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MULTIPLE ITEMS, ASCENDING PRICE AUCTIONS: AN EXPERIMENTAL EXAMINATION OF ALTERNATIVE AUCTION SEQUENCES

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SOCIAL SCIENCE WORKING PAPER #1406

APRIL 16, 2015

Multiple Item, Ascending Price Auctions: An Experimental Examination of Alternative Auction Sequences

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April 2015

Abstract

The paper investigates the revenue and efficiency of different ascending price auction architectures for the sale of three items and five bidders. Four architectures are studied: two different sequences of single item auctions, simultaneous auctions with a common countdown clock, and simultaneous auctions with item specific countdown clocks. A countdown clock measures the time until the auction closes but resets with each new bid. The environment contains independent private values, no uncertainty about own preferences, no information about other's preferences, and a one unit budget constraint. The Nash equilibrium best response with straight forward bidding fits both dynamic and outcome data well. When non-unique Nash equilibria exist as in the case of simultaneous markets with a common clock, the social value maximizing Nash equilibrium emerges as the equilibrium selection. Both total revenue and efficiencies depend on the architecture as predicted by the Nash model, with the exception of the independent clocks architecture, which performs poorly on all dimensions.

1. Introduction and overview

The study is motivated by a simple question. What sequence of auctions should an auctioneer use when selling multiple items using ascending price auctions? This question decomposes into three questions. (i) What theory will help answer the question for the typical conditions an auctioneer might face when posing the question? (ii) What does the theory suggest as an answer? (iii) How should one approach finding an answer to the previous two questions? Of course, the answers depend on many features of the environment and details of the auction architectures. Our approach to the third question is to focus on a specific experimental environment with fixed bidder preferences and information. We then vary the auction architectures within a class of ascending price auctions and ask which features of theory help explain auction results. If successful, the theory or its extensions can then be used to offer answers to the questions for

[1] The financial support of the Gordon and Betty Moore Foundation is gratefully acknowledged. Special acknowledgement is given to class members Sarah Brandsen, Wen Min Chen, Rebecca Hu, and Emily Jensen who contributed to the early development of the research. We thank Matthew Elliot, Ben Gillen, Dave Grether, Katrina Scherstyuk, Kirill Pogorelskiy, Fabio Michelucci and Robert Sherman for their many helpful comments.

specific cases including different preferences, different information, and perhaps different forms of ascending price auctions. Or, the theory might indicate what needs to be known in order for the questions to be answered.

We will focus on a specific environment. The auction environment we study has the following major features:

- Three distinct items and only one unit of each item is for sale.
- Five bidders participate with different preferences.
- A bidder's own preferences for all items are known with certainty.
- A bidder can purchase no more than one item so synergies are not an issue.[2]
- No public information is available about bidder preferences. Bidders know only their own preferences.
- Bids are anonymous. Leading bidders know if their bid is leading.

The details of preferences and preference inducements are discussed in Section 3**.**

Four different auction architectures are studied. The common features of all architectures are:

- ascending bids,
- a small increment requirement,
- countdown clock(s) that resets with new bids and resumes a countdown,[3]
- auction ends when the countdown clock reaches zero, and
- item allocation to the leading bidder at the time of bid close.

The differences among the architectures considered are:

- whether the items are auctioned in sequence,
- whether the auctions for items auctions are conducted simultaneously with a common countdown clock,

[2] When synergies and complementarities exist the continuous, ascending price structure can be used successfully and efficiently when the bids are packages. For the performance of such an auction in experiments and in the field, see Plott, Lee, and Maron (2014).

[3] The resetting countdown clock has been used routinely in market experiments since the early 1970's when the auctioneer/experimenter used a watch and added seconds to the end time as new bids and asks were tendered. Later, this became a large physical clock that would reset and countdown when the auctioneer pressed a button in response to new bids. This became a resetting countdown clock on the computer. This type of auction ending was called a "soft close" as opposed to a "hard close" that ended the auction when a preset time was reached. Additional types of clocks are used in complex auctions, such as combinatorial auctions, in order to encourage the pace of bidding. See Plott, Lee and Maron (2014).

- whether the item auctions are conducted concurrently with independent countdown clocks, and
- if auctioned in sequence, the sequence in which the items are auctioned.

The details of the auction architectures studied are discussed in Section 4. The basic architecture employed here has a family resemblance to other types of ascending price auctions. Clock auctions (Japanese auctions) are based on prices that move automatically and all bidders are in the auction unless they indicate that they are drop out. Rounds auctions proceed in stages in which all bidders place bids at the same time. Auctioneers "call" auctions by announcing prices and seeking bidders who will agree to the bid. Open outcry auctions have bidders tendering bids as governed by an increment rule and a closing rule. The auctions architectures we study are a class often used by auctioneers and are based on an continuous open outcry feature that prevents simultaneous submissions, an increment requirement and a closing rule dictated by the countdown clock.

Section 5 is a discussion of theory and models. Our analysis is focused primarily on "market level" variables such as prices and efficiency. The basic model tested and applied is a multiple item generalization of best response dynamics and resulting Nash equilibria that is routinely applied to describe behavior in the single item case. When applied to the sequential auction case, the classical model is tested with the bidder with the highest value winning at a price equal to the value of the second highest bidder. The theory when applied to the simultaneous auction is more complex due to multiple Nash equilibria. The allocation that maximizes the social value (the sum of the valuations of item holders) can be supported as a Nash equilibrium of the simultaneous auction. The experimental environment used to test the models is outlined in Section 6.

The results of the experiments are contained in Section 7. Strong support exists for the Nash equilibria. Support also exists for the best response dynamic model in which bidders exhibit a "straight forward" bidding strategy in which strength of preference appears to play a role. Dynamic exceptions appear as jump bids, which seem to be motivated by attempts to speed the auction and by what we will call inertia. In the simultaneous auctions condition performed under a common clock, the welfare maximizing Nash equilibrium emerges as a reasonably accurate model.

The summary of conclusions is Section 8. Both theory and results demonstrate a clear answer to our primary question. The auction sequence does make a difference. However, in order to select the revenue maximizing sequence the auctioneer would need enough information about preferences to determine the Nash equilibria of the competing architectures.

2. Related Research

A long history of theory about behavior of ascending price auctions rests on the hypothesis that bidders will follow a "simple", "straight forward", "best response strategy", Nash response or Nash best response, when engaged in single item auctions; see Brewer (1999), Milgrom (2000).[4] The model holds that given the existing bids on items a bidder will place a bid that maximizes profits should it win. The possible reactions of other bidders are not used in the bid decision and indeed, the preferences of others need not be known as is the case as the model is used here. While exceptions to the model can exist, especially in the form of jump bidding, the observed patterns of bidding and the final outcomes often tend to conform to those of the "straight forward bidding" model, and the model naturally extends to explain patterns that appear to be exceptions (Salmon, Isaac, and Zillante (2005, 2007).

Whether or not the model naturally extends to predict behaviors of multiple item auctions including those where bidders must operate within constraints are not questions that are extensively explored in the literature. Early experiments with simultaneous, ascending price multiple item experiments were conducted by Plott (1997) as part of the testing of auction architectures subsequently used by the FCC in the spectrum auctions. The auctions studied budget constrained bidders using bidding round rules similar to those subsequently employed by the FCC. The evidence suggests that convergence was to an efficient competitive equilibrium but neither the Nash properties nor the dynamic path were studied. The experiments reported here embody both features, multiple items and individual constraints. The models extend themselves naturally to more complex environments so the question is not so much about what version of theory might be applied as it is about how accurate the theory might be when it is applied. Extensive tests do not exist in the literature but the results from Plott and Salmon (2004) suggest

[4] For a review of single item, multiple unit auctions see Kwasnica and Shertyuk (2013).

that natural extensions of the model will work well to capture much of the behavior of multi item environments. The principle requires that bidders follow a best response when switching from one item to another just as they use the principle when deciding whether to place a bid in a market with only one item. Given the bids of others, place a bid in order to become the item's leading bidder and do so on profit maximizing terms.

Our basic question is which sequence produces more revenue. The answer to our question seems to be suggested by the "revenue equivalence theorem" (Myerson, 1981). Namely, in an expectations sense the revenue produced by architectures in a class of environments is expected to be the same. We are not addressing that model and do not attempt to test it here or to implement conditions under which the theorem might apply. Instead, we explore conditions that might be faced by an auctioneer that lacks information about preferences. The beliefs that bidders hold about the values of other bidders or the expectations that bidders might hold about item prices are unknown to the auctioneer. Bidder beliefs about the strategies of other bidders are unknown to the auctioneer. In addition, bidders are constrained to purchase only one unit of any item and the items are different. Indeed it is known that item sequence can make a difference and sometimes in surprising ways, see Grether and Plott (2009). The revenue equivalence theorem is a useful benchmark that supports a need to investigate special cases as explored in this paper.

3. The Basic Economic Environment

All experiments studied were constructed from a common basic set of parameters. Experiments study the behavior of five types of bidders $\{a, b, c, d, e\}$ assigned to five bidders who compete for three items $\{A, B, C\}$. The items are different and only one unit of each item is offered for auction. A bidder can acquire at most one item. Thus, a "type" is a list of three numbers assigned to a bidder. The types rotate among subjects over the course of the experiment but the facts of rotation and a fixed set of types is unknown to subjects. For each agent $i \in \{a, b, c, d, e\}$ and item $x \in \{A, B, C\}$, consider the quasi-linear utility function of the form $U^i(x) = V^i(x) + M^i - p(x)$, where $V^i(x)$ is the value the agent places on x, $p(x)$ is the price of the item, and M^i is the money held by the individual, where M^i is sufficiently large to be non-binding.

Table 1: Basic Preference Parameters

BASIC				
	type	A	B	C
	a	211	70	70
	b	130	80	29
	c	183	90	43
	d	150	43	56
	e	225	143	14

Suppose the values of the bidders are as shown in Table 1, which are used as the basic parameters for experiments, as will be explained in later sections. A significant feature of this table is the allocation that would yield the maximum possible "social value" from the distribution of the three items given that bidders can have only one item each. The maximum occurs at $(\{A, a, 211\}, \{B, e, 143), \{C, d, 56\})$ where the notation is {item, type, value}, and the maximum possible social value is 410 (= 211 + 143 + 56). This maximum possible value, often referred to as the social maximum, plays a role in the computation of performance efficiency, efficiency = [surplus at outcome/ maximum possible surplus]. The allocation plays a basic role in the selection of an outcome prediction from among the many possible Nash equilibria.

4. INSTITUTIONAL ENVIRONMENT: Auction Architectures and Rules

We study four architectures that are commonly used. Conspicuously absent is the architecture used by eBay, which uses a fixed time for the auction ending. By contrast, all auctions studied here remain open as long as the bidding remains sufficiently active, as imposed by countdown clocks.

All auctions studied have ascending bids with a minimum increment requirement of 1. The auctions are continuous in the sense that a bid can be tendered at any time for any item open for bidding. When a bid is tendered in a continuous auction the bidder knows the state of the system, all existing bids, at the time of the bid. A bid higher than the current leading bid for the item resets the auction's countdown clock (20 seconds in these auctions) that reports the time remaining until the auction closes. The auction's clock then resumes the countdown. Thus, the auction only ends when sufficient time has elapsed from the last bid.

An important property of auction architectures is the nature of the sequence with which items are offered for bidding. Single item auctions are those in which the items are offered for auction one at a time in some predetermined, public sequence. Each of the different possible sequences is, in essence, a different architecture. The bidding does not start on an item until the auction of the previous auction has closed. Suppose the items are offered in the sequence A, B, C. Then A is offered for auction and the bid clock starts counting down. With each bid on A the bid clock resets and starts the countdown. When the countdown clock reaches zero, the leading bidder wins item A and pays the bid price. After the auction for item A has closed the bidding starts for item B. The process continues until all items are sold.

Simultaneous auctions with a common clock have all items open for bidding simultaneously. Bids can be tendered at any time for any item. A common bid clock operates to measure the remaining time in the auction. The clock resets when a bid is tendered for any item. When the time on the clock reaches zero, the auction is closed and the leading bidder on each item wins the item and pays the bid price for the item.

Simultaneous auctions with independent clocks are continuous auctions, each of which is controlled by its independent clock. All items are open for bidding at the same time but the bid clock on an item does not change until a bid is placed on the item. When the item receives a bid, its bid clock resets to a predetermined number of seconds and starts the countdown. As will become clear, the size of the reset is an important variable. The clock for a given item is only reset by bids on the specific item as opposed to any other item as is the case with the common clock. The auction on an item is closed when its bid own bid clock reaches zero at which time the leading bidder for the item becomes the winner and pays the bid price. Notice that the auctions on items can close at different times. For example, if the pace of bidding on and item is steady, then the item remains open for bidding. However, if the pace becomes slow, the auction for the item can close while the auctions for other items remain open.

5. Models, General Theory and Predictions

The modelling effort has a limited scope. The basic model is Nash best response model with agents who have limited information and follow simple or straight forward strategies. In all cases, no information is distributed about competitor preferences or individual competitor decisions so the terms Nash best response or Nash equilibrium is appropriate even though it is inconsistent with an often used convention of assuming preference information is public when applying the Nash equilibrium theory. We hope that our departure from this convention is not confusing. Three forms of the model are applied. The first is an "unconditional" model that has bidding based on the parameters specified in the experimental environment and not modified by "local" circumstances, the history of play or expectations. The equilibria are calculated along an equilibrium path. Given the parameters as initially specified all efficiencies, prices and incomes can be calculated and predicted. It is a natural model to apply if nothing is known about events that take place during the auction. For the "unconditional" application, Table 2 contains for each set of experimental conditions the predictions of the Nash best response equilibrium model for item prices, total revenue from the auction and efficiency of the auction. Since the model predicts the winner and the prices, the incomes can be deduced from the known types.

The second version of the model is a "local", "myopic" or "conditional" version that has bidding decisions based on the narrow circumstances that exist for the bidder at the time of decision. Previous decision, including errors can shape the decisions at any instant for which the model is being applied. For example, in the sequential auction the winners of previous auctions are not present in current auctions and thus the parameters of a current auction are influenced in a fundamental way by what happened in previous auctions. The model is applied to make predictions for each auction conditional on what happened in the past. A "mistake" in the first auction can influence the outcome of all subsequent auctions and the model can be applied to predict what will happen. Of course, the model has limited uses given the task of deciding among auction architectures since the predictions of the model depend on events that cannot be known beforehand. However, it is possible to examine the data after the auction is over to determine if the behavior was consistent with the model and that is the application applied here.

When applied as a dynamic model, especially for the sequential auctions, an unobserved, expectations parameter can be added to the Nash best response. At each point in the auction, each agent $i \in \{a, b, c, d, e\}$ has some expected, non-negative, gain G^i from later auctions if the agent does not win the current item, based on the private information and the information gained from the previous bids. Natural properties to impose on G^i are that (1) it can fluctuate according to any additional information gained from the patterns observed in the bids, that (2) it decreases as each item is sold, since fewer opportunities of gain remain, and that (3) $G^i = 0$ for the last item, since there is no item to be sold after the last item. With the expectations parameter added, each agent stops bidding once the price reaches $V^i(x) - G^i$, as opposed to $V^i(x)$ as predicted by the unmodified Nash best response model. While the consequences for the simultaneous auction architectures are unknown, when applied to the sequential auction architectures the model suggests that the error of the unmodified Nash best response model will decrease with each item due to property (2) and that the error from the auction of the last item to be zero due to property (3).

As shown in Table 2, the equilibrium predictions of the unconditional model will be considered for four different auction architectures. The conditional applications of the model will be considered in the results section. The first two architectures are sequences chosen to reflect different predictions. The third and fourth architectures are variations in which all items are open simultaneously. The details of the rules all architectures will be explained in later sections. Here we will just state the differences in the equilibria that result from auction rules that will be explained later. Architecture (1): The items are auctioned in the sequence ACB, meaning that A is auctioned first, then item C is auctioned and then item B is auctioned. Architecture (2): The items are auctioned in the sequence BAC. Architecture (3): The auction for all three items is opened simultaneously with a "common clock". Architecture (4): The auction for all three items is opened simultaneously with "independent clocks" that might close the item auction separately.

Table 2: Predictions from Nash Model

	total Revenue	efficiency	Price			Winner		
architecture	*model*	*model*	*A*	*B*	*C*	*A*	*B*	*C*
ACB(ABC)	*347*	*0.94*	211	80	56	e	c	a
BAC	*316*	*1*	183	90	43	a	e	d
common Clock fast response independent clocks	*316*	*1*	183	90	43	a	e	d
Slow response independent Clocks	213	.88	211	1+	1+	e	b	d

Architecture (1): Sequence ACB

For auctions in which the items are offered in sequence, the predictions of the model can be easily computed. The individual with the highest value among the set of allowed bidders will be the leading bidder and the price will be the value of the agent with the second highest value. Since all agents have different value this model leads to a unique (agent, item) pairs as predictions for any given sequence for which the items are offered for auction. Total revenues will depend on the sequence with which the items are offered.

Architecture ACB. Item A is auctioned first. The Nash model with simple/straight-forward bidding holds that the individual with the highest value will win the item and pay a price equal to the value of the second highest. Thus, according to the model the item goes to type e, who has a value of 225 and pays a price equal to the second highest value of 211 that is held by type a. When item C is auctioned next, type e can no longer buy, having bought item A in the first auction. Item C will be purchased by type a at a price of 56. Item B will be auctioned to one of the remaining types {b,c,d} and will be sold to type c, whose value is 90, at a price of 80, the value of type b who has the second highest value of the three remaining bidders. Thus, the winning {item, types, personal value and price} for all items are ({A, e, 225, 211}, {B, c, 90, 80}, {C, a, 70, 56}). The total revenue will be the sum of the item prices, 211+80+56 = 347. The predicted surplus (efficiency) is 385 (0.939). If the expectations term exists in the bidder behavior then the errors of the Nash best response model are expected to decrease with each item and to become zero for the last item.

Architecture (2) Sequence BAC

In this architecture, item B is auctioned first, followed by the auction of item A and then the auction of item C. The winner of item B is type e who pays a price of 90, which it the value of the second highest bidder. Item A is auctioned among the remaining bidders, types a,b,c, and d. The winner of item A is type a with a value of 211 and who pays a price of 183. Item C is the last to be auctioned to the remaining bidders' types b, c, and d. The winner is type d who pays a price of 43. Thus, the outcome of the entire auction in terms of {item, winner, value, price} are ({A,a, 211,183}, {B,e, 225, 90), {C, d, 56, 43}). The total revenue will be the sum of the item prices, 183+90+43 = 316. The predicted surplus (efficiency) is 410 (1.00). If the expectations term exists in the bidder behavior then the errors of the Nash best response model are expected to decrease with each item and to become zero for the last item.

Architecture (3): Simultaneous Auctions with Common Clock

For the common clock auction, there are typically many Nash equilibria for the environment we study. An increment requirement exists; only one unit of each item is sold and each agent is restricted to at most one item. An individual that tenders something other than a best response can find himself/herself a leader on an item and unable to bid on some other item that promises more profit.

The most efficient Nash equilibrium in this environment is closely related to a competitive equilibrium which can be found by a linear program. Find the allocation that maximizes the sum of the values of those receiving an item subject to no more than one of each item sold and no agent receives more than one item. The dual to this optimization problem can be viewed as competitive equilibrium prices and under our parameters the allocation and associated prices interpreted as bids can support the allocation as a Nash equilibrium to the auction. [5]

The surplus maximizing allocation as outlined above is ({A, a, 211}, {B, e, 143), {C, d,56}) and the competitive equilibrium prices are (183,90,43) for items A,B and C respectively. Thus, the predicted revenue is 316. The predicted surplus (efficiency) is 410 (1.00). The predicted outcome is the same as that of Architecture BAC but the application of theory is different.

In order to demonstrate that the prediction is a Nash equilibrium (in the best response sense), one needs to only check to see if a strategy exists for any bidder to improve his payoff given the bids of others. That no improvements exist is demonstrated by the following set of equations where $V^i(x)$ is the value of the item found the Table 1 and $P(x)$ is the equilibrium price of the item.

[5] See Kranton, R. and D. Minehart (2000), Milgrom (2009). See also G, Demange, D. Gale and M. Sotomayor (1986) who demonstrate convergence to near a competitive equilibrium by a Nash best response form of auction.

Type a: wins A : $V^a(A) – P(A) = 28 \rho V^a(C) – P(C) = 27 \rho V^a(B) – P(B) = -20$

Type b: wins 0 : $0 \rho V^b (B) – P(B) = -10 \rho V^b (C) – P(C) = -14 \rho V^b(A) – P(A) = -53$

Type c: wins 0 : $0 = V^c(A) – P(A) =0 \rho V^c(C) – P(C) = 0 \rho V^c(B) – P(B) = 0$

Type d: wins C : $V^d(C) – P(C) = 13 \rho V^d(A) – P(A) = -33 \rho V^d(B) – P(B) = -47$

Type e: wins B : $V^e(B) – P(B) = 53 \rho V^e(A) – P(A) = 42 \rho V^e(C) – P(C) = -29$

Given the equilibrium prices, no agent can improve utility by purchasing any item they are not allocated.

The predicted outcome is one among many possible Nash equilibria when the auction is simultaneous, ascending price with a common clock. What might be called a "support" for Nash equilibria consists of three different outcomes [{A,e,211},{B,c,80},{C.a.57}] ; [{A,c,151},{B,e,90},{C,a,57}]; [{A,a,183},{B,e,90},{C,d,43}]. Each of these three serves as an index for many different prices that will be Nash equilibriums. Indeed, for each of the equilibria in the support set almost every price from the base equilibrium price up to the value of the item leader can be part of a Nash equilibrium. The dynamics that might lead to the establishment of the alternative (path dependent) equilibria could reflect jump bids, mistakes, bluffs, attempts to create expectations in other bidders or other elements of strategic behavior as providing appropriate behavior to get to any of these equilibriums; see, Avery (1998); Isaac, Salmon, and Zillante (2007); Raviv (2006,2008); Salmon, Isaac, and Zillante (2005,2007).

Architecture (4): Simultaneous Auctions with Independent Clocks

The independent clock architecture allows auctions to close at different times, which can influence revenue and the time at which the clocks stop the auction depend on the speed with which bidders shift from one item to another. For example, an item could receive a bid at the open of the auction. Its clock would start. If the potential profits from other items remain attractive sufficiently long, attracting the bids of other bidders, the clock of the item in question can run down and the item sell for the opening bid.

If bidder response is fast, all clocks will remain counting and the auction will behave the same as the simultaneous common clock auction. If bidder response is not fast, then the results are sensitive to the speed with which they place bids. An extreme case on the impact revenues if bidders are slow can be developed by following the values in Table 1. The auction starts with all bidders bidding for item A, which is the most valuable for all of them. As the price of item A increases to 50, type b becomes indifferent between bidding 51 on A with potential profit 130-51 and bidding 1 on item B with potential profit 80 -1. The Nash response requires that b would place a bid on B, setting the price of B at 1 and causing the clock for B to set to 20 and count down. The next bidder that will be forced away from A to item B would be type e but that event will not happen until the price of A reaches 82. If there is as much as a 1 second delay between bids, the B clock will reach zero before the price of A will ascend sufficiently to force type e to place a bid on B and reset the clock. Thus, the auction for item B would close and the price of B would be 1. A similar dynamic could occur at any point in the auction.

The outcome of the independent clock architecture also depends on the unknown speed with which bidders react to the bids of others. If bidders react quickly, then the independent clock architecture behaves like the common clock and thus has the same equilibrium, revenue and efficiency as the common clock and will be called the "fast response" case. If the response of bidders to the bids of others is sufficiently slow, an auction can close while bidders who might be able to make profitable bids are busy bidding on other items. At the other extreme is the slowest possible bidding while remaining consistent with the best response model, the (slowest) equilibrium.

In summary, the simultaneous independent clock architecture has two extremes: The fast response time assumption yields: ({A, a, 211, 183}, {B, e, 143, 90}, {C, d, 56, 43}) revenue 316, social value (efficiency), 410 (1.00). The slow response time model yields: ({A,e,225,211}, {B,b,80,1},{C,d,56,1}), revenue 227, social value (efficiency), 361 (0.88).

6. Experimental Environment

Preferences, types, and values (increment, clocks -timing and stopping, information). All experiments are conducted with five bidders competing for three items, A, B, and C. Only one of each of the items is sold and a bidder can win at most one item. The experiments and models are built from a base parameter set shown in Table 1. The items are indexed as A, B, and C. Induced values are constructed in terms of a unit of currency called "francs" that are worth $0.20 each.

Individual incentives are defined in terms of a "type" that defines a value in francs for each of the three items. There are five types {a,b,c,d,e}. As shown in Table 1, type a places values 211, 70, 70 francs on the three items A,B,C, respectively; type b has franc values 130, 80, 29 on A,B, C respectively. For any auction, each bidder is assigned one of the five types of incentives. The assignment of bidders to types differs from auction to auction with a bidder never being assigned the same type twice except on rare occasions near the end of a session.

Table 3: Experimental Design

session	Sequence_ types allocation to IDs (low to high)
1	ACB_abcde
2	BAC_ceadb
3	ACB_bcdea
4	BAC_ebcad
5	CClk_cdeba
6	IClk_eacdb
7	ACB_abcde repeat 1
8	CClk_cdeba repeat 5

Experiments were conducted as a series of auction with the architecture and incentives changing in each. When subjects logged into the experiment, they were assigned a personal ID that was maintained throughout the experiment and known only to the subject. The sequence with which the architectures were deployed in the experiments were fixed across experiments and are listed in Table 3. The first auction in an experiment was ACB_abcde as listed in the table. This means that the items were auctioned sequentially in the order A, then C, and then B. The letters abcde means that in this auction the person who logged into the experiment first was assigned type a for the auction. The second person who logged in to the experiment was assigned type b for this auction, etc. The second auction in an experiment was BAC_ceadb. In this auction, the items were auctioned in the sequence B, then and then C and the types were assigned c,e,a ... to the first, second and third person who logged in, etc. The auction CClk_cdeba was a common clock auction and the auction IClk_eacdb was an independent clock auction. The lower case letters indicate the assignment of types to individuals according to original login sequence.

The subjects were all students from the California Institute of Technology. The subjects were instructed about the auction rules and the methods of tendering bids, the clocks and other features of the software by watching a short instructional video. To induce item preferences, each subject was assigned a set of three "redemption values" for the items in each period. These redemption values were those of the appropriate type for the subject for that auction (refer to appendix for experimental procedures and instructions). A total of 45 experiments were conducted. A list of experiments is the content of Table 4.

7. Results

The price formation paths for the three architectures are illustrated in Figures 1a, 1b and 1c. Figure 1a is the time series from a typical experiment in which the auctions are conducted sequentially. The first item auctioned is item A, followed by an auction for item C and then an auction for item B. The time series in each market illustrates the nature of the convergence path of the separate auctions including jumps and hesitations as the price moves up to near the equilibrium. Figure 1b is an example taken from an experiment in which the auctions were conducted simultaneously with a common clock. The figure illustrates a typical pattern with the bidders attracted to market 1 (item A) first because all bidders value it the most. As the price for item A increases the bidding shifts to market 2 (item B) and finally to market 3 (item C). Bidding activity shifts among markets until all have equilibrated. Figure 1c is an experiment in which the auctions were conducted with independent clocks. The earlier bidding is attracted to item A, as is the case with other architectures. However, other items receive early bids as a strategy reflecting the possibility that the clock would run down and the item would sell cheaply. The time structure reflects lags as bidders in other markets move their activity to reset the clock before it runs down and the opportunity to buy is lost. In some cases the clock does run down without new bids resetting the clock and the item is sold cheaply. Notice, for example, that item C is sold at a price of 24 in the simultaneous, independent clock auction of Figure 1c while the item is sold for a price of 31 in the sequential auction for Figure 1a, where it is sold second, and a price of 42 in the simultaneous auction with common clock in Figure 1b.

Table 4: List of all 45 experimental sessions conducted over six days. Experimental sessions with a given group of subjects typically lasted a consecutive two hours either in the morning or in the afternoon of a given day. Subjects saw items labeled as markets 1, 2 and 3 as opposed to A,B and C so switching of items to markets helped mask any similarity of parameters across sessions.

All Experiments (Forty Five Experiments)

Date	Session	Architecture	Parameters
20141204	18	Sequential	ACB_abcde
20141204	19	Sequential	BAC_ceadb
20141204	20	Sequential	ACB_bcdea
20141204	21	Sequential	BAC_ebcad
20141204	23	Common Clock	CClk_cdeba
20141204	24	Independent Clocks	IClk_eacdb
20141206_1	26	Sequential	ACB_abcde
20141206_1	27	Sequential	BAC_ceadb
20141206_1	28	Sequential	ACB_bcdea
20141206_1	29	Sequential	BAC_ebcad
20141206_1	32	Independent Clocks	IClk_eacdb
20141206_3	40	Sequential	ACB_abcde
20141206_3	41	Sequential	BAC_ceadb
20141206_3	42	Sequential	ACB_bcdea
20141206_3	43	Sequential	BAC_ebcad
20141206_3	45	Common Clock	CClk_cdeba
20141206_3	46	Independent Clocks	IClk_eacdb
20141206_3	47	Sequential	ACB_abcde
20141206_3	49	Common Clock	CClk_cdeba
20141209	52	Sequential	ACB_abcde
20141209	53	Sequential	BAC_ceadb
20141209	54	Sequential	ACB_bcdea
20141209	55	Sequential	
20141209	56	Common Clock	CClk_cdeba
20141209	57	Independent Clocks	IClk_eacdb
20141209	58	Sequential	ACB_abcde
20141209	60	Common Clock	CClk_cdeba
20141210	61	Sequential	ACB_abcde
20141210	63	Sequential	BAC_ceadb
20141210	64	Sequential	ACB_bcdea
20141210	65	Sequential	BAC_ebcad
20141210	66	Common Clock	CClk_cdeba
20141210	67	Independent Clocks	IClk_eacdb
20141210	69	Sequential	ACB_abcde
20141210	70	Common Clock	CClk_cdeba
20141210	71	Independent Clocks	IClk_eacdb
20141212	72	Sequential	ACB_abcde
20141212	73	Sequential	BAC_ceadb
20141212	74	Sequential	ACB_bcdea
20141212	75	Sequential	BAC_ebcad
20141212	76	Common Clock	CClk_cdeba
20141212	77	Independent Clocks	IClk_eacdb
20141212	78	Sequential	ACB_bcdea
20141212	79	Common Clock	CClk_cdeba
20141212	80	Independent Clocks	IClk_eacdb

Rotation of types are assigned to subjects in the subject order 1,2,3,4 and 5. Thus, the notation sequential ACB_abcde means the item A was sold in market 1, item C in market 2 and item B in market 3. Subjects were unaware of the item A,B,C labels used in the experimental design and instead knew the items only by the label of the market. Thus, except for the sessions in which the design called for an exact replication, even if a subject was the same type in two auctions the relationship would be difficult for the subject to detect because of this rotation of labels and because different architectures were employed in all instances. The dates and sessions, (x,y), in which type assignments were the same are: {20141206_3:(40,47) (45,49)},{20141209:(52,58) (56,60)}, {20141210: (61,69) (66,70) (67,71)}, {20141212:(72,78)(67,79)(77,80)}. Comparisons of the sessions suggest that the masking of types between sessions was successful.

Figure 1a. Bidding prices vs time of session 28 (sequential ACB).

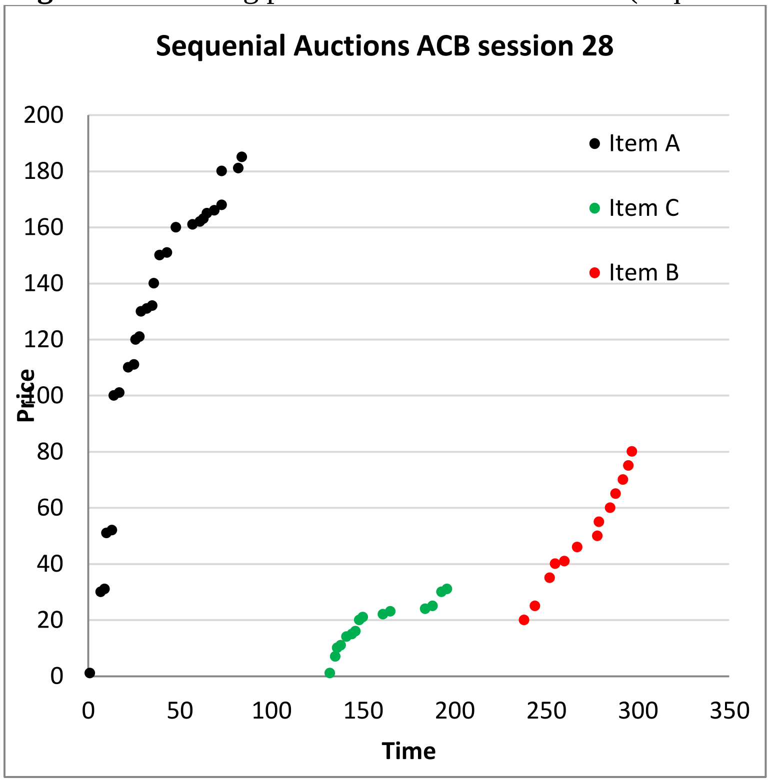


Figure 1b. Bidding prices vs time of session 70 (common clock).

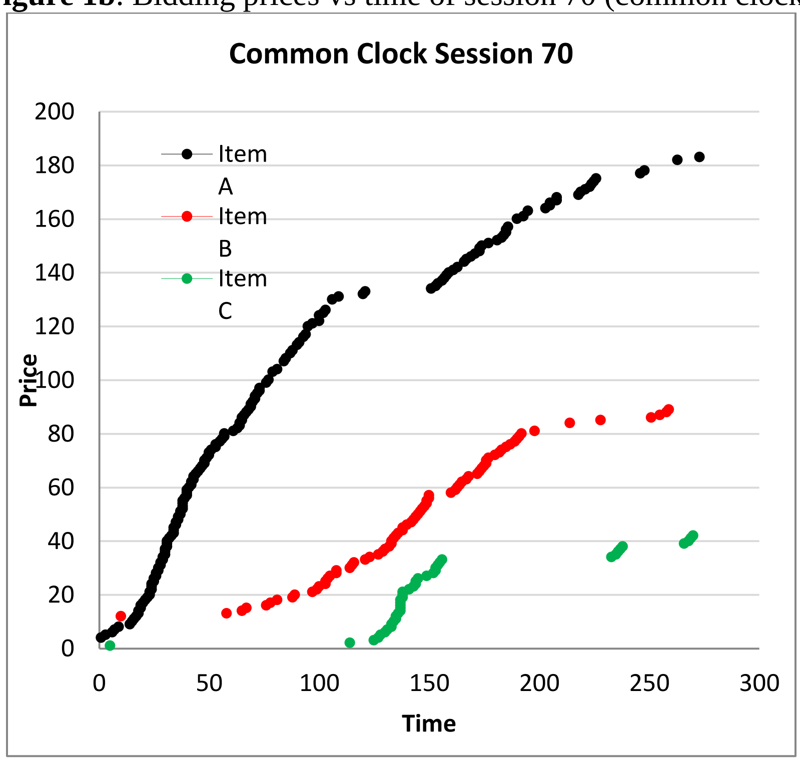


Figure 1c. Bidding prices vs time of session 24 (independent clock).

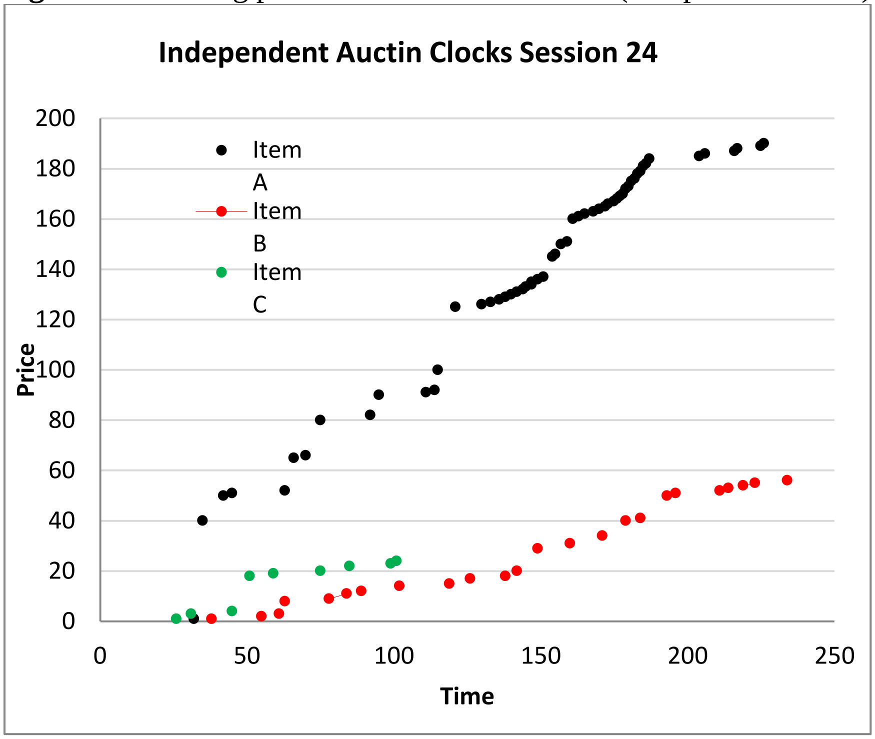


Table 5: Results model prediction tests, where if p < .05 reject the null that the mean of the error is zero.

		tot Rev			efficiency	tot rev	efficiency	income	Theory P
architecture	parameters	model	Total R	efficiency	model	error	error	error	minus P
ACB	mean	347	328	0.942	0.939	-19.0	0.00366	10.8	5.46
	SD		16.4	0.0271		16.4	0.0271	13.6	11.0
	n	16	16	16	16	16	16	80	48
	p	-			-	$3.51\text{x}10^{-4}$	0.597	$1.65\text{x}10^{-9}$	0.00126
BAC	mean	316	317	0.987	1.00	0.583	0.0128	17.6	-0.194
	SD		23.4	0.0243		23.4	0.0243	23.0	13.2
	n	12	12	12	12	12	12	60	36
	p	-			-	0.933	0.0947	$1.70\text{x}10^{-7}$	0.930
common C	mean	316	318	0.996	1.00	2.11	0.00379	18.1	-0.704
	SD		3.79	0.0114		3.79	0.0114	19.6	14.4
	n	9	9	9	9	9	9	45	27
	p	-			-	0.1332	0.347	$1.75\text{x}10^{-7}$	0.801
independent C		use fast response cc model							
	mean	316	282	0.981	1.00	-33.8	-0.0192	24.0	11.3
	SD		28.0	0.0309		28.0	0.0309	25.4	18.1
	n	8	8	8	8	8	8	40	24
	p	-			-	0.0114	0.122	$5.63\text{x}10^{-7}$	0.00580

Result 1. The order in which items are offered for sale in the single item, ascending price auctions conducted in sequence makes a difference to revenue and to efficiency. The difference is on the border of statistical significance reflecting, in part, the fact that the theory predicts only a small difference relative to prices (see Result 2).

Support:

(i) The appropriate statistics comparing revenues are in Table 5. The revenue from the sale in the order ACB is (328) which is more than the revenue produced from the order BAC, which is (317). The difference is significant at the 0.10 level with p = 0.07.

(ii) The appropriate statistics comparing efficiencies are in Table 5. The efficiency of the auction of the item order ACB is (0.937) which is less efficient than the order BAC (0.987). However, the difference in means is only slightly significant at T = 1.10 and p = 0.142 or a significant difference of approximately 0.15.

Notice that the higher revenue does not imply higher efficiency.

The next result assesses the auction relative to the purely theoretical Nash equilibrium bidding model, where $EQ - x = \varepsilon$, EQ is the prediction of either the unconditional or the conditional application of Nash equilibrium model, x is the observation and ε is the error. The Nash equilibrium, when used for modeling purposes requires a tight conformance between theory and behavior. If applied unconditionally to a sequence of auctions, the model has almost zero flexibility in accommodating patterns of behavior that deviate from those predicted by the theory. The analysis proceeds through the examination of the unconditional application, a conditional application and the addition of an expectations term.

The next two results indicate that the data for all three major measures for the sequential architectures are near or are indistinguishable from the predictions of the Nash equilibria of the appropriate model.

Result 2. The patterns of (i) revenues, (ii) efficiencies, (iii) incomes, and (iv) prices of single item, ascending price auctions conducted in sequence are explained by the simple Nash best response equilibrium model. The unconditional application of the model is very accurate and is supported by almost all measures but the conditional model is even more accurate because it is based on more information.

Support:

Two different analyses are required to capture different conditions under which the model is applied. The first is the unconditional application of the model. It examines the prediction of the model based on initial parameters and the assumption that the auction proceeds along the entire equilibrium path of decisions. The second application is conditioned on the auction behavior at various stages and compares the predictions at each instant modified by outcome of earlier auctions.

A. Unconditional application. Perhaps the unconditional applications best capture the overall accuracy of the model for the purpose of evaluation of auction architecture. Table 5 contains the appropriate data for all auctions with the items sold in sequence. Since the model applies regardless of the sequence the data for both sequence ACB and sequence BAC are comparable.

(i) Auction revenues are approximately the Nash equilibrium. As shown in the table, the average of the model for the sequence ACB is -19.0 or with SD 16.4, which is statistically distinguishable from zero, $p = 3.51 \times 10^{-4}$ but the error is very small, on the order of only 5% of the predicted value. For the auction sequence BAC, the mean error for the Nash equilibrium model when applied to total revenue is 0.58 with standard deviation 23.4. The hypothesis that the error is zero cannot be rejected, p =0.933.

(ii) The efficiencies of the allocation of the two sequences are those predicted by the Nash equilibrium model. For the sequence ACB, the error of the Nash efficiency prediction is 0.00244 which is close to 0 (p=.796). The error of the Nash efficiency prediction for the sequence BAC is 0.0128, which could be interpreted as marginally significant (p=0.094) due to a small SD = 0.024. However, the interpretation is clouded by the fact that the error can only be non-negative (the predicted efficiency is 1). Nevertheless the data suggest the model is accurate with nine of twelve observations of efficiency equal to 1.

(iii) Individual incomes are accurately modeled by the Nash equilibrium model for both the sequence ACB and the sequence BAC. The model error for ACB is 10.8 (SD13.6) with $p = 1.65 \times 10^{-9}$ and the error for BAC of 17.6 (SD 23.0) with $p = 1.70 \times 10^{-7}$ are distinguishably different from zero.

(iv) Prices for all three items are near predictions with errors of 5.46 (11.0) and -0.194 (13.2) relative to the average of predicted prices of 115 for sequence ACB and 105 for sequence BAC respectively. The prices for ABC are near the equilibrium levels but the error rate is distinguishable from 0 (p =0.001) but the error rate for BAC is equal zero statistically (p = 0.93).

B. The conditional application of the model measures the accuracy of the model when on predictions are conditioned on otherwise unpredicted events. Specifically, the conditional model defines active bidders as those who have not purchased in a previous an item in a previous auction. It predicts that items should always be sold to the agent with the top value of active bidders but if not then it will be sold to the agent with the second highest value from among the active bidders. If the agent with the highest value decides not to buy, then the buyer should be the bidder with the highest value of those remaining. Which agent has the highest value depends on the bidding that took place for the previously auction items.

The conditional prediction of the Nash best response model is that in any auction in a sequence of auctions, the item is sold to either the active bidder with the highest value or the active bidder with the second highest value. An examination of all sequential auctions reveals that the prediction holds 100% of the time. Specifically in the ACB auctions, the agent with the highest value wins for A, B and C respectively 81.25%, 81.25%, and 93.75% and in all cases in which the top value was not the winner, the second highest bidder was the winner. In the BAC auctions, the highest value wins for B,A and C respectively 83.33%, 1.00% and 91.66% and if the highest value agent did not win then the agent with the second highest value did win.

While the Nash best response model does capture the preponderance of the data in the sequential auctions, the difference between the conditional predictions and the unconditional predictions indicates the possible role of expectations. As demonstrated by the success of the conditional predictions, bidders who have the highest value for an item and thus should be the winner, have a slight tendency to stay in the auction until near the end and then stop bidding and save their capacity to buy for a subsequent auction. The bidding patterns suggests themselves as a measure of the expectations term, G, as discussed in Section 5, which should decrease from the first of the three items to the last at which point the expectation term should be zero.

Table 6: The error of the Conditional Nash best response equilibrium model decreases as the auction advances through a sequence of items thus producing evidence of bidding decisions influenced by expectations.

		sequence ACB			sequence BAC		
		A	C	B	B	A	C
model error	mean	8.375	3.937	2.875	9.000	0.583	-3.333
	STDEV	10.500	9.889	7.745	9.954	7.204	7.820
model absolute error	mean	10.625	6.188	3.250	9.000	4.417	3.500
	STDEV	8.041	8.573	7.585	9.954	5.567	7.740
	t (mean = 0)	0.000	0.011	0.107	0.009	0.019	0.146

Result 3. When expectation terms are added to the conditional application of the Nash best response model the added terms behave consistent with theory. The magnitude of expected gain from waiting falls with each item in sequential auctions and approaches zero in the final auction.

Support:

The tendencies of the discrepancies of winning prices from expected strongly supports the observation that the error should decrease from the first of the three items to the last at which point should be zero. Since we care about the dispersion from the model's prediction, it suffices to compute the absolute differences between the actual results and the predicted results. For the ACB-sequence auctions, the average errors were 10.6 for the first item (SD 8.04), 6.12 for the second item (SD 8.57), 3.25 for the third item (SD 7.59). For the BCA-sequence auctions, the average errors were 9.00 for the first item (SD 9.95), 4.42 for the second item (SD 5.57), 3.50 for the third item (SD 7.74). Notice that both sequences show a decrease, satisfying what was theoretically expected about the expectation term. Table 6 shows the t-test results. Notice that the t-tests for the first items rejects the null hypothesis of mean = 0 even at the strictest conventional level of 0.01. Both t-tests for the second items reject the null hypothesis under the 0.05 level, but fails to reject the null hypothesis under the 0.01 level. For the third items, the null hypothesis cannot be rejected under any conventional level (even for 0.10). This indicates that the differences have a tendency to decrease, and that for the third item, it is essentially indistinguishable from 0, as was proposed by the characteristic of the expectation term.

Result 4. The patterns of (i) revenues, (ii) efficiencies, (iii) incomes and (iv) prices of the simultaneous ascending price auctions with a common clock are predicted by the general competitive equilibrium model and are supported by the Nash equilibrium model appropriately generalized to apply to simultaneous markets.

Support:

The data are presented in Table 5

(i) The total revenue produced by the auction is near that predicted by the models. The model predicts revenue of 316 and actual revenues are on average 318 with an error of 2.11 (SD 3.8) or less than 1%. However, the low variance produces a $p = 0.1332$, which is statistically different from 0 at the 0.10 level. Models with such accuracy (low variance) are most appropriately tested relative to other models.

(ii) The efficiencies of the allocations are near 100% as predicted. The error is 0.004 with $p = 3.5$ which is statistically indistinguishable from 0.

(iii) Individual incomes are accurately predicted. The error is 18 which is very close to zero in a statistical sense, $p = 1.75 \times 10^{-7}$ which indicates that the hypothesis of zero error cannot be rejected.

(iv) The hypothesis that prices equal predicted prices cannot be rejected. The error is -0.7 (SD 14.4) relative to the average of predicted prices of 105. The $p = 0.8$ indicates that the hypothesis of zero error cannot be rejected.

Result 5. The auctions with independent clocks have lower revenue and lower efficiency than do the auctions with the common clock. However, efficiencies and revenue are always higher than those predicted by the very slow bidder speeds model.

Support:

As shown in Table 5, the average revenue from the independent clock auctions is 282 as compared with the 213 predicted by the slow bidding model and the 316 predicted by the fast bidding model. Efficiencies of the independent clock experiments are 0.98 on average as compared with 0.88 predicted by the slow bidding model and 1.00 predicted by the fast bidding model. Of the eight auctions conducted five had efficiencies of 1.00 and three had an average efficiency of 0.943.

The results above demonstrate that the patterns of winning items predicted by the unconditional or the conditional Nash best response theory closely conform to the predictions of theory. . A natural question addresses the degree to which the dynamics of the adjustment is captured by a best response model of bidding behavior. In the sequential auction environment, the Nash response is not challenged since only one item can receive a bid. All bids appear as Nash best responses unless the bid is above item value. However that is not the case when multiple markets are operating.

Result 6 and Result 7 ask about the proportion of bids that were a best response in the sense that no other bid by the bidder would have yielded a higher potential profit. If a bidder placed a bid, was it the best bid the bidder could have placed? Examination of the data reveals two types of strategies. The first is a clear choice of a most profitable option. The second suggests a bidder who had been bidding on a most profitable option but did not switch bidding immediately when the price of some other option changed to produce an alternative bidding strategy. The phenomenon is labeled as “inertia” and is measured by a continuation of at least three bids before a bid change. The third are bids that are neither best response nor inertia. Since all markets are open in the common clock auctions the expectations of future prices that appears in the sequential auctions should not be at work. The proportions of the three bidding patterns are shown in Figure 2 for the common clock case and in Figure 3 for the individual clock case. As can be seen, a large part of the bidding is a best response but there are substantial differences between the common clock and the individual clock auctions.

Figure 2. Proportion of Bids that are Best Response (Nash) and Inertial: Simultaneous Auctions Common Clock Sessions

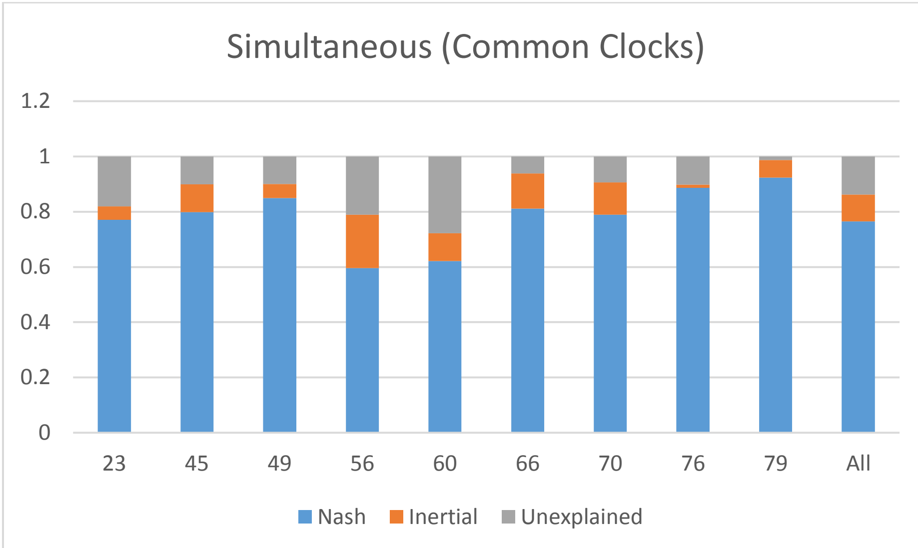


Figure 3. Proportion of Bids that are Best Response (Nash) and Inertial: Simultaneous Auctions Independent Clock Sessions

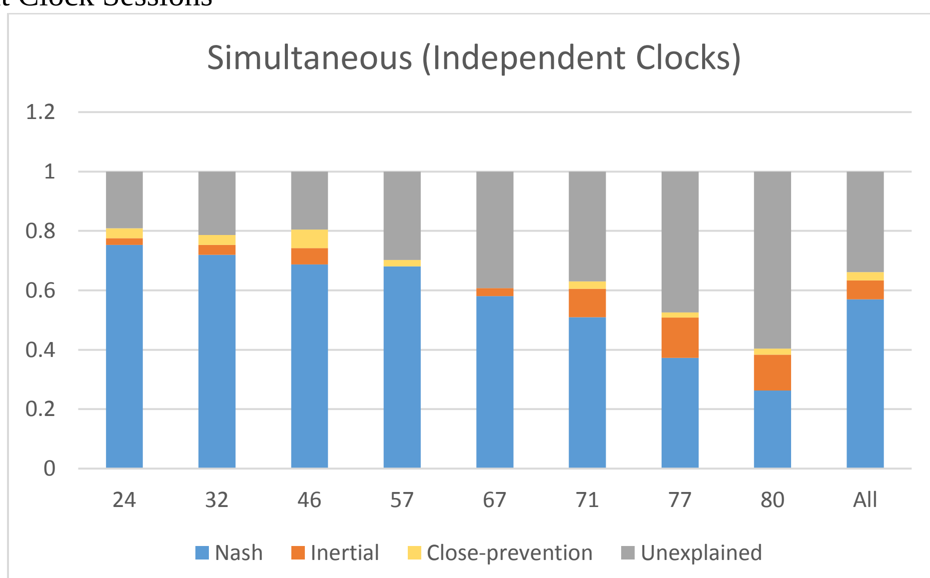


Table: 7: Proportion of Bids that are Best Response (Nash) and Inertial: Simultaneous Auction Common Clock Sessions

Session	Nash	Inertial	Unexplained
23	0.771	0.0482	0.181
45	0.798	0.101	0.101
49	0.850	0.0500	0.100
56	0.596	0.193	0.211
60	0.622	0.101	0.278
66	0.811	0.128	0.0612
70	0.790	0.116	0.0942
76	0.886	0.0114	0.102
79	0.924	0.0633	0.0127
All	0.765	0.0972	0.138

Result 6. The common clock bidding path follows the Nash best response model. Model errors are due to “inertia” or lack of attention.

Support:

Table 7 contains the proportion of best responses, the inertia responses and the unexplained responses in the common clock auctions. Of the bids placed in the simultaneous common clock auction, on average 76 of the bids are best response; 9.7 % are inertial and 13.8% are unexplained.

While the Nash best response model does capture the preponderance of the data in the simultaneous common clock auctions, a proportion is remains unexplained. The departure from equilibrium does not seem to be due to expectations because all prices are public at all times in the common clock auctions. We are at a loss for any explanation other than "inertia".

The next result reflects the facts illustrated in Figure 2 and 3. The independent clocks behave differently from the common clocks. Not only are fewer bids consistent with best responses, a new motive for bidding appears that is motivated by the nature of the independent clocks. Recall each market has its own clock that resets with bids, but this means that a market can close leaving some bidders with no purchases if the prices of the other markets increase enough to exclude them and would buy the item with the closed market had the market remained open. In particular, bids appear to be placed as an attempt to prevent an auction from closing. This can be the case if a bidder is competing for one item but the market for a different item is close to closing. The bidder can place a bid in the near- close market, which is quickly beat by the competitor in that market, but the bid keeps the auction open while the bidder returns to the original market.

Result 7. The best response model is consistent with more than half of the bids placed in the independent clock auctions. However, the best response model explains more data in the common clock auctions than in the independent clock auctions. In addition, inertial bids are present in the independent clock auctions (but a smaller proportion than in the common clock auctions). About 2.8% of the bids appear to be placed to stop a market from closing immediately, indicating a type of strategic behavior. Overall, the proportion of unexplained bids is 34% in the independent clock auctions (greater than the 13.8% that is the case with the common clock).

Support:

Table 8 contains the data. As reported in the Table 8, 56.9% of the bids are best response. The inertial bids account for 6.4% of the bids and the strategic bids to keep a market open represent 2.8% of the bids.

Table: 8 Proportion of Bids that are Best Response (Nash) and Inertial: Simultaneous Independent Clock Sessions

Session	Nash	Inertial	Close-prevention	unexplained
24	0.753	0.0225	0.0337	0.1908
32	0.719	0.0337	0.0337	0.2136
46	0.688	0.0547	0.0625	0.1948
57	0.681	0	0.0213	0.2977
67	0.580	0.0268	0	0.3932
71	0.510	0.0955	0.0255	0.369
77	0.373	0.136	0.0169	0.4741
80	0.263	0.121	0.0202	0.5958
All	0.569	0.0641	0.0282	0.3387

The analysis now turns to a more detailed study of the path of bids. Theory suggests that the dynamic path is important for the selection of the actual outcome from among the multiple Nash equilibria. Evidence exists that market adjustments can be related to a Marshallian path (Barner, Feri, and Plott (2005); and Plott, Roy, and Tong (2013)) in which the relative speed of market actions, bidding in our case, is influenced by potential profitability. That is, at any instant the agents with the highest potential profits have the earliest to bid. Our experiments provide an opportunity to collect evidence. We examine the bids tendered for the first item auctioned in the two sequential architectures ACB and BCA. We examine all bids that are tendered up until the first bidder drops out. During that time, when the price is below 130, which is the value of the type with the lowest value for the item (type b), the number of bidders is the same across all experiments. We measure the bids of each bidder as a proportion of the total of total bids tendered during those periods.

The proportion of bids tendered during a fixed period of time is related to speed of bidding. In these experiments bids arrive quickly so time between bids is not a useful measure. However, proportion of bids placed during the fixed time period is an appropriate measure.

Two approaches are taken. First the data are examined in relation to the magnitude of the redemption value (RV) of the item. The question posed is whether the proportion of bids is positively related to the redemption value (RV) of the bidder. Secondly we ask if the rank of the redemption value (RV) relative to those bidding is related to the relative proportion of bids tendered. Figure 4 summarizes the results. It shows that the proportion of bids placed by agents in negatively related to rank. As the rank of the bidder in terms of size of redemption value goes up the proportion of bids placed by the bidder goes down.

The measures are contained in Tables 9, 10 and 11. The result is summarized as Result 8.

Result 8. The relative speed with which buyers tender bids is positively related to the relative profitability of the bidder. Specifically, the bidder with the largest value places more bids than those with values with lesser magnitude.

Support:

> First consider the size of the redemption values (RV). Table 9 contains the average proportion of bids tendered by the holder of each of the possible redemption values for each of the segments for which measurements of proportion is appropriate. The mean (SD) of these averages for the highest value (ranked 1) the second highest (ranked 2) is

contained in Table 9. The close relationship between rank and proportion of bids tendered is shown in Figure 4.

Table 9: Mean proportions of total bids in the respective segments and their corresponding redemption values (RV). Note that only segments where the number of bidders ≥ 4 is considered for consistency and accuracy.

RV	BAC_ ItemB _1-43	RV	BAC_ ItemB _44-70	RV	ACB_ ItemA _1-130	RV	ACB_ ItemA _131-150	RV	BAC_ ItemA _1-130	RV	ACB_ ItemC _1-29
143	0.275	143	0.289	225	0.280	225	0.231	211	0.260	211	0.294
90	0.201	90	0.283	211	0.165	211	0.222	183	0.146	183	0.306
80	0.193	80	0.191	183	0.138	183	0.168	150	0.331	150	0.225
70	0.145	70	0.236	150	0.244	150	0.229	130	0.263	130	0.175
43	0.187	-	-	130	0.164	-	-	-	-	-	-

	1	2	3	4	5
1st	X	0.00146	0.000540	0.000959	$7.73\text{x } 10^{-5}$
2nd	X	X	0.323	0.426	0.0706
3rd	X	X	X	0.604	0.143
4th	X	X	X	X	0.0412
5th	X	X	X	X	X

Table 10: Rank of RV among those bidding in a segment: largest RV = 1, second largest = 2, etc. This table was based from the previous table and used to generate the figure as shown above.

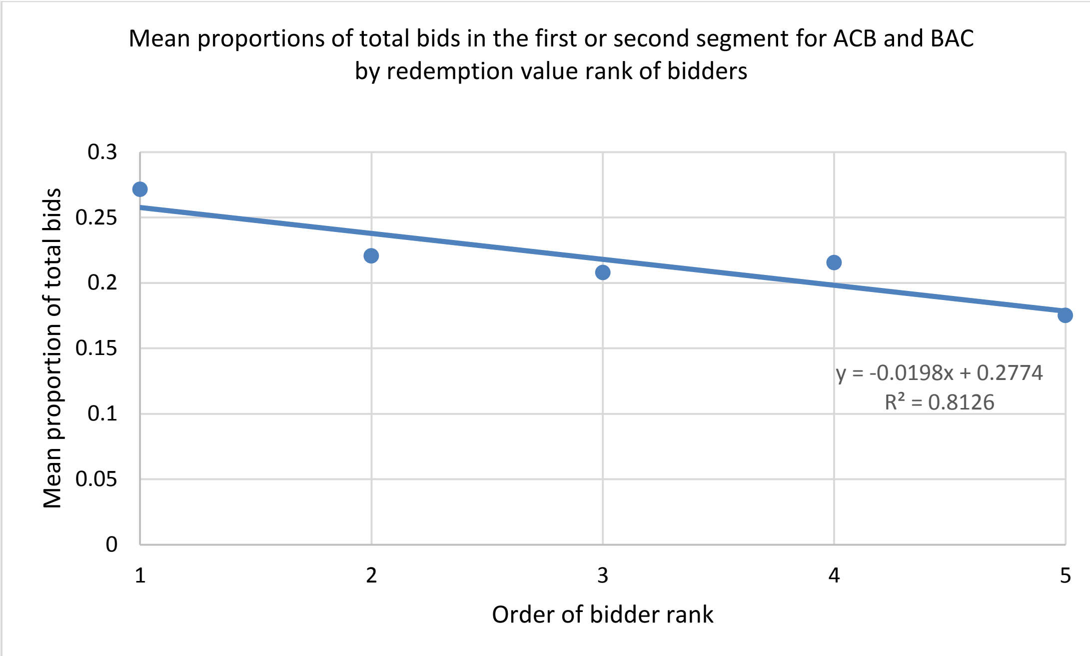
Bidder rank	Average of the mean proportions	Standard deviation
1st	0.272	0.0231
2nd	0.220	0.0636
3rd	0.208	0.0668
4th	0.215	0.0454
5th	0.175	NA*

*since there are only two values, R is not able to compute the standard deviation.

Table 11: p-values computed by R. The p-values are based on one-sided t-tests. The first row, for example, given the data of bidder rank 1, the null hypothesis is where the average of the mean proportions of bidder rank 1 is equal to that of bidder rank 2, and the alternative hypothesis is where the average of the mean proportions of bidder rank 1 is greater than that of bidder rank 2. If p-value is < 5%, we reject the null.

	1	2	3	4	5
1st	X	0.00146	0.000540	0.000959	$7.73\text{x } 10^{-5}$
2nd	X	X	0.323	0.426	0.0706
3rd	X	X	X	0.604	0.143
4th	X	X	X	X	0.0412
5th	X	X	X	X	X

Figure 4: Pooled data of mean proportions of total bids in the segments for ACB (Items A and C) and BAC (Items B and A) where number of bidders ≥ 4.



Tables 10 and 11 contain the results of statistical tests. The tests asks if bidders with rank v place a higher proportion of bids than bidders with rank v+k. For v = 1 the test of equality is rejected for all k. That is, the bidder with the highest value bids more aggressively in all cases that any other bidder. The data do not support the hypothesis that the second or third highest value bidder bids more rapidly than those ranked lower.

The next result is focused on jump bidding, which has been examined in ascending price auctions and presented itself as a challenge to auction theory in general and Nash equilibrium theory in particular. Studies of individual bidding behavior have revealed that bidders in ascending price auctions deviate from straight forward bidding by submitting jump bids. Rather than submit bids according to the minimum increment required, bidders tender bids for more than the minimum. Theoretical speculation about the reasons for the jump bids has centered on two motivations: (i) signaling a willingness to bid above competitors' values and thus attempt to discourage competitors from bidding and (ii) impatience in the sense of saving time and resolving the auction quickly. [6] Early

[6] Additional motives are isolated and analyzed by D. Grether, David Porter, and Matthew Shum (forthcoming) who conduct a field experiment with on line automobile auctions. By controlling for levels of potential jump bids, the

studies suggested that the phenomena would result in diminished auction performance in terms of reduced revenue and efficiency (see Avery (1998), Daniel and Hirshleifer (1997), and Banks, et.al. (2003). The theory holds that the jump bidding will cause the outcome to be path dependent and end with less efficient allocations that would be the case with straight forward bidding. However, a definitive study by Isaac, Salman and Zillante (2005) suggests that jump bids are part of a Markovian equilibrium strategy and do not have the deleterious effects reported as possibilities in the literature. A recent paper by Ettinger and Michelucci (forthcoming) develops a model in which jump bids are motivated by the structure of asymmetric information about payoffs and an incentive to prevent bids through which information can be conveyed.[7]

Our results are consistent with and provide additional support to the conclusions of Isaac, et.al. The issues are focused on two explanations for jump bidding: signaling and impatience. We examine the data under conditions which signaling can be expected to play no role and under conditions under which signaling could have an impact. The absence of asymmetric information about payoffs in our environment removes associated strategic behavior as an explanation for the jump bids contained in our data. Table 12 contains the relative frequencies of jump bids of different sizes for item A in the sixteen ACB auctions when it is the first item auctioned under the sequential architecture. The Table also reports the relative frequencies of jump bids for item A when it is auctioned under the simultaneous, common clock architecture. Each auction is divided into two time segments. The "early" bidding is defined as bids that take place before the price reaches 130, the value of the bidder with the lowest value. Prior to that time, the auction will (theoretically) consist of five bidders in the sequential auctions, and after that price, the number of bidders begins to shrink. In the simultaneous common clock auctions, all bidders typically bid on item A during the first part of the auction. It is the most valuable for all bidders. The types of jumps are partitioned into (i) "no jump", a case in which the bid was exactly the

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study is able to measure an influence of jump bids on auction prices. However, the mechanism through which this takes place is not clear due to the presence of seller strategies that appear as attempts to manipulate prices.

[7] Ettinger and Michelucci (forthcoming) is a theoretical study of a Japanese ascending clock auction in which individual decisions to drop out are public and carry information about own value to some of the participants. It is a model in which jump bids are used as strategies that prevent the public revelation of dropouts and thus preventing competitors from using the information. While our auction institutions reveal neither the identity of buyers nor the number that might be actively bidding, one can imagine that some information about dropouts or the number of active bidders remaining might be contained in the speed of bids and the jumps themselves.

minimum required; (ii) bids for which the francs above the minimal bid were in the interval 2β jump β 20 and (iii) 20 < jump.

The latitude for the possible influence of jump bids differs between the two auction formats. In the sequential auction, there is little reason to suspect that jump bids can signal values in a way that would influence the behavior of competitors. In the sequential auctions, the bidders have no place to go because bidders will be able to bid on other items only after the auction for the first item is over. Bidders are fully informed about the information they will receive during the auction and they are fully informed about the item price until the auction ends. The resetting clock provides sufficient opportunity to respond to the final bid. The cost of bidding is low so bidders have little incentive to stop bidding until they know they cannot acquire the unit profitably. Thus, a presumption exists that under the sequential auction the only incentive to jump bid is to terminate the auction quickly. However, under the simultaneous auction with a common clock bidders do have the opportunity to bid on other items. A bidder has an incentive to change bidding strategies from one item to another if there is reason to believe that bids on the first will not be successful. If jump bids carry such signals, bidders could be discouraged from bidding on an item and the jumps might be successful. Thus, there is reason to suspect that if the motive for jump bidding to end the auction quickly, the jumps will exist in both the sequential environment and the simultaneous environments. If jumps are motivated by signaling, they would be observed with greater frequency and intensity in the common clock and perhaps the independent clock environments.

Result 9. (i) Jump bidding is observed. (ii) Observed jump bidding cannot be explained by attempts to signal or intimidate other bidders. (iii) Observed jump bidding is consistent with impatience or a desire to speed the end of the auction. (iv) Jump bidding has no effect on final prices or efficiency.

Support:

(i) Jump bidding is observed. Our proportions of jump bids are similar to those observed in other auctions, e.g. the same as ISZ, ranging from 30 -50 percent of all bids.

(ii) Motivations for intimidation through signaling are removed in the auction for item A under the sequence ACB. The increment requirements are minimal. Bidders have no alternatives other than bidding on the item at auction that might benefit from an exit prior to the end of the auction. As reported in Table 12, the instances of jump bidding are not greater when the incentive for signaling exists, as is the case in the simultaneous auction. Neither the frequencies nor the magnitudes of jump bids respond to the existence of incentives for bidders to discourage the participation of other bidders through a process of using bids to signal values.

(iii) Jump bidding takes place early in the auction. Frequency and size fall as the price ascends. This is true of both measures of jump size. The proportion of straight forward bidding (non-jump bids) increases as the action progresses for both the sequential auction and the simultaneous auctions. Since signaling is eliminated as an explanation the remaining explanation is impatience, or attempts to speed the auction. The actions are consistent with impatience and a desire to end the auction.

(iv) There is little or no effect of jump bids on efficiency or price. Of the sixteen auctions all but three ended with the item going to the predicted bidder, with an average efficiency of .988 of the predicted level and an average price of 202 compared to a predicted price of 211.

Table 12. Proportion of Jump Bids in Sequentially Auctioned Items and Simultaneous Common Clock Auction by Size of Jump. For sequential auctions, the jump bidding dynamics of item A (5 bidders) in ACB was studied. Likewise, for comparison, the jump bidding dynamics of item A in both the common clock and independent clock auctions were studied

	Sequential price 130 or less	Sequential price more than 130	Common clock price 130 or less	Common clock price more than 130	Independent clock price 130 or less	Independent clock price more than 130
Non jump (increase =1)	51.7	66.6	66.2	78.5	59.7	73.8
Small jump (increase 2 up to 20)	34.0	31.2	19.2	20.0	20.8	13.7
Large jump (more than 20)	14.2	2.2	14.6	1.6	19.5	12.5

Jump bidding appears to have no effect leading to inefficient or path dependent outcomes. The bids may not be Markov, as claimed by Isaac, et.al. but the divergence from complete best response bidding seems to be limited to a type of "inertia" in which bids do not stop where theory suggests. Failure to bid according to best response in the sequential auctions could reflect a decision to save buying power for a subsequent auction. As price goes up the likelihood grows that a subsequent auction might be more profitable. In simultaneous auctions, the inertia could reflect a type of “inattention” in which “excessive” bidding continues on an item because the opportunity cost presented by other markets goes unnoticed. For now, such theoretical possibilities remain as speculations fueled only by hints in the data.

8. Summary of Conclusions

The auction architecture has an impact on both revenues and efficiencies. Higher revenue when using an architecture does not mean the auction operated with higher efficiency. The simultaneous ascending auction with a common clock consistently has high efficiency. However, the simultaneous ascending auction with common clock need not produce the highest revenue. The overall results suggest that the simultaneous auction with independent clock should be avoided. The independent clock architecture is dominated by the common clock in all dimensions of comparison.

Perhaps the most important result is that the best response Nash equilibrium model is a very powerful predictor of performance in all auction architectures and a strong predictor of all performance measures studied. The answer to the question about which architecture an auctioneer should use is that the auctioneer should seek such information as needed to determine what the Nash equilibrium model predicts. A good bet, from the point of view of the auctioneer is that the auction will follow a best response path and end up at the Nash equilibrium (unless expectations can be manipulated).

Two additional results are worthy of note. First, a tendency exists for the agent with the largest potential profit to tender bids more quickly than other bidders. This tendency has been observed in other forms of markets and is recognized as the Marshallian path. The fact that it is observed in our experiments suggests it is guided by a principle that remains inexplicable. Secondly, we observe jump bidding, which is inconsistent with a best response dynamic but the phenomenon should be interpreted as a sequence of rapid bids intended to speed the auction as opposed to signals or attempts to follow some form of psychological intimidation. The broad patterns of dynamics are largely best response and the outcomes are those predicted by the Nash equilibrium model. Thus, while the best response model can be rejected in a literal sense, the system itself has many of the characteristics that would be expected if the best response model was followed perfectly by the bidders.

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APPENDIX

Auction Architectures

Rules:

Talking and collaborating were strictly prohibited during the auctions. Before each auction began, the participants were shown a brief (~3 minute) instructional video explaining the rules and mechanisms of the auction. Three items were open for bidding during each period. In the first four periods, the items were opened sequentially. In other words, only one item was available to bid on at any given time. In the next two periods, the items were opened simultaneously. That is, all three items were available to bid on at the same time, and subjects were allowed to bid on the items in any order. The simultaneous rounds either had a common clock governing the bidding time or individual clocks for each item. Most of the auctions then concluded with a repeat of the first and fifth period. In each period, each subject was only allowed to win one item of the three total items. During the simultaneous periods, once a subject was the leading bidder on an item, he/she was not allowed to bid on any other item. The unit of currency in the auctions was the “Franc.” Each Franc was equivalent to $0.20 to $0.30. The subjects were informed of the currency exchange rate before the auctions began. A subject’s earnings for any item was defined as the difference between the redemption value for the item (induced in the subject, see later discussion) and the price paid (bid value).

Bidding mechanism:

At the very top of the auction page was the subject’s ID number. Immediately below was the current period of the auction. The redemption values were given in a separate window after clicking on the link in the top right hand corner of the page.

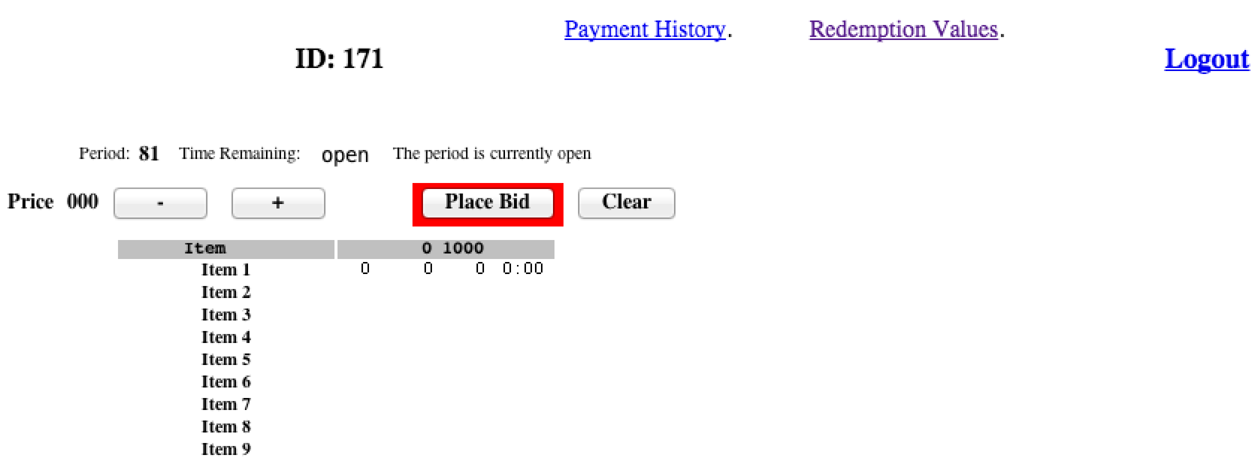
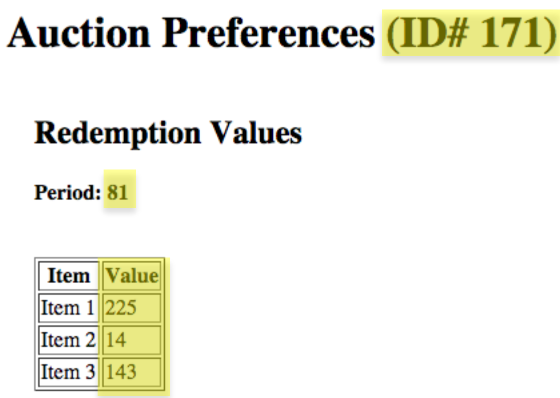


Figure A1. *Screenshot of auction page.* The locations of ID #, period and redemption values are highlighted in yellow.



Auction Preferences (ID# 171)

Redemption Values

Period: 81

Item	Value
Item 1	225
Item 2	14
Item 3	143

Figure A2. *Redemption values for one period.* These values are obtained by clicking on the "Redemptions Values" link in the top right corner of the auction page.

To place a bid, the subjects first had to click on the column to the right of the item such that it was highlighted in yellow. The subjects could then modify the bid value (in increments of 1 Franc) by clicking on the "+" and "-" signs to the right of "Price." To place a bid, the subjects then clicked on the "Place Bid" button. The current highest bid and the time remaining until each item closed were given in the column to the right of the item number. Each time a bid was placed on an item, the timer was reset to 20 seconds. During the simultaneous, common clock auctions, the time remaining was given at the top of the auction page instead of in the column to the right of the items. Once an item's auction was closed, the column to the right was highlighted either in red or gold, indicating whether the subject won the item (gold) or did not win the item (red). There was a short pause between each individual item auction. The subjects were verbally notified when the next item was open for bidding.

[Screen shot – right column in yellow, time remaining and common clock time, red vs. gold]