behavior, that sellers have a strong incentive to match the recommendations of other sellers in the market.

The paper is organized as follows. Section II presents our formal model of buyer behavior, taking the link between states of the world and seller recommendations as given. Section III uses the results of section II to establish properties of buyer and seller behavior. A concluding section offers several comments respecting limitations and extensions of our current model, as well as potential applications of the entire methodology.

II. A Formal Model of Consumer Behavior

Consider a situation in which a consumer demands one of two services, a or b. Any seller in the market can provide either service. The consumer's problem is to decide which service he or she needs and to purchase that service at a low price. The solution to this problem is modeled as a stopping rule. In order to focus on the effects of asymmetric information regarding the underlying states of the world, we assume no price variation across sellers. Let p_a = the price of service a and p_b = the price of service b. As a further simplifying assumption we assume there are precisely two states of the world, A and B. The relationship between states of the world and the value of services will be made precise below, but the fundamental assumption of the model is that sellers have information not possessed by the consumer regarding the true state of the world. The consumer may or may not wish to use this information, depending on how he or she feels about sellers' abilities and/or motives.

When state of the world A is realized we will say the consumer is in demand state A. When state of the world B is realized we will say the consumer is in demand state B. Let q = the consumer's subjective estimate that he or she is in demand state A and 1 - q = the consumer's subjective estimate that he or she is

in demand state B. Information supplied by a seller comes in the same form; that is, we let p = a seller's subjective estimate that the consumer is in demand state A and 1 - p = a seller's subjective estimate that the consumer is in demand state B. Further, we let $g(p|y)$ represent the consumer's beliefs regarding the likelihood a seller will predict the pair (p,1-p) given the true state of demand is y (where $p \in [0,1]$ and $y \in \{A,B\}$).[2]

Several variations of this model are obvious. For example, one could assume the consumer has a prior distribution over q and 1 - q, and that he or she uses information supplied by sellers to update this prior to yield a posterior distribution. We have implicitly assumed that the only admissable class of such distributions are degenerate. This has yielded a number of strong results and the additional predictive power of variations has not appeared worth their costs.[3,4]

The payoff to the consumer from purchasing either service depends on the consumer's true state of demand. Let $v(x,p_x|y)$ = the indirect utility attained by purchasing service x at price p_x when the true state of demand is y (where $x \in \{a,b\}$ and $y \in \{A,B\}$). Finally, let the unit cost of visiting sellers be constant at c, measured in the same units as v.

Make the following assumptions.

A1) $\partial g(p|A)/\partial p > 0$ and $\partial g(p|B)/\partial p < 0$ for all $p \in [0,1]$.
Moreover, $0 < g(0|A) < g(0|B) < 1$ and $0 < g(1|B) < g(1|A) < 1$.

A2) $v(a,p_a|A) > v(a,p_a|B)$; $v(b,p_b|B) > v(b,p_b|A)$.

A3) $c \geq 0$.

Assumption 1 implies a type of monotonicity with respect to the information provided by sellers: a seller is more likely to predict a high probability that the consumer is in demand state A if the consumer is in fact in demand state A than if the consumer is in fact in demand state B. Similarly, a seller is more likely to predict a high probability that the consumer is in demand state B if the consumer is in fact in demand state B than if the consumer is in demand state A. Assumption 1 also implies $g(p|y) > 0$ for $p \in [0,1]$ and $y \in \{A,B\}$. This is stronger than we need but it simplifies several proofs.

Assumption 1 is based on the underlying assumption that the link between demand states and predictions is imperfect. This can be due to two factors. First, sellers may have to base their predictions on clues which are themselves randomly linked to demand states (as in the experiments studied in Plott and Wilde [1980]. Second, sellers may find it in their interest to not make "sure-thing" predictions (i.e., $p = 0$ or $p = 1$) even if they feel certain of the true demand state. This suggests that ultimately the form of g should be endogenous. That is, sellers should respond to consumer information acquisition and evaluation strategies in making their predictions. Section 3 discusses this issue in more detail. However, we emphasize here that Assumption 1 is the single most important assumption made in this paper.

Assumption 2 simply states that p_a and p_b are such that it is always preferable (from the consumer's point of view) to purchase service a in demand state A and service b in demand state B. Assumption 3 is obvious.

The final element of our model of consumer behavior links the consumer's prior subjective estimates of the probabilities of being in demand state A or B with seller provided information. Let $F(p,q)$ be the consumer's "posterior" subjective estimate of being in demand state A given a seller predicts that probability to be p when the consumer's prior subjective estimate of being in demand state A is q.

A4) $$F(p,q) = \frac{g(p|A)q}{g(p|A)q + g(p|B)(1-q)} .$$

Assumption 4 is based on the premise that the consumer acts as a classical Bayesian in forming new expectations based on seller information. The following three lemmas follow directly from (A1) through (A4) and are stated without proof. They will be useful later in deducing properties of seller behavior (e.g. the corollaries on page 17).

<u>Lemma 1</u>: $F(p,q)$ is increasing in q. It is concave when $g(p|A) < g(p|B)$, linear when $g(p|A) = g(p|B)$, and convex when $g(p|A) > g(p|B)$.

<u>Lemma 2</u>: $F(p,q) \gtreqless q$ as $g(p\ A) \gtreqless g(p|B)$.

<u>Lemma 3</u>: $F(p_2,F(p_1,q)) \gtreqless q$ as $g(p_1|A)g(p_2|A) \gtreqless g(p_1|B)g(p_2|B)$.

Formally, the consumer is assumed to maximize expected utility net of search costs. This is done by the appropriate choice of a stopping rule. The functional equation associated with this choice is defined by

$$W(p,q) = -c + \max\{S(F(p,q)),\ EW(P,F(p,q))\}.^{5} \tag{3}$$

In this equation $W(p,q)$ is the expected value of following an optimal policy when a prediction of p has been received, given q is the consumer's prior subjective probability of being in demand state A and $S(F(p,q))$ is the expected value of stopping and purchasing service a or b (depending on which yields a higher expected payoff) when the consumer's posterior subjective estimate of being in demand state A is $F(p,q)$.

Define $S_x(t) \equiv v(x,p_x|A)t - v(x,p_x|B)(1-t)$ for $x \in \{A,B\}$ and $t \in [0,1]$. This function gives the expected value of stopping and purchasing service x when the consumer's subjective estimate of being in demand state A is t. We make one final assumption to keep the problem nontrivial.

A5) $v(a,p_a|B) < v(b,p_b|B);\ \ v(b,p_b|A) < v(a,p_a|A)$.

<u>Lemma</u> 4: There exists $t^* \in (0,1)$ such that

$$S(t) = \begin{cases} S_B(t) & \text{if } t \le t^* \\ S_A(t) & \text{if } t \ge t^* \end{cases}$$

<u>Proof</u>: (A2) implies S_A and S_B are both linear with $S_A' > 0$ and $S_B' < 0$. (A5) guarantees their intersection is interior to the unit interval.

q.e.d.

Figure 1 illustrates one possible form of $S(t)$ along with t^*. Note

[Figure 1]

that Assumption 2 and Assumption 5 together imply it is always better for the consumer to match purchases with demand states. They do not imply anything about $v(a,p_a|A)$ vis-à-vis $v(b,p_b|B)$ or $v(a,p_a|B)$ vis-à-vis $v(b,p_b|A)$.

The infinite horizon functional equation (3) is the key to understanding optimal consumer behavior in this model. To analyze it directly is difficult, though, so we begin with a sequence of finite horizon problems. Define

$$W_0(p,q) = -c + S(F(p,q)). \tag{4}$$

This function gives the expected value of following an optimal policy when no further sampling is possible, a prediction of p has just been received, and the consumer's prior subjective estimate of being in demand state A is q. In a similar way, we define

$$W_n(p,q) = -c + \max\{S(F(p,q)),\ EW_{n-1}(P,F(p,q))\} \tag{5}$$

for $n \ge 1$. Consider first W_0.

The form of W_0 obviously depends crucially on the form of $S(F(p,q))$. Little can be said about $S(F(p,q))$, however, since S' can be positive or negative, and F is concave in q when $g(p|A) \ge g(p|B)$ and convex in q when $g(p|A) \le g(p|B)$ (see Lemma 1). Fortunately, the important function is not $W_0(p,q)$, but rather $EW_0(P,q)$.

For notational convenience define

$$h(p,q) \equiv g(p|A)q + g(p|B)(1-q). \tag{6}$$

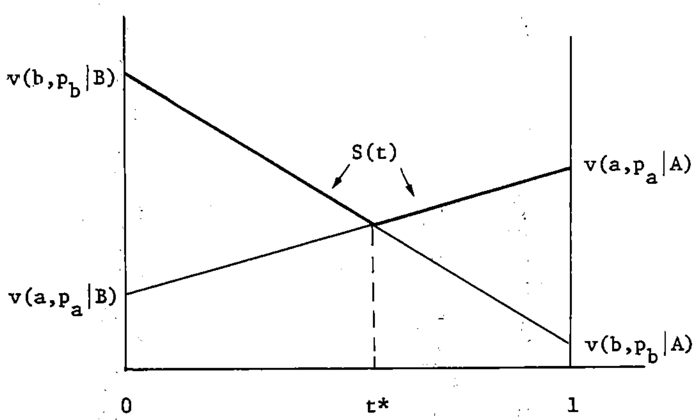

Figure 1: S(t) and t*

Then (4) implies

$$EW_0(p,q) = -c + \int_0^1 S(F(p,q))h(p,q)dp. \tag{7}$$

Lemma 5: $EW_0(P,q)$ is convex in q. Moreover, it is piecewise linear.

Proof: The proof of this lemma is straightforward but tedious. It is presented in the appendix. q.e.d.

Remark 1: Define a "continue set" by $\Gamma_0 \equiv \{q \in [0,1] \mid EW_0(P,q) \geq S(q)\}$. Lemma 5 plus the facts that $\partial EW_0(P,0)/\partial q > S'(0)$ and $\partial EW_0(P,1)/\partial q < S'(1)$ imply Γ_0 is compact and connected (if it is nonempty). Moreover, $EW_0(P,0) = S(0) - c$ and $EW_0(P,1) = S(1) - c$ so that for c small enough (but not necessarily zero), $\Gamma_0 \neq \phi$. The proof of this claim is straightforward and can be found in the appendix.

[Figure 2]

Figure 2 illustrates the properties of Γ_0. Also obvious from Figure 2 (and Remark 1) is the following.

Remark 2: If c is small enough that $\Gamma_0 \neq \phi$, then $t^* \in \Gamma_0$.

We now turn to the case where a prediction of p has just been received, the consumer's prior subjective estimate of being in demand state A is q, and at most one more seller can be sampled.

Define, as in (5),

$$W_1(p,q) = -c + \max\{S(F(p,q)), EW_0(P,F(p,q))\}. \tag{8}$$

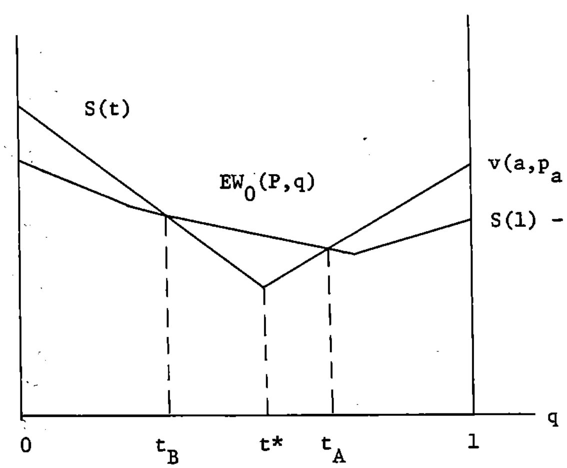

Figure 2: Properties of Γ_0

In order to proceed to the infinite horizon via an induction argument, we need to establish properties of EW_1 similar to those established for EW_0. Consider the case when $\Gamma_0 \neq \phi$.[6] Let $t_A = \max\{q \in \Gamma_0\}$ and $t_B = \min\{q \in \Gamma_0\}$ (see figure 2). Define $\pi_A(q)$ via $F(\pi_A(q),q) = t_A$ and $\pi_B(q)$ via $F(\pi_B(q),q) = t_B$. Then

$$EW_1(P,q) = \int_0^{\pi_B(q)} S_B(F(p,q))h(p,q)dp + \int_{\pi_B(q)}^{\pi_A(q)} E'W_0(P,F(p,q)) + \int_{\pi_A(q)}^{1} S_A(F(p,q))h(p,q)dp. \qquad (9)$$

Hence

$$\frac{\partial EW_1(P,q)}{\partial q} = \int_0^{\pi_B(q)} [S_B'(F(p,q))F_2(p,q)h(p,q) + S_B(F(p,q))h_2(p,q)]dp$$
$$+ \int_{\pi_B(q)}^{\pi_A(q)} [\frac{\partial EW_0(P,F(p,q))}{\partial q} F_2(p,q)h(p,q) + EW_0(P,F(p,q))h_2(p,q)]dp$$
$$+ \int_{\pi_A(q)}^{1} [S_A'(F(p,q))F_2(p,q)h(p,q) + S_A(F(p,q))h_2(p,q)]dp.$$

Differentiating again,

$$\frac{\partial^2 EW_1(P,q)}{\partial q^2} = \int_0^{\pi_B(q)} S_B'(F(p,q))[F_{22}(p,q)h(p,q) + 2F_2(p,q)h_2(p,q)]dp$$

$$+ \int_{\pi_B(q)}^{\pi_A(q)} \frac{\partial^2 EW_0(P,F(p,q))}{\partial q}[F_{22}(p,q)h(p,q) + 2F_2(p,q)h_2(p,q)]dp$$

$$+ \int_{\pi_A(q)}^{1} S_A'(F(p,q))[F_{22}(p,q)h(p,q) + 2F_2(p,q)h_2(p,q)]dp$$

$$+ [S_B'(t_B)F_2(\pi_B(q),q)h(\pi_B(q),q) + S_B(t_B)h_2(\pi_B(q),q)]\pi_B'(q)$$

$$+ [\frac{\partial EW_0(P,t_A)}{\partial q}F_2(\pi_A(q),q)h(\pi_A(q),q) + EW_0(P,t_A)h_2(\pi_A(q),q)]\pi_A'(q)$$

$$- [S_A'(t_A)F_2(\pi_A(q),q)h(\pi_A(q),q) + S_A(t_A)h_2(\pi_A(q),q)]\pi_A'(q)$$

$$- [\frac{\partial EW_0(P,t_B)}{\partial q}F_2(\pi_B(q),q)h(\pi_B(q),q) + EW_0(p,t_B)h_2(\pi_B(q),q)]\pi_B'(q).$$

The first three terms in $\partial^2 EW_1(P,q)/\partial q^2$ are clearly all zero. Hence

$$\frac{\partial^2 EW_1(P,q)}{\partial q^2} = \pi_B'(q)\Big[(S_B'(t_B) - \frac{\partial EW_0(P,t_B)}{\partial q})F_2(\pi_B(q),q)h(\pi_B(q)$$

$$+ (S_B(t_B) - EW_0(P,t_B))h_2(\pi_B(q),q)\Big]$$

$$+ \pi_A'(q)\Big[\Big(\frac{\partial EW_0(P,t_A)}{\partial q} - S_A'(t_A)\Big)F_2(\pi_A(q),q)h(\pi_A(q),q)$$

$$+ (EW_0(P,t_A) - S_A(t_A))h_2(\pi_A(q),q)\Big].$$

But by definition $S_A(t_A) = EW_0(P,t_A)$ and $S_B(t_B) = EW_0(P,t_B)$. Hence

$$\frac{\partial^2 EW_1(P,q)}{\partial q^2} = \pi_B'(q)\Big[(S_B'(t_B) - \frac{\partial EW_0(P,t_B)}{\partial q})F_2(\pi_B(q),q)h(\pi_B(q),q)\Big]$$

$$+ \pi_A'(q)\Big[\Big(\frac{\partial EW_0(P,t_A)}{\partial q} - S_A'(t_A)\Big)F_2(\pi_A(q),q)h(\pi_A(q),q)\Big].$$

It is clear from Remark 1 that $S'_B(t_B) - \partial EW_0(P,t_B)/\partial q < 0$ and $\partial EW_0(P,t_A)/\partial q - S_A'(t_A) < 0$. Moreover $F_2(\pi_x(q),q)h(\pi_x(q),q) > 0$ for $x \in \{A,B\}$. Thus, if $\pi_x'(q) < 0$ for $q \in [0,1]$ and $x \in \{A,B\}$, we would have $EW_1(P,q)$ piecewise convex. In fact this is guaranteed by (A1).

<u>Remark 3</u>: (A1) is somewhat stronger than it might appear. It implies, for example, that $g(p|q)$ is continuous and differentiable for all $p \in [0,1]$ and $q \in \{A,B\}$. These facts guarantee $EW_1(P,q)$ is differentiable and is therefore convex (see the appendix for a formal proof of this assertion).

<u>Remark 4</u>: While $EW_1(P,q)$ is convex in q, it is not piecewise linear. However, as in Remark 1, it is easy to show that $\partial EW_1(P,0)/\partial q = \partial EW_0(P,0)/\partial q > S'(0)$ and $\partial EW_1(P,1)/\partial q = \partial EW_0(p,1)/\partial q > S'(1)$. Hence the "continue set" for the one-period problem, $\Gamma_1 \equiv \{q \in [0,1] | EW_1(P,q) \geq S(q)\}$, is compact and connected (if it is nonempty). Also, $EW_1(P,0) = EW_0(P,0) = S(0) - c$ and $EW_1(P,1) = EW_0(p,1) = S(1) - c$.

<u>Remark 5</u>: It is trivial that $EW_1(P,q) \geq EW_0(P,q)$ for all $q \in [0,1]$. Thus if $\Gamma_1 \neq \phi$, $t^* \in \Gamma_1$.

Based on the above lemmas and remarks, an induction argument which extends those results to any finite problem as defined by equation (5) is straightforward. We state the following theorem without proof. Note the continue set for the n-period problem is defined as $\Gamma_n \equiv \{q \in [0,1] | EW_n(P,q) \geq S(q)\}$.

<u>Theorem 1</u>: Assume (A1) through (A5) hold. Then the following is true for all $n \geq 0$.

1) $EW_n(p,q)$ is convex for all $q \in [0,1]$.

2) $W_{n+1}(p,q) \geq W_n(p,q)$ for all $p \in [0,1]$ and $q \in [0,1]$.

3) $EW_n(P,0) = S(0) - c$; $EW_n(P,1) = S(1) - c$.

4) $\partial EW_{n+1}(P,0)/\partial q = \partial EW_0(P,0)/\partial q > S'(0)$;
$\partial EW_{n+1}(P,1)/\partial q = \partial EW_0(P,1)/\partial q < S'(1)$.

5) Γ_n is compact and connected; $\Gamma_n \subseteq \Gamma_{n+1}$; if $\Gamma_n \neq \phi$ then $t^* \in \Gamma_n$.

The ultimate usefulness of the results given in Theorem 1 is in establishing properties of the solution to the infinite horizon problem defined by the functional equation (3). We would like to show (i) a solution exists, (ii) the solution is unique, and (iii) its expected value (with respect to P) taken as a function of q is convex.

Consider the sequence of functions $\{W_n(p,q)\}_{n=0}^{\infty}$. This sequence is increasing and bounded above. Hence it converges pointwise to some limit function, say $W(p,q)$. We claim $W(p,q)$ is a solution to (3), it is the unique continuous solution, and it has

similar properties as those ascribed to the solutions of the finite problems in Theorem 1. Note the continue set for the infinite problem is defined by $\Gamma \equiv \{q \in [0,1] | EW(P,q) \geq S(q)\})$.

<u>Theorem 2</u>: Assume (A1) through (A5) hold. Let $W(p,q) = \lim_{n\to\infty} W_n(p,q)$ for all $(p,q) \in [0,1] \times [0,1]$. Then $W(p,q)$ is the unique continuous solution to (3). Moreover:

1) $EW(P,0)$ is convex for all $q \in [0,1]$.

2) $EW(p,0) = S(0) - c$; $EW(P,1) = S(1) - c$.

3) $\partial EW(P,0)/\partial q = \partial EW_0(P,0)/\partial q > S'(0)$.
$\partial EW(p,1)/\partial q = \partial EW_0(P,1)/\partial q < S'(1)$.

4) Γ is compact and connected. If $\Gamma \neq \phi$, then $t^* \in \Gamma$.

<u>Proof</u>: That $W(p,q)$ exists and solves (3) is trivial (simply take the limit on both sides of (5) and note $\lim_{n\to\infty} EW_n(P,q) = EW(p,q)$ by the Lebegue Dominated Convergence Theorem). The proof of uniqueness follows as a straightforward extension of a result originally due to MacQueen and Miller [1960]. Properties (1) through (4) are obvious.

q.e.d.

[Figure 3)

Theorem 2 describes the fundamental properties of the consumer's optimal strategy (illustrated in Figure 3). While the arguments used herein to establish properties (1) through (4) rely heavily on the assumptions that $\partial g(p|A)/\partial p > 0$ and $\partial g(p|B)/\partial p < 0$ for $p \in [0,1]$, the theorem holds for more specialized cases. For

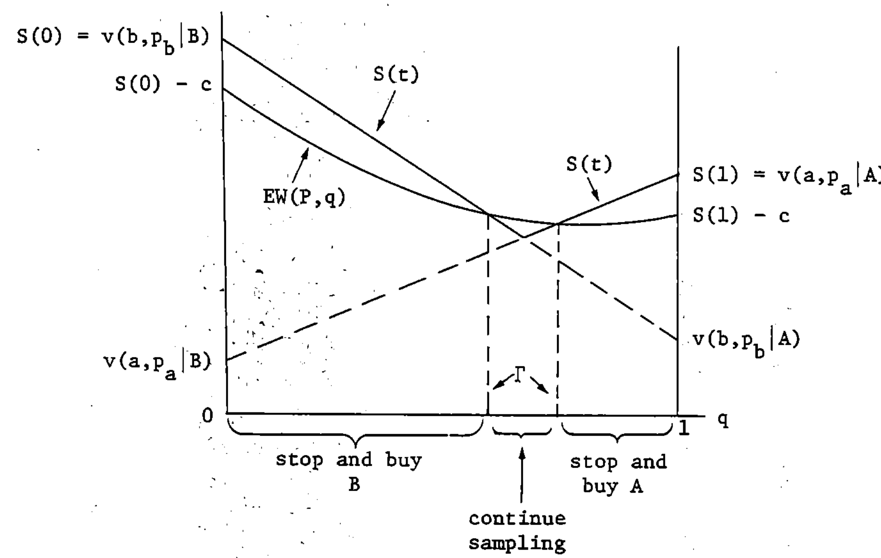

Figure 3: The Optimal Strategy

example, if $g(p|y) = 0$ for all $p \in (0,1)$ and $y \in \{A,B\}$, then $EW(p,q)$ is still convex so that our primary qualitative results still hold. This latter case is interesting because it describes a situation in which only "sure-thing" predictions are made by sellers.[7]

We close this section by stating two corollaries of Theorem 2 which reveal some nonintuitive properties of the consumer's optimal strategy.

Corollary 1: Assume $q \in \Gamma$. Then $S(F(p,q)) \geq EW(P,F(p,q))$ implies $S(F(p,q)) = S_B(F(p,q))$ iff $g(p|A) \leq g(p|B)$ and $S(F(p,q)) = S_A(F(p,q))$ iff $g(p|A) \geq g(p|B)$.

This corollary has a natural interpretation. In it we assume the consumer has a prior subjective estimate of the probability of being in demand state A such that it pays to sample another seller. However, the consumer next receives a prediction which leads him or her to stop sampling. The service purchased will then be consistent with the final prediction in the following way: Service a is preferred (strictly) to service b if and only if the prediction "favors" demand state A in the sense that $g(p|A) > g(p|B)$, the consumer is indifferent between service a and service b if and only if $g(p|A) = g(p|B)$,[8] and service b is preferred (strictly) to service a if and only if the prediction "favors" demand state B in the sense that $g(p|B) > g(p|A)$.

Corollary 2: Assume $q \in \Gamma$ and $F(p_1,q) \in \Gamma$. Then $S[F(P_2,F(P_1,q))] \geq EW[P,F(P_2,F(p_1,q))]$ implies $S[F(p_2,F(p_1,q))] = S_B[F(p_2,F(p,q))]$ iff $g(p_1|A)g(p_2|A) < g(p_1|B)g(p_2|B)$ and $S[F(p_2,F(p_1,q))] = S_A[F(p_2,F(p_1,q))]$ iff $g(p_1|A)g(p_2|A) > S(p_1|B)g(p_2|B)$.

Corollary 2 is an extension of Corollary 1 which, like it, has a natural interpretation. We again assume the consumer has a prior subjective estimate of the probability of being in demand state A such that it pays to sample another seller. In this case, however, the consumer receives a prediction which leads him or her to sample yet again. The second sample yields a prediction which induces the consumer to stop (i.e. $F(p_1,q) \in \Gamma$ but $F(p_2,F(p_1,q)) \geq EW[P,F(p_2,F(p_1,q))]$ for some p_1 and p_2). In this case the service purchased will be consistent with the final two predictions in the following manner: the consumer prefers service a if and only if the two predictions "favor" service a in the sense that $g(p_1|A)g(p_2|A) > g(p_1|B)g(p_2|B)$, the consumer is indifferent between purchasing service a and service b if and only if $g(p_1|A)g(p_2|A) = g(p_1|B)g(p_2|B)$,[9] and the consumer prefers service b if and only if the final two predictions "favor" service b in the sense that $g(p_1|A)g(p_2|A) < g(p_1|B)g(p_2|B)$.

In the case where $g(p|y) = 0$ for all $p \in (0,1)$ and $y \in \{A,B\}$, Corollary 2 implies that if the consumer stops and buys service a after sampling at least two sellers, the last two predictions are either $<1,1>$ or $<0,1>$ with the latter being the case only if $g(0|A)g(1|A) > g(0|B)g(1|B)$. But this inequality holds if and only if $g(1|A) < g(0|B)$. In other words, if sellers make only "sure-thing" predictions, then the sequence $<0,1>$ followed by a purchase of a is possible only if sellers are believed (by the consumer) to be less likely to be correct when predicting demand state A than demand state B. This seems unintuitive, but notice that $g(1|A) < g(0|B)$ if and only if $g(0|A) > g(1|B)$. That is, if sellers tend to be <u>less</u> likely to be correct when predicting demand state A than demand state B, then they are also <u>more</u> likely to be incorrect when predicting demand state B than demand state A.

Similarly, if the consumer stops and buys b after sampling at least two sellers, the last two predictions are either $<0,0>$ or $<1,0>$ with the latter being the case only if $g(1|A) > g(0|B)$, or, equivalently, $g(0|A) < g(1|B)$. These conditions are just the opposite of those noted above, implying that the two situations are mutually exclusive.

III. Properties of Buyer and Seller Behavior

In the introduction to this paper we offered two reasons for developing the model of section II. The first reason was to understand why consumers might not wish to engage in much shopping in markets such as the one we have modeled. The three primary explanations which emerge from the analysis of section II are (1) high search costs, (2) little information content in "recommendations" and (3) little value to becoming informed.

(1) The effects of high search costs are obvious. An increase in c shifts $EW(P,q)$ down relative to $S(q)$ and reduces the size of Γ. This is as one might expect. Other things constant, an increase in c will reduce the expected number of sellers sampled prior to purchase.

(2) The effects of a change in the indirect utility functions are also straightforward: an increase in $v(a,p_a|B)$ or $v(b,p_b|A)$, or a decrease in $v(b,p_b|B)$ or $v(a,p_a|A)$ will all decrease Γ and reduce shopping activity (see figure 3). This is because all four decrease the benefits of being "better" informed, or at least of being more certain of the true demand state.

(3) Finally, consider changes in $g(p|A)$ or $g(p|B)$. If $g(p|A) = 1 = g(p|B)$ for all $p \in [0,1]$ then $\Gamma = \phi$; there would be no search because search is absolutely uninformative. At the other extreme, if all the weight of $g(p|A)$ were massed at $p = 1$ and all the weight of $g(p|B)$ were massed at $p = 0$, $\Gamma = (0,1)$ but it would take only one observation for the consumer to decide which service to purchase. As $g(p|A)$ and $g(p|B)$ begin to shift from the former case to the latter (in some appropriately "smooth" way), Γ would get monotonically larger, but the expected number of sellers sampled might rise initially and then fall back to one.

The second reason for developing the model of section II was to understand ways in which sellers might be able to take advantage of buyers in markets such as the one we have modeled. To do this most effectively requires a full equilibrium model in which sellers have some control over the veracity of their individual predictions. Developing such a model is a very ambitious task and will not be undertaken herein. However, some initial results can be gleaned from the model of section II.

The first result concerns "second opinions". Suppose a buyer comes to a seller, and the seller knows the buyer has already sampled another seller. This means $q \in \Gamma$. In this case it never pays the seller to make a prediction interior to [θ,1]; that is, only "sure-thing" predictions should be made. Whether $p = 0$ or $p = 1$ is more desirable (from the seller's point of view) depends on the relative profits obtainable by selling service a or selling service b.

Determining the optimal prediction (on the basis of expected profit maximization) when the seller does not know whether $q \in \Gamma$ or $q \notin \Gamma$ is more difficult. It depends on the seller's estimate of the distribution of priors across buyers and the relative profits obtainable by selling service a or selling service b. This is one major problem with developing a full equilibrium version of this model, its implications ultimately depend on ad hoc assumptions concerning the distribution of priors.[10]

Some speculations concerning possible equilibria can be obtained, however, by using Corollary 1, Corollary 2, and the discussion following them. Consider, for example, the case in which only "sure-thing" predictions are made. In that case, if a sequence of predictions ending in <0,1> results in a purchase of a then it is <u>not</u> possible that a sequence of predictions ending in <1,0> results in a purchase of b. In effect, <u>not</u> matching the previous prediction is a very risky strategy unless the seller knows whether the buyer believes $g(0|A) > g(1|B)$ or $g(0|A) > g(1|B)$; 50 percent of the time it is guaranteed to <u>not</u> yield a sale! The best way to get a sale is to reinforce (i.e. match) the last prediction. If all sellers use the same strategies to link demand states with recommendations, then deviating from these strategies is unlikely to

increase expected profits for any given seller. In other words, there is likely to be a tendency toward "convergent" recommendations, whether or not they are veracious.

IV. <u>Conclusion</u>

It is a common characteristic of markets in which consumers depend upon producers of a good or service for information which has an impact on their demand for that good or service that search costs are high. High search costs are a special problem in these markets because "quality" is just as important, perhaps even more important than price. Hence if there is not enough shopping to get competitive prices, it is hard to believe than there is enough shopping to get a competitive outcome with respect to quality. The important policy issue here is whether more shopping would yield socially preferable market outcomes, with respect to both price and quality.

Once again, our lack of a full equilibrium model limits the precision with which we can address this issue, but we have gained some useful insights. First, even though we have assumed prices for both products are the same across all sellers, it is clear that there is little connection between prices and the veracity of predictions. In this case, "quality" is not obviously improved by increased shopping. Nevertheless, in a more general model, in which price was allowed to vary, one would expect prices to fall in response to increased shopping. Thus the following question arises: if consumers shop primarily to gather information, but that shopping activity has a stronger effect on prices than on the quality of information, is increasing shopping activity in these markets a desirable goal?

To answer this question we need to look once again at the factors which influence consumer behavior in these markets. In particular, the linkage function $g(p|y)$ is crucial. We know from section III that shopping for quality will be at a minimum when this function is either totally uninformative or perfectly informative, and that shopping for quality will be maximized at some intermediate level. In other words, shopping for quality will be maximized when consumers believe that producers have some useful information at their disposal, but that this information is not perfect. The problem is that while shopping for quality is maximized under such circumstances, welfare may not be. Since any shopping consumes resources, and since it is not obviously related to the quality of information provided by the market, it can only be justified if it has a substantial impact on prices. Moreover, even if this can be established, it is no answer to the problem of obtaining a competitive outcome with respect to quality.

These conclusions are, of course, somewhat tentative. However, they do suggest that the kind of markets we have been considering in this paper may operate in fundamentally different ways than other markets in which information is not tied so closely to product.

FOOTNOTES

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1. Darby and Karni [1973, p. 68].

2. That is, $g(p|y)$ is a conditional probability density function.

3. See Calvert [1979] who explores some of these variations of this model and uses a similar methodology to describe voting behavior.

4. We also assume the link between true states of demand and sellers' predictions, $g(p|y)$, is stable. One generalization would be to let this function be sensitive to the actual sequence of seller predictions. Another generalization is to let $g(p|y)$ be seller specific. That is, the consumer could sample a producer, observe that producer's entire linkage function, and then decide whether to purchase a recommendation from that producer.

5. An equivalent formulation of the functional equation is to write

$$V(p,q) = \max\{S(F(p,q)),\ EV(P,F(p,q) - c\}.$$

It is easy to see that $V(p,q) = W(p,q) - c$.

6. If c is large enough that $\Gamma_0 = \phi$ the problem is trivial since $W_1(p,q) = -c + S(F(p,q)) = W_0(p,q)$ in that case.

7. See the comments following Theorem 2 and Section III.

8. Note this case could never arise since $g(p|A) = g(p|B)$ means that $F(p,q) = q$ so that the consumer would never choose to stop and purchase either a or b under these conditions.

9. Again, $g(p_1|A)g(p_2|A) = g(p_1|B)g(p_2|B)$ means that $F(p_2,F(p_1,q)) = q$ so that the case of indifference would never arise.

10. For a discussion of this problem in the context of a price-search model see Wilde and Schwartz [1979].

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APPENDIX

Lemma 5: $EW_0(P,q)$ is convex in q. Moreover, it is piecewise linear.

Proof of Lemma 5: Using (7),

$$\frac{\partial EW_0(P,q)}{\partial q} = \int_0^1 [S'(F(p,q))F_2(p,q)h(p,q) + S(F(p,q))h_2(p,q)]dp,$$

$$\frac{\partial^2 EW_0(P,q)}{\partial q^2} = \int_0^1 S'(F(p,q))[F_{22}(p,q)h(p,q) + 2F_2(p,q)h_2(p,q)]dp.$$

Define

$$\sigma_p(q) = F_{22}(p,q)h(p,q) + 2F_2(p,q)h_2(p,q).$$

From (2),

$$F_{22}(p,q) = \frac{2g(p|A)g(p|B)[g(p|B) - g(p|A)]}{[g(p|A)q + g(p|B)(1-q)]^3}$$

$$= \frac{-2g(p|A)g(p|B)h_2(p,q)}{[g(p|A)q + g(p|B)(1-q)]^2\, h(p,q)}.$$

Hence

$$\sigma_p(q) = \frac{-2g(p|A)g(p|B)\, h_2(p,q)}{[g(p|A)q + g(p|B)(1-q)]^2} + 2F_2(p,q)\, h_2(p,q)$$

$$= 2h_2(p,q)\left[F_2(p,q) - \frac{g(p|A)g(p|B)}{[g(p|A)q + g(p|B)(1-q)]^2}\right]$$

$$= 0,$$

and thus

$$\frac{\partial^2 EW_0(P,q)}{\partial q^2} = \int_0^1 S'(F(p,q))\sigma_p(q)dp = 0.$$

The argument that $EW_0(P,q)$ is convex is straightforward but tedious. Define

$$Q_A = \{q \in [0,1] \mid F(p,q) > t^* \quad \text{for all } p \in [0,1]\}$$

$$Q_B = \{q \in [0,1] \mid F(p,q) < t^* \quad \text{for all } p \in [0,1]\}$$

$$Q_{AB} = \{q \in [0,1] \mid q \notin Q_A \text{ and } q \notin Q_B\}.$$

The set Q_A includes those prior probabilities for which <u>any</u> prediction will yield a posterior probability that would induce the consumer to purchase service a were he or she to stop sampling sellers. The set Q_B is defined in an analogous way. The set Q_{AB} includes those prior probabilities which are not in Q_A or Q_B (i.e. those prior probabilities for which seller provided information truly matters). It is clear that

$Q_A \neq \phi \neq Q_B$ since $F(p,0) = 0$ for all $p \in [0,1]$ and $F(p,1) = 1$ for all $p \in [0,1]$. Moreover, $Q_{AB} \neq \phi$ since $t^* \in Q_{AB}$.

Also define

$$a_1 = v(a,p_a|A) - v(a,p_a|B) > 0$$

$$a_2 = v(a,p_a|B) > 0$$

$$b_1 = v(b,p_b|A) - v(b,p_b|B) < 0$$

$$b_2 = v(b,p_b|B) > 0.$$

Then by definition, $S_A(q) = a_1 q + a_2$ and $S_B(q) = b_1 q + b_2$. Finally, define $\Sigma_p(q) = F_2(p,q)h(p,q) + F(p,q)h_2(p,q)$. Since $h_{22}(p,q) = 0$ for all $q \in [0,1]$, $\Sigma'_p(q) = \sigma_p(q) = 0$ for all $q \in [0,1]$. In fact straightforward computation shows $\Sigma_p(q) = g(p|A)$.

<u>Claim 1</u>:

$$\frac{\partial EW_0(P,q)}{\partial q} = \begin{cases} a_1 & \text{if } q \in Q_A \\ a_1 + [(b_1 - a_1) + (b_2 - a_2)]G(\pi(q)|A) \\ \qquad + (b_2 - a_2)G(\pi(q)|B) & \text{if } q \in Q_{AB} \\ b_1 & \text{if } q \in Q_B \end{cases}$$

where $\pi(q)$ is defined by $F(\pi(q),q) = t^*$ and $G(p|x)$ is the c.d.f. associated with $g(p|x)$.

<u>Proof of Claim 1</u>:

Consider $q \in Q_A$. Here

$$EW_0(P,q) = \int_0^1 S_A(F(p,q))h(p,q)dp$$

so that

$$\frac{\partial EW_0(p,q)}{\partial q} = \int_0^1 [S'_A(F(p,q))F_2(p,q)h(p,q) + S_A(F(p,q))h_2(p,q)]dp$$

$$= a_1 \int_0^1 [F_2(p,q)h(p,q) + F(p,q)h_2(p,q)]dp + a_2 \int_0^1 h_2(p,q)dp$$

$$= a_1 \int_0^1 \Sigma_p dp + a_2 \int_0^1 [g(p|A) - g(p|B)]dp$$

$$= a_1$$

The result for $q \in Q_B$ follows analogously. The result for $q \in Q_{AB}$ follows as the above with the observation that $p \leq \pi(q)$ implies $S(F(p,q)) = S_B(F(p,q))$ and $p \geq \pi(q)$ implies $S(F(p,q)) = S_A(F(p,q))$.

Since $EW_0(p,q)$ is continuous in q, the final step of the proof of the Lemma is given by the observations that $a_1 > 0 > b_1$ and $b_2 > a_2 > 0$.

q.e.d.

Remark 1: Besides being convex and piecewise linear, $\partial EW_0(P,0)/\partial q > S'(0)$ and $\partial EW_0(P,1)/\partial q < S'(1)$. Hence $\Gamma_0 \equiv \{q \in [0,1] \mid EW_0(P,q) \geq S(q)\}$ is compact and connected (if it is nonempty). Moreover, $EW_0(P,0) = S(0) - c$ and $EW_0(P,1) = S(1) - c$ so that for c small enough (but not necessarily zero). $\Gamma_0 \neq \phi$.

Proof of Remark 1: Consider $q = 0$. By definition

$$EW_0(P,0) = -c + \int_0^1 S(F(p,0))h(p,0)dp$$

$$= -c + S(0)\int_0^1 g(p|B)dp$$

$$= -c + S(0)$$

Also, from Lemma 5,

$$\frac{\partial EW_0(P,q)}{\partial q} = \int_0^1 [S'(F(p,0))F_2(p,0)h(p,0) + S(F(p,0)h_2(p,0)]dp$$

$$= \int_0^1 [S'(0)g(p|A)g(p|B) + S(0)(g(p|A) - g(p|B))]dp$$

$$= b_1\int_0^1 [g(p|A) + g(p|A)g(p|B) - g(p|B)]dp$$

$$= b_1\int_0^1 g(p|A)g(p|B)dp.$$

But $b_1 < 0$ and

$$0 < \int_0^1 g(p|A)g(p|B)dp < 1.$$

Hence $\partial EW_0(P,0)/\partial q > b_1 = S'(0)$.

The result for q = 1 follows analogously. Properties of Γ_0 are obvious from inspection of figure 2. q.e.d.

It was claimed on page 14 that $EW_1(p,q)$ is differentiable

Proof of Claim: Consider the following: $EW_1(P,q+\delta) - EW_1(P,q) =$

$$\int_0^{\pi_B(q+\delta)} S_B(F(p,q+\delta))h(p,q+\delta)dp + \int_{\pi_B(q+\delta)}^{\pi_A(q+\delta)} EW_0(P,F(p,q+\delta))h(p,q+\delta)dp$$

$$+ \int_{\pi_A(q+\delta)}^{1} S_A(F(p,q+\delta))h(p,q+\delta)dp - \int_0^{\pi_B(q)} S_B(F(p,q))h(p,q)dp$$

$$- \int_{\pi_B(q)}^{\pi_A(q)} EW_0(P,F(p,q))h(p,q)dp - \int_{\pi_A(q)}^{1} S_A(F(p,q))h(p,q)dp$$

$$= \int_0^{\pi_B(q+\delta)} [S_B(F(p,q+\delta)) - S_B(F(p,q))h(p,q)]dp - \int_{\pi_B(q+\delta)}^{\pi_B(q)} S_B(F(p,q))h(p,q)dp$$

$$+ \int_{\pi_B(q)}^{\pi_A(q+\delta)} [EW_0(P,F(p,q+\delta))h(p,q+\delta) - EW_0(P,F(p,q))h(p,q)]dp$$

$$+ \int_{\pi_B(q+\delta)}^{\pi_B(q)} EW_0(P,F(p,q+\delta))h(p,q+\delta)dp - \int_{\pi_A(q+\delta)}^{\pi_A(q)} EW_0(P,F(p,q)$$

$$+ \int_{\pi_A(q)}^{1} S_A(F(p,q+\delta))h(p,q+\delta) - S_A(F(p,q))h(p,q)]dp$$

$$- \int_{\pi_A(q+\delta)}^{1} S_A(F(p,q+\delta))h(p,q+\delta)dp.$$

Hence, $\lim_{\delta \to 0} \frac{EW_1(P,q+\delta) - EW_1(P,q)}{\delta} =$

$$\int_0^{\pi_B(q)} [S_B'(F(p,q))F_2(p,q)h(p,q) + S_B(F(p,q))h_2(p,q)]dp$$

$$+ \int_{\pi_B(q)}^{\pi_A(q)} \left[\lim_{\delta \to 0} \frac{EW_0(P,F(p,q+\delta))h(p,q+\delta) - EW_0(P,F(p,q))h(p,q)}{\delta}\right]dp$$

$$+ \int_{\pi_A(q)}^{1} [S_A'(F(p,q))F_2(p,q)h(p,q) + S_A(F(p,q))h_2(p,q)]dp.$$

But $EW_0(P,F(p,q))$ is constant for $p \in [\pi_B(q), \pi_A(q)]$. Hence $\partial EW_1(P,x)/\partial q$ is well-defined as x goes to q from above or below. The desired result then follows from the analogous observation that

$$\lim_{\delta \to 0} \frac{EW_1(p,q) - EW_1(p,q)}{\delta} =$$

$$\int_0^{\pi_B(q)} [S_B'(F(p,q))F_2(p,q)h(p,q) + S_B(F(p,q))h_2(p,q)]dp$$

$$+ \int_{\pi_B(q)}^{\pi_A(q)} \left[\lim_{\delta \to 0} \frac{EW_0(P,F(p,q))h(p,q) - EW_0(P,F(p,q-\delta))h(p,q-\delta)}{\delta}\right]dp$$

$$+ \int_{\pi_A(q)}^{1} [S_A'(F(p,q))F_2(p,q)h(p,q) + S_A(F(p,q))h_2(p,q)]dp.$$