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THE DYNAMIC FREE RIDER PROBLEM: A LABORATORY STUDY

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Abstract

Most public goods are durable and have a significant dynamic component. In this paper, we report the results from a laboratory experiment designed explicitly to study the dynamics of free riding behavior in the accumulation of a durable public good that provides a stream of discounted benefits over a potentially infinite horizon. The dynamic fee-rider problem differs from static ones in fundamental ways and implies several economically important predictions that are absent in static frameworks. We consider two cases: economies with reversibility (RIE), where the agents' voluntary contributions to the public goods can be positive or negative; and economies with irreversibility (IIE), where contributions are non negative. For both economies, we characterize the unique Markov perfect equilibrium. The evidence supports the main predictions from the theory: behavior is generally consistent with stationary, forward-looking behavior; both in RIE and IIE the accumulation path is inefficiently slow and the public good under-provided; and RIE induces significantly higher public good contributions than IIE. A number of interesting deviations from the theoretical predictions are observed: both in RIE and in IIE we have over-investment in the early rounds of the game; in RIE over-investment is followed by periods in which negative contributions correct the stock, bringing it back to the predicted steady state; in IIE over-investment tends to decline approaching zero. To test the Markovian assumption, we compare the predictions of the Markov equilibrium with the prediction of the most efficient subgame perfect equilibrium and propose a novel experimental methodology that relies on the comparison between the behavior in the dynamic game and the behavior in a one-period reduced-form version of the dynamic game.

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1 Introduction

There is a vast literature addressing questions related to the provision of public goods in *static* environments. This includes hundreds of theoretical papers in the lineage initiated by Samuelson's (1954) seminal paper, presaged by the classical treatises on public finance by Wicksell and Lindahl.[1] It also includes hundreds of experimental papers based on one variation or another of Samuelson's theoretical model (Ledyard 1995). The typical motivating examples are national defense, public health, transportation infrastructure, pollution abatement, and so forth. What is striking is that essentially all economically important examples are public goods that take years to accumulate, provide streams of benefits over the long term, and require ongoing expenditures in order to improve or even maintain their levels. In other words, most public goods one can think of are *durable goods* and hence dynamics are an important component of their provision. In spite of this, remarkably little research has addressed the durable public goods problem from a dynamic perspective, either in the theoretical and experimental literatures.

We are mainly interested in three questions: How serious is free riding in the provision of durable public goods? What new issues emerge from the dynamic nature of the investment process? How do the answers to these questions depend on the degree to which investment decisions are reversible over time?

Dynamic free-rider problems differ from static in subtle but important ways. In dynamic environments, we not only have the familiar free rider problem present in static public good provision, but also present is a second *dynamic free rider* phenomenon that further erodes incentives for efficient provision. In these games strategies depend on the accumulated level of the public good, the state variable of the game: an increase in current investment by one agent typically triggers a reduction in future investment by all agents, in what is essentially a dynamic crowding-out effect. Such dynamic crowding out is especially severe if agents coordinate on *stationary* equilibria where strategies depend only on the accumulated level of the public good. On the other hand, the infinite horizon of the game generates a plethora of non-stationary equilibria that provide strategic opportunities to endogenously support cooperative outcomes using carrot-and-stick strategies. In principle, this could completely overcome both the static and the dynamic free rider problems. Thus, it is an open empirical question whether or not the free rider problem is exacerbated or ameliorated in the case of dynamic provision of durable public goods, as compared to one-shot public goods problems.

Dynamic free-rider problems, moreover, offer a number of economically important predic-

[1] An excellent account of the development of the theory of public goods is Silvestre (2003).

tions that cannot be assessed (or even stated) with static frameworks because they depend on the *durability* of the public good. First, regarding the storage technology, a public good is reversible if players can either increase it or decrease it transforming it back to private consumption; a public good is irreversible, if players cannot decrease it. Most investments are partially reversible, and the degree (or cost) of reversibility varies widely.[2] What is the effect of irreversibility on contributions? Second, regarding the accumulation process, how are investment strategies going to depend on the state variable? If players use the state as a reference point, then the steady state may depend on the investments in the first periods: a good start with over-investment (compared to the equilibrium level) may induce a permanent increase in the steady state. If agents instead are anchored to a given equilibrium steady state target, then players should be expected to correct "anomalous" contributions: over-investment in the early periods should be corrected with underinvestment later on.

In this work, we make a first attempt to answer the questions raised above by studying the theoretical predictions of a simple dynamic public good game in a laboratory experiment. The economy we study has n individuals. In each period, each individual is endowed with w units of input that can be allocated between personal consumption and contribution to the stock of durable public good. Utility is linear in consumption of the private good and concave in the accumulated stock of the durable public good. Total payoffs for a player in the game are the discounted sum of utility over an infinite horizon of the game, where the discount factor is δ. We characterize the efficient accumulation path as a function of w, n, and δ.[3] We study the Markov perfect equilibria of the game under two different assumptions about reversibility: full reversibility and irreversibility. We prove that investment is always higher in the irreversible case, and this theoretical property of our model is the basis for the main theoretical treatment in our experiment: reversibility vs. irreversibility. We also have a secondary treatment dimension, which is the number of individuals in the game: we

[2] For example, the art collection at the Louvre, which took centuries to accumulate, could be sold off to private collectors and the proceeds distributed as transfer payments to the citizens of France. Cobblestone roads have been dug up and the stones used to build private dwellings. Military vehicles and aircraft can be (and have been) privatized and converted to civilian use. Publicly owned open space, even with conservation easements, are routinely converted to the private development of shopping malls, ski resorts, or new residential communities. Decades of sustainable management of fisheries, forests or other re-plenishable resources can be rapidly reversed by over-harvesting or poaching.

[3] To keep the experimental design simple, there is no depreciation, so at time t the stock of the public good is simply the sum of individual investments across all periods up to time t. Battaglini, Nunnari, and Palfrey (2012a) also characterize the efficient path and the equilibrium accumulation paths for arbitrary depreciation rates, $d \in [0, 1]$.

compare $n = 3$ and $n = 5$. Thus, the experiment has four different treatments depending on n and whether investments are reversible.

The main comparative static prediction of the model is that in a Markov equilibrium there should be greater contributions and a higher equilibrium steady state level of public good in the irreversible investment economy than in the reversible investment economy. In contrast, the model predicts no significant difference in public good levels as a function of n. The data are consistent with these predicted treatment effects (or non-effects, with respect to n): behavior is generally consistent with stationary, forward-looking behavior; both in RIE and IIE the accumulation path is inefficiently slow and the public good under-provided; and RIE induces significantly higher public good contributions than IIE; public good accumulation does not seem to be significantly affected by n. We do, however, observe some differences between the finer details of the theoretical predictions and the data, mainly with respect to the path of convergence to the steady state. In equilibrium, convergence should be monotonic. That is, the stock of public good should gradually increase over time until the steady state is reached after which investment is zero. Instead, there is a tendency for initial over-investment in the early periods, compared to the equilibrium investment levels. In the treatment with reversibility, this is followed by a significant reversal, i.e., negative investment, with the stock of public good gradually declining in the direction of the equilibrium steady state. After several periods of play, the stock of the public good is very close to the Markov equilibrium of the game. When disinvestment is not feasible, investment steadily decreases but the initial over-investment cannot be corrected and the long run level of the public good remains above the equilibrium steady state.

To test for the equilibrium predictions we follow two approaches. First, we compare the predictions of the Markov equilibrium with the predictions of the best subgame perfect equilibrium (a solution concept often used in applied work). The comparative static predictions implied by the two equilibrium concepts are completely opposed with respect to the effect of reversibility on investment: the Markov equilibrium predicts higher investments in a irreversible economy, the most efficient subgame perfect equilibrium the opposite. As said, the data supports the comparative statics of the Markov equilibrium.

Second, we construct a new approach to test for Markovian behavior in equilibrium. The idea of the new experimental test consists in designing a one-period experiment where subjects payoffs from the public good are given by the equilibrium value function of the unique concave Markov perfect equilibrium of the game with reversibility. In this reduced-form version of the game, the individual incentives to contribute in the public good are exactly the same as in the fully dynamic game (under the assumption that subjects condition their strategies only on the public good stock), but there is no possibility to sustain a higher public good outcome through the non-stationary strategies that can arise in a repeated game. We observe no systematic difference in contributions between this reduced form of the dynamic game and the fully dynamic game. We conclude that observed behavior in the dynamic game is well approximated by the predictions of a purely forward looking Markov equilibrium, rather than by an equilibrium in which agents use more complicated history-dependent strategies to punish uncooperative behavior or reciprocate cooperative behavior by other members of the group.

Our work is related to three distinct strands of research. First, naturally, the experimental literature on public good provision in static environments. This literature has explored voluntary contributions under a variety of conditions. The early experiments focused primarily on free riding in environments where there was a dominant strategy for all individuals to contribute zero to the public good. Variations on these early dominant strategy public goods games have been conducted in the laboratory under many different assumptions about utility functions and technology, different subject pools, asymmetric endowments and preferences, different information conditions, different public good mechanisms, variable group sizes, and so forth. Many of these variations are discussed at length in Ledyard's (1995) comprehensive survey of the seminal work in this area.[4] The dynamic environment we study is fundamentally different from the static environments studied in these papers. We have already mentioned some of these important differences and will discuss this issue in greater detail Section 5, in the context of the results from our experiment.

The second literature to which our work is related consists in the work on sequential mechanisms for the provision of static public goods. Although in this literature players play a dynamic game, the purpose of the game is the determination of a one-shot provision of a discrete public good (Harrison and Hirschleifer 1989; Dorsey 1992; Duffy, Ochs, and Vesterlund 2007; Choi, Gale and Kariv 2008; Diev and Hichri 2008; Noussair and Soo 2008; Cho, Gale, Kariv, and Palfrey 2011).[5] In the contribution games studied in these papers, agents have the opportunity to revise their initial contributions over time, and observe the cumulative level of contributions at each moment. Contrary to our setup, the public good does not provide any benefit until the game ends. Moreover, when payoffs from the public good are a continuous function of cumulative contributions, the unique equilibrium of these

[4] See Chaudhuri (2011) and Vesterlund (2012) for more recent (and more selective) surveys.

[5] There are also public goods voluntary contribution experiments with reduced-form one-shot payoff functions that are motivated by common pool resource problems. See Ostrom (1999) for a survey.

mechanisms is not different from the corresponding one-shot games (that is, no contribution). Only when a certain threshold guarantees a discrete benefit, agents might achieve the provision of this discrete public good, through history-dependent trigger strategies.

Finally, our work is related to the emerging experimental literature on dynamic stochastic games in which a state variable provides a strategic link across periods. Early contributions are Lei and Noussair (2002) and Noussair and Matheny (2000) who experimentally study single agent dynamic optimization problems. Herr et al. (1997) present a model of resource utilization in a finitely repeated environment in which players' actions have externalities on the preferences of current and future players. Battaglini and Palfrey (2007) test a dynamic model of pure redistribution (in which the state variable is the status quo distribution of resources and the amount of resources is constant over time). Battaglini, Nunnari and Palfrey (2012b) study the effect on investments of voting rules in a model of public good accumulation in which investment is chosen through a non-cooperative bargaining process. To our knowledge Battaglini, Nunnari and Palfrey (2009) is the first paper to presents an experimental study of a dynamic public good game in which players make voluntary contributions. The results of this working paper are now incorporated in our current paper.[6] Following this paper, Vespa (2012) has provided an alternative test of Markovian behavior in a game of resource exploitation similar to Herr et al. (1997).[7] The results of this paper confirm our finding that the Markov equilibrium is a good model of behavior in the laboratory. Interestingly, however, Vespa observes that the feasibility of cooperation with non-stationary strategies may depend on the complexity of the action space: cooperation may be possible with two actions, but not possible already with three. This may suggest that the Markov equilibrium can do well in environments like ours (with a continuum of actions) because players find it difficult to deal with non stationary strategies when the action space is non trivial.

From a theoretical point of view, our work draws on Battaglini Nunnari and Palfrey (2012a) who first characterized the equilibrium in the dynamic public good game that we study with and without reversibility.[8] In our paper we use the characterization presented

[6] In Battaglini Nunnari and Palfrey (2009) we studied voluntary contribution only in the reversible case. The results presented in this working paper are now incorporated in the current expanded paper that includes also the irreversible case.

[7] A related experiment is presented by Saijo et al. (2009) who focus on an environment with static, state independent strategies.

[8] Previously, work on the reversible case was done in a framework of a linear differentiable game with quadratic preferences (Fershtman, C. and S. Nitzan 1991 and Duckner and Long 1993 among others). Contribution games with irreversibility are studied in the literature on monotone games, see Matthews (2012)

there as a basic prediction for the players' behavior in the laboratory, integrating it with an analysis of other non stationary equilibria for completeness.

The reminder of this paper is organized as follows. In Section 2 we present the model and its solutions: the first best solution, the equilibrium when the public good is reversible, the equilibrium when the public good is irreversible and the equilibrium predictions with non-stationary subgame perfect equilibria. In Section 3 we describe the experimental design. Section 4 discusses the results of the experiment. Section 5 compares the results in our dynamic environment with the results of previous experiments on static public good games. Section 6 concludes.

2 The Model

Here we describe a simplified version of the model in Battaglini, Nunnari, and Palfrey (2012a), which we will use in our experimental design. Consider an economy with n agents who interact for an infinite number of periods. There are two goods: a private good x and a public good g. The level of consumption of the private good by agent i in period t is x_t^i, the level of the public good in period t is g_t. We refer to $z_t = (x_t, g_t)$ as the allocation in period t. The utility U^j of agent j is a function of $z^j = (x_\infty^j, g_\infty)$, where $x_\infty^j = (x_1^j, ..., x_t^j, ...)$, and and $g_\infty = (g_1, ..., g_t, ...)$. We assume that the future is discounted at a rate δ and that U^j can be written as:

$$U^j(z^j) = \sum_{t=1}^{\infty} \delta^{t-1} \left[x_t^j + 2\sqrt{g_t} \right]$$

There is a linear technology by which the private good can be used to produce public good, with a marginal rate of transformation $p = 1$. The private consumption good is nondurable, the public good is durable and does not depreciate between periods. Thus, if the level of public good at time $t-1$ is g_{t-1} and the total investment in the public good is I_t, then the level of public good at time t will be

$$g_t = g_{t-1} + I_t.$$

It is convenient to distinguish the state variable at t, g_{t-1}, from the policy choice g_t. In the remainder, we denote $y_t = g_{t-1} + I_t$ as the new level of public good after an investment I_t when the last period's level of the public good is g_{t-1}. The initial stock of public good is $g_0 \geq 0$, exogenously given. Public policies are chosen as in the classic free rider problem.

(and the references cited there) for a recent comprehensive analysis. This work however assume the players have a dominant strategy and it can be applied only to special cases that do not include our environment.

In each period, each agent j is endowed with $w = W/n$ units of private good and chooses on its own how to allocate its endowment between an individual investment in the public good (which is shared by all agents) and private consumption, taking as given the strategies of the other agents. The key difference with respect to the static free rider problem is that the public good can be accumulated over time. The level of the state variable g, therefore, creates a dynamic linkage across policy making periods.

We consider two alternative economic environments. In a *Reversible Investment Economy* (RIE), the level of individual investment can be negative, with the constraint that $i_t^j \in [-g_t/n, W/n]\ \forall j$, where $i_t^j = W/n - x_t^j$ is the investment by agent j.[9] In an *Irreversible Investment Economy* (IIE), an agent's investment cannot be negative and must satisfy $i_t^j \in [0, W/n]\ \forall j$.

The RIE corresponds to a situation in which the public investment can be scaled back in the future at no cost. An example can be an art collection, land for common use, the level of global warming, or less tangible investments like "social capital." The IIE corresponds to situations in which once the investment is done it cannot be undone. This seems the appropriate case for investments in public infrastructure (for example, a bridge or a road). In this environment, private consumption cannot be negative and the total economy-wide investment in the public good in any period is given by the sum of the agent investments.

2.1 The Planner's Solution

As a benchmark with which to compare the equilibrium allocations, we first analyze the sequence of public policies that would be chosen by a benevolent planner who maximizes the sum of utilities of the agents. This is the welfare optimum because the private good enters linearly in each agent's utility function.

Denote the planner's policy as $y_P(g)$ and consider first an economy with reversible investment. As shown by Battaglini, Nunnari, and Palfrey (2012a), the objective function of the planner's is continuous, strictly concave and differentiable and a solution of its maximization problem exists and is unique. The optimal policies have an intuitive characterization. When the accumulated level of public good is low, the marginal benefit of investing in g is high, and the planner finds it optimal to invest as much as possible: in this case $y_P(g) = W + g$ and $\sum_{j=1}^{n} x^i = 0$. When g is high, the planner will be able to reach the level of public good

[9] This constraint guarantees that (out of equilibrium) the sum of reductions in g can not be larger than the stock of g. The analysis would be similar if we allow each agent to withdraw up to g. In this case, however, we would have to assume a rationing rule in case the individuals withdraw more than g.

y_P^* that solves the planner's unconstrained problem:

$$y_P^* = \left(\frac{n}{1-\delta}\right)^2 \tag{1}$$

The investment function, therefore, has the following simple structure. For $g < y_P^* - W$, y_P^* is not feasible: the planner invests everything and $y_P(g) = g + W$. For $g \geq y_P^* - W$, instead, investment stops at $y_P(g) = y_P^*$. This investment function implies that the planner's economy converges to the steady state $y_P^o = y_P^*$. In this steady state, without loss of generality, we can set $x^i(g) = (W + g - y(g))/n \ \forall i$.[10]

The planner's optimum for the IIE case is not very much different. The planner finds it optimal to invest all resources for $g \leq y_P^* - W$. For $g \in (y_P^* - W, y_P^*)$, the planner finds it optimal to stop investing at y_P^*, as before. For $g \geq y_P^*$, y_P^* is not feasible, so it is optimal to invest 0, and to set $y_P(g) = g$ This difference in the investment function for IIE, however, is essentially irrelevant for the optimal path and the steady state of the economy. Starting from any g_0 lower than the steady state y_P^*, levels of g larger or equal than y_P^* are impossible to reach, and the irreversibility constraint does not affect the optimal investment path.

2.2 Reversible Investment Economies

We first describe equilibrium behavior when the investment in the public good is reversible. We focus on continuous, symmetric Markov-perfect equilibria, where all agents use the same strategy, and these strategies are time-independent functions of the state, g. A strategy is a pair $(x(\cdot), i(\cdot))$: where $x(g)$ is an agent's level of consumption and $i(g)$ is an agent's level of investment in the public good in state g. Associated with any equilibrium is a value function $v_R(g)$ which specifies the expected discounted future payoff to a legislator when the state is g. The optimization problem for agent j if the current level of public good is g, the agent's value function is $v_R(g)$, and other agents' investment strategies are given by $x_R(g)$, can be represented as:

$$\max_{y,x} \left\{ \begin{array}{c} x + 2\sqrt{g} + \delta v_R(y) \\ s.t \quad x + y - g = W - (n-1)x_R(g) \\ W - (n-1)x_R(g) + g - y \geq 0 \\ x \leq g/n + W/n \end{array} \right\} \tag{2}$$

Contrary to the planner, agent j cannot choose y directly: it chooses only its level of private consumption and the level of its own contribution to the public investment. The

[10] Indeed, the planner is indifferent regarding the distribution of private consumption.

agent, however realizes that given the other agents' level of private consumption $(n-1)x_R(g)$, his/her investment ultimately determines y. It is therefore *as if* agent j chooses x and y, provided that he satisfies the feasibility constraints. The first constraint is the resource constraint: it requires that total resources, $W+g$, are equal to the sum of private consumption, $(n-1)x_R(g)+x$, plus the public investment y. The second constraint requires that private consumption x is non negative. The third constraint requires that no agent can reduce y by more than his share g/n.

In a symmetric equilibrium, all agents consume the same fraction of resources, so agent j takes as given that in state g the other agents each consume:

$$x_R(g) = \frac{W+g-y_R(g)}{n},$$

where $y_R(g)$ is the equilibrium level of investment in state g. Substituting the first constraint of (2) in the objective function, recognizing that agent j takes the strategies of the other agents as given, and ignoring irrelevant constants, the agent's problem can be written as:

$$\max_y \left\{ \begin{array}{c} 2\sqrt{y} - y + \delta v_R(y) \\ y \leq \frac{W+g}{n} + \frac{n-1}{n} y_R(g),\ y \geq \frac{n-1}{n} y_R(g) \end{array} \right\} \tag{3}$$

where it should be noted that agent j takes $y_R(g)$ as given.[11] The objective function shows that an agent has a clear trade off: a dollar in investment produces a marginal benefit $\frac{1}{\sqrt{g}} + \delta v_R'(y)$, the marginal cost is -1, a dollar less in private consumption.[12] The first constraint shows that at the maximum the agent can increase the investment of the other players (i.e., $\frac{n-1}{n} y_R(g)$) by $\frac{W+g}{n}$. The second constraint makes clear that at most the agent can consume his endowment W/n and his share of g, g/n.

We restrict attention to equilibria in which the objective function in (3) is strictly concave, and we refer to these equilibria as *concave equilibria*. Depending on the state g, the solution of (3) falls in one of two cases: the first case corresponds to the situation where the first constraint in (3) is binding, so all resources are devoted to investment in the public good. In this case, $x_R(g) = 0$, $y_R(g) = W + g$, and investment by each agent is $i_R(g) = \frac{W}{n}$. In the second case, private consumption is positive, that is $x_R(g) > 0$, and the solution is characterized by a unique public good level $y_R^* = \left(\frac{n}{n-\delta}\right)^2$. In this second case, the investment by each agent is equal to $i_R(g) = \frac{1}{n}\left[y_R^* - g\right]$ and per capita private consumption is $x_R(g) =$

[11] Since $y_R(g)$ is the equilibrium level of investment, in a symmetric equilibrium $(n-1)y_R(g)/n$ is the level of investment that agent j expects from all the other agents, and that he/she takes as given in equilibrium.

[12] For simplicity of exposition we assume here that $v_R(g)$ is differentiable. This is essentially without loss of generality and we refer to the proofs in Battaglini, Nunnari, and Palfrey (2012a) for the details.

$\frac{W+g-y_R^*}{n} > 0$. The first case is possible only if and only if $W \leq y_R^* - g_R$, that is, if g is below some threshold g_R defined by: $g_R = \max\{y_R^* - W, 0\}$. We summarize this in the following proposition, which also proves the existence of an equilibrium and its uniqueness when $v_R(g)$ is strictly concave:

Proposition 1. *In the game with reversible investment, a strictly concave equilibrium exists and it is unique. In this equilibrium, public investment is:* $y_R(g) = \min\{W + g, y_R^*\}$ *where* $y_R^* = \left(\frac{n}{n-\delta}\right)^2 < y_P^*$.

Proof. See Appendix A.

The public good function $y_R(g)$ is qualitatively similar to the corresponding planner's function $y_P(g)$. The main difference is that $y_R^* < y_P^*$ and $g_R < g_P$, so public good provision is typically smaller (and always smaller in the steady state). This is a dynamic version of the usual free rider problem associated with public good provision: each agent invests less than is socially optimal because he/she fails to fully internalize all agents' utilities. Part of the free rider problem can be seen from (3): in choosing investment, legislators count only their marginal benefit, $u'(y) + \delta v_R'(y)$, rather than $nu'(y) + \delta n v_R'(y)$, but all the marginal costs (-1). In this dynamic model, however, there is an additional effect that reduces incentives to invest, called *the dynamic free rider problem.* A marginal increase in g has two effects. An immediate effect, corresponding to the increase in resources available in the following period: g. But there is also a delayed effect on next period's investment: the increase in g triggers a reduction in the future investment of all the other agents through an increase in $x_R(g)$: for any level of $g > g_R$, $y_R(g)$ will be kept at y_R^*. In a symmetric equilibrium, if agent j increases the investment by 1 dollar, he will trigger a reduction in future investment by all agents by $1/n$ dollars; the net value of the increase in g for j will be only δ/n.

2.3 Irreversible Investment Economies

When the stock of the public good cannot be reduced, the optimization problem of an agent can be written like (2), but with an additional constraint: the individual level of investment cannot be negative or, in other words, each agent's private consumption cannot exceed his endowment, $x_i(g) \leq W/n$. Following similar steps as before, we can write the maximization problem faced by an agent as:

$$\max_y \left\{ \begin{array}{c} 2\sqrt{y} - y + \delta v_{IR}(y) \\ y \leq \frac{W+g}{n} + \frac{n-1}{n} y_{IR}(g), \; y \geq \frac{g}{n} + \frac{n-1}{n} y_{IR}(g) \end{array} \right\} \tag{4}$$

where the only difference with respect to (3) is the second constraint. To interpret it, note that it can be written as $y \geq g + \frac{n-1}{n}\left[y_{IR}(g) - g\right]$: the new level of public good cannot be lower than g plus the investments from all the other agents.

As pointed out in Section 2.1, when public investments are efficient, irreversibility is irrelevant for the equilibrium allocation. The investment path chosen by the planner is unaffected because the planner's choice is *time consistent*: he never finds it optimal to increase g if he plans to reduce it later. In the concave equilibrium characterized in the previous section, the investment function may be inefficient, but it is weakly increasing in the state. Agents invest until they reach a steady state, and then they stop. It may seem intuitive, therefore, that irreversibility is irrelevant in this case too, but this intuition is not correct. To the contrary, irreversibility destroys the concave equilibrium we characterized for reversible investment economies and induces the agents to significantly increase their investment, leading to a significantly higher unique steady state. Intuitively, the reason is that the agents no longer have to worry about the dynamic free rider problem: the irreversibility constraint creates a "commitment device" for the future; the agents know that g cannot be reduced by the others (or their future selves).

Proposition 2. *In an economy with irreversible investment, a weakly concave equilibrium exists. Any weakly concave equilibrium, moreover, is associated to the same unique steady state equal to* $y^*_{IR} = \left(\frac{1}{1-\delta}\right)^2$. *This steady state level is strictly greater than* y^*_R *and strictly smaller than* y^*_P *for any* $n > 1$ *and any* $\delta \in [0, 1)$.

Proof. See Appendix A.

The first part of Proposition 2 follows directly as a special case of Propositions 4 and 5 in Battaglini, Nunnari, and Palfrey (2012a), where it is established that the dynamic free rider game with irreversibility admits an equilibrium with standard concavity properties. The second part, uniqueness of the steady state, is established in Appendix A. In this steady state, the public good stock is strictly smaller than the one accumulated by a benevolent planner, but strictly higher than the one accumulated in the unique concave equilibrium of RIE. This steady state, $y^*_{IR} = \left(\frac{1}{1-\delta}\right)^2$, is exactly the same level that an agent alone would accumulate and it is independent of n.

The equilibrium selection based on Markov strategies therefore leads to a clear prediction that the irreversibility of investment will generate a higher level of investment in each period, as well as a higher steady state of the public good. The intuition for this is straightforward: the impossibility to convert today's investment back into private consumption at a future dates, eliminates worries about future agents' incentives to plunder the current public good

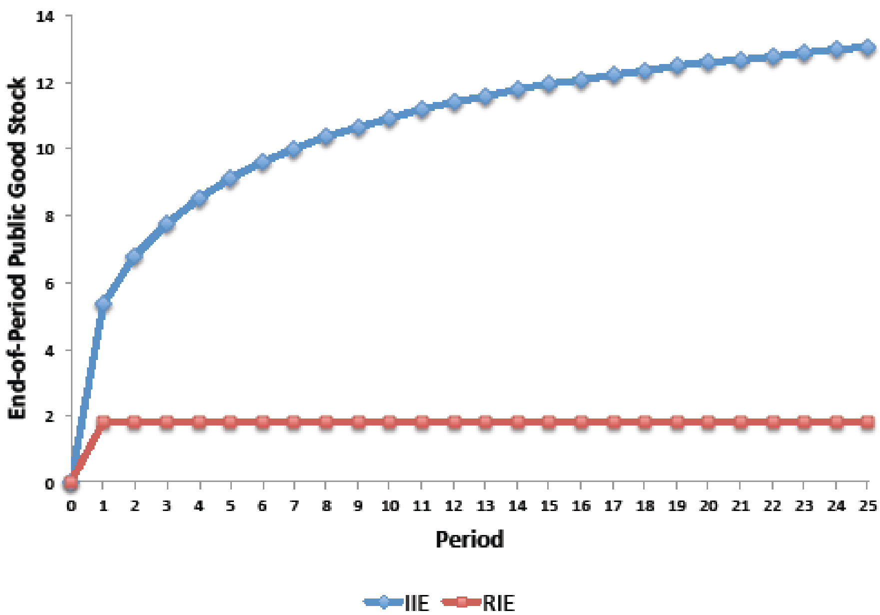

Figure 1: Predicted Time Path of the Stock of g, RIE vs. IIE.

investments. In other words, irreversibility mitigates the dynamic free rider problem. Moreover, the investment function in the equilibrium described in Proposition 2 is different than the one for the reversible investment case, where the agents would either find it optimal to invest everything, or just enough to reach the steady state. In contrast, in the irreversible investment case, the investment function increases gradually over time[13], agents' investment efforts are strategic complements, and the steady state is reached only asymptotically.

Figure 1 shows the evolution of the stock of the public good as a function of the time period in the concave MPE, for the parameters of one of our experimental treatments (with three members groups), for the reversible investment (RIE) and irreversible investment (IIE) case.

2.4 Cooperation Using Non-Stationary Strategies

We have restricted our attention to symmetric Markov perfect equilibria. However, the voluntary contribution game we study is an infinite horizon dynamic game with many subgame

[13] This property of gradually increasing contributions in our model is reminiscent of a property of the repeated game equilibria found elsewhere in the literature (see, for example, Marx and Matthews 2010), but the intuition behind it is quite different. Here gradualism is needed in order to smooth out the value function of the Markov equilibrium at the steady state, while elsewhere gradualism follows from the non-Markov repeated game strategies that are used to enforce efficient equilibria.

perfect equilibria. The Markovian assumption of stationary strategies is very restrictive and it is possible that some other equilibria can sustain more efficient outcomes through the use of history-dependent strategies that use punishments and rewards for past actions. As we show below, in economies with reversible investment, the optimal solution can indeed be supported as the outcome of a subgame perfect equilibrium:

Proposition 3. *There is a $\widehat{\delta}_R \in [0, 1)$ such that, for all $\delta > \widehat{\delta}_R$, the efficient investment path characterized by the optimal solution is a Subgame Perfect Nash Equilibrium of the voluntary contribution game with reversible investment.*

In Appendix A, we derive non-stationary strategies for the voluntary contribution game with reversible investment whose outcome is the efficient level of public good (the optimal solution), and show that these strategies are a subgame perfect Nash equilibrium.[14]

The strategy for each agent is to allocate the optimal level of investment to public good production, $(y_P^*(g) - g)/n$, and to consume the remainder. A deviation from this investment behavior by any agent is punished by reversion to the unique concave Markov perfect equilibrium characterized in Section 2.2. This is a simple strategy that involves the harshest individually rational punishment for deviation from cooperation: whenever $g > y_R^*$ and a deviation is observed, the public good will revert to y_R^* and it will stay at this level for all future periods.

When investment is irreversible, the efficient outcome cannot be sustained with strategies similar to the ones proposed above for environments with reversible investment. Matthews (2012) shows that, with discounting, no subgame perfect equilibrium of a general family of dynamic contribution games is efficient, in the sense of supporting the optimal public good stock in each period. In particular, that result applies to our environment, implying the following proposition as corollary.

Proposition 4. *There is no $\widehat{\delta}_{IR} \in [0, 1)$ such that, for all $\delta > \widehat{\delta}_{IR}$ the optimal investment strategies are a Subgame Perfect Nash Equilibrium of the voluntary contribution game with irreversible investment.*

The intuition behind Proposition 4 is that the potential for punishment is significantly dampened by the irreversibility constraint. Whenever $g > y_{IR}^*$ and a deviation is observed, agents cannot disinvest down to y_{IR}^* and the harshest punishment is characterized by no investment and a constant stock in all periods following the first deviation.

[14] Our goal is to show that the optimal solution is the outcome of some subgame perfect Nash equilibria of the game. We do not claim that the strategies proposed in the proof of Proposition 3 are the best punishment schemes, and there may be different non-stationary strategies that work for lower δ.

Since many multiple equilibrium generally exist, a refinement is always needed for comparative statics or policy evaluation. It is standard practice in applied work to use as solution concept the most efficient subgame perfect equilibrium (in our model the solution with the highest investment in g). The propositions presented above are important because they allow us to cleanly separate the time path of investment behavior implied by the Markov equilibrium discussed in the previous section from the time path of investment behavior in the best subgame perfect (non-Markov) equilibrium. Let $g_t^{M,R}$ and $g_t^{M,IR}$ denote the equilibrium stock of accumulated public good at time t in the Markov equilibria discussed in the previous section with reversibility and irreversibility, respectively. Let $g_t^{S,R}$ and $g_t^{S,IR}$ corresponding stock of accumulated public good observable at time t in the best subgame perfect equilibrium. We have:

Corollary 1. *There is a* $\delta^* \in [0,1)$ *such that, for* $\delta > \delta^*$ *we have:*

- $g_t^{M,R} < g_t^{M,IR}$ on the equilibrium path.
- $g_t^{S,IR} \leq g_t^{S,R}$ on the equilibrium path.

The first bullet point can be established using Propositions 1 and 2, while the second bullet point follows from Propositions 3 and 4. Corollary 1 establishes that the comparative static predictions implied by the two different equilibrium concepts (Markov vs. best SPE) are completely opposed with respect to the effect of reversibility vs. irreversibility on investment. This theoretical insight thus provides two starkly opposite predictions about efficiency that will be useful for interpreting the results of the experiment.

3 Experimental Design

The experiments were all conducted at the Social Science Experimental Laboratory (SSEL) using students from the California Institute of Technology. Subjects were recruited from a pool of volunteer subjects, maintained by SSEL. Eight sessions were run, using a total of 105 subjects. No subject participated in more than one session. Half of the sessions were for Reversible Investment Economies and half for Irreversible Investment Economies. Half were conducted using 3 person committees, and half with 5 person committees. In all sessions the discount factor was $\delta = 0.75$, and the current-round payoff from the public good was proportional to the square root of the stock at the end of that period, that is $u(g) = 2\sqrt{(g)}$. In the 3 person committees, we used the parameters $W = 15$, while in the

5 person committees $W = 20$. Payoffs were renormalized so subjects could invest fractional amounts.[15] Table 1 summarizes the theoretical properties of the equilibrium for the four treatments. It is useful to emphasize that, as proven in the previous sections, given these parameters the steady state is uniquely defined both for the RIE and IIE game and for all treatments: so the theoretical predictions of the convergence value of g is independent of the choice of equilibrium.

Treatment	**n**	**W**	$\mathbf{y}^*_{MPE}$	$\mathbf{y}^*_P$
RIE	3	15	1.77	144
RIE	5	20	1.38	400
IIE	3	15	16	144
IIE	5	20	16	400

Table 1: Experimental parameters, equilibrium and planner steady states

Discounted payoffs were induced by a random termination rule by rolling a die after each period in front of the room, with the outcome determining whether the game continued to another period (with probability .75) or was terminated (with probability .25). The $n = 5$ sessions were conducted with 15 subjects, divided into 3 groups of 5 members each. The $n = 3$ sessions were conducted with 12 subjects, divided into 4 groups of 3 members each.[16] Groups stayed the same throughout the periods of a given match, and subjects were randomly rematched into groups between matches. A match consisted of one multi-round play of the game which continued until one of the die rolls eventually ended the match. As a result, different matches lasted for different lengths (that is, for a different number of periods). Table 2 summarizes the design.

Treatment	**n**	**# Groups**	**# Subjects**
RIE	3	70	21
RIE	5	60	30
IIE	3	80	24
IIE	5	60	30

[15] We do this in order to reduce the coarseness of the strategy space and allow subjects to make budget decisions in line with the symmetric Markov perfect equilibrium in pure strategies. This is particularly important for the RIE where the steady state level of the public good is 1.77 for n=3 and 1.38 for n=5, and the equilibrium level of individual investment is, respectively, 0.59 and 0.28 in the first period and 0 in all following periods.

[16] One of the $N = 3$ sessions used 9 subjects.

Table 2: Experimental design

Before the first match, instructions[17] were read aloud, followed by a practice match and a comprehension quiz to verify that subjects understood the details of the environment including how to compute payoffs. The current period's payoffs from the public good stock (called *project size* in the experiment) was displayed graphically, with stock of public good on the horizontal axis and the payoff on the vertical axis. Subjects could click anywhere on the curve and the payoff for that level of public good appeared on the screen. Subjects received information about the total investment in the public good as well as about the individual investments of other subjects in their group, at the end of each period.

At the end of the last match each subject was paid privately in cash the sum of his or her earnings over all matches plus a show-up fee of $10. Earnings ranged from approximately $20 to $50, with sessions lasting between one and two hours. There was considerable range in the earnings and length across sessions because of the random stopping rule.

4 Experimental Results

4.1 Public Good Outcomes

We start the analysis of the experimental results by looking at the long-run stock of public good by treatment. We consider as the *long-run* stock of public good, the stock reached by a group after 10 periods of play.[18] Table 3 compares the theoretical and observed levels of public good by treatment. In order to aggregate across groups, we use the median level of the public good from all groups in a given treatment at period 10 ($\mathbf{y}_{mdn}^{10}$). Similar results hold if we use the mean or other measures of central tendency.[19] We compare this to the stock predicted by the Markov perfect equilibrium of the game after 10 periods (y_{MP}^{10}), and to the stock accumulated in the optimal solution after 10 periods (y_P^{10}).

[17]Sample instructions are reported in Appendix D. Subject decisions, interactions, and feedback were implemented in a computer network using the open source interactive game software, Multistage (http://software.ssel.caltech.edu/).

[18]In the experiment, the length of a match is stochastic and determined by the roll of a die. No match lasted longer than 17 periods and we have very few observations for periods 11-17.

[19]In Appendix B, we report averages, medians and standard errors of the stock of the public good by period for each treatment. The statistical tests in the remainder of this section compare average stocks between different treatments using t-tests and their underlying distributions using Wilcoxon-Mann-Whitney tests.

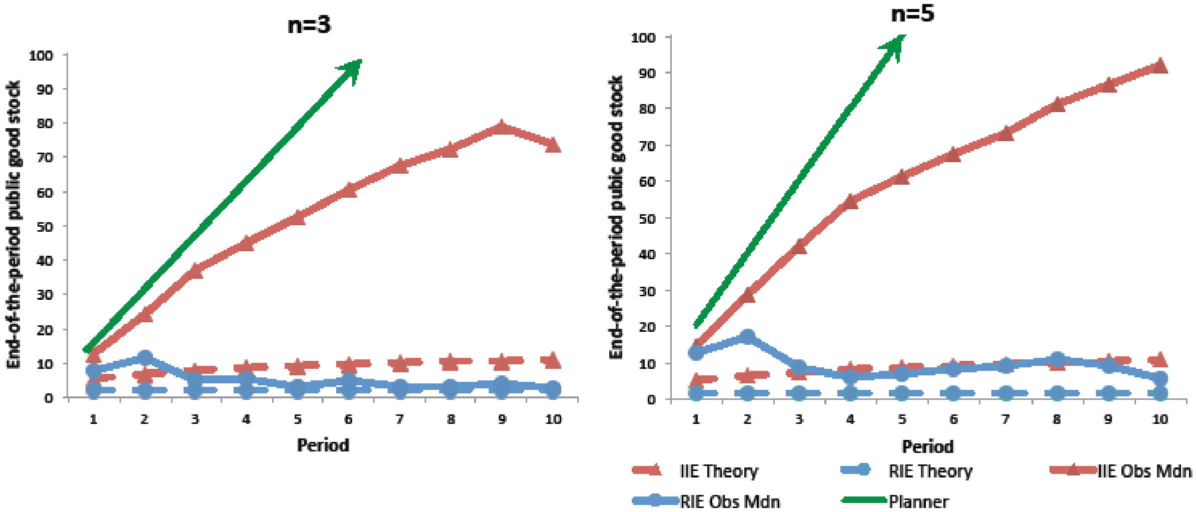

Figure 2: Median time paths of the stock of g, RIE vs. IIE

Treatment	n	$\mathbf{y}^{10}_{mdn}$	y^{10}_{MP}	y^{10}_{P}
Reversible Investment (RIE)	3	2.5	1.38	144
Reversible Investment (RIE)	5	5.5	1.77	200
Irreversible Investment (IIE)	3	73.38	10.91	144
Irreversible Investment (IIE)	5	91.75	10.57	200

Table 3: Long-Run Stock of Public Good, Theory vs. Results by Treatment

How do groups get to these stocks of public good? Figure 2 gives us a richer picture, showing the time series of the stock of public good by treatment.[20] The horizontal axis is the time period and the vertical axis is the stock of the public good. As in Table 3, we use the median level of the public good from all groups in a given treatment. Superimposed on the graphs are the theoretical time paths (represented with dashed lines), corresponding to the Markov perfect equilibria and to the optimal solution.

Table 3 and Figure 2 exhibit several systematic regularities, which we discuss below in comparison with the theoretical time paths.

[20]These and subsequent figures show data from the first ten periods. Data from later periods (11 for IIE with $n = 5$, 11-13 for RIE with $n = 5$ and $n = 3$, and 11-17 for IIE with $n = 7$) are excluded from the graphs because there were so few observations. The data from later periods are reported in Appendix B and included in all the statistical analyses.

FINDING 1. Irreversible investment leads to higher public good production than reversible investment. According to t-tests and Wilcoxon-Mann-Whitney tests[21], the average stock of public good is significantly lower in RIE than in IIE in every single period for both group sizes. This difference is statistically significant at the 1% level ($p<0.01$) for periods 1-10 for both group sizes. Not only are the differences statistically significant, but they are large in magnitude. The median stock of public good is around four times greater in the IIE treatment than in the RIE treatment, averaged across all periods for which we have data (38.75 in IIE vs. 10.75 in RIE for $n = 5$ and 44.25 in IIE vs. 7.25 in RIE for $n = 3$). The differences between the two treatments are relatively small in the initial period, but they increase sharply as more periods are played. By period 10, the differences are very large (73.38 vs. 2.5 for $n = 3$, and 91.75 vs. 5.5 for $n = 5$).

FINDING 2. Both reversible and irreversible investment lead to significantly inefficient long-run public good levels. The optimal steady state is y*=400 for $n = 5$ and y*=144 for $n = 3$, and the optimal investment policy is the fastest approach: invest W in every period until y* is achieved. After 10 periods, the median stock of public good achieved with the optimal investment trajectory is 200 for $n = 5$ and 144 for $n = 3$. In the experiments, the median stock of public good levels out at about 3 ($n = 5$) or 6 ($n = 3$) under reversible investment economies, while it keeps growing, but at an inefficiently slow pace, under irreversible investment. The median stock averages 8.38 in periods 7-10 in RIE with $n = 5$, 3.25 in periods 7-10 in RIE with $n = 3$, 81 in periods 7-10 in IIE with $n = 5$, and 70.13 in periods 7-10 in IIE with $n = 3$. In all treatments the average stock of public good in the last periods (rounds 8 on) is significantly smaller than the level predicted by the optimal solution (the level attainable investing W each period) according to the results of a t-test on the equality of means ($p<0.01$).

FINDING 3. Public good accumulation is higher in five members groups than in three member groups. This difference, however, is statistically significant only in the initial periods. For the same accumulation mechanism (reversible or irreversible investment), the average and median stock of public good is higher with five members groups than with three members groups in every single period. However, this difference is small in magnitude (especially for the earlier periods and for the reversible investment games) and,

[21] The p-values associated with these tests are reported in the Appendix B. The null hypothesis of a t-test is that the averages in the two samples are the same. The null hypothesis of a Wilcoxon-Mann-Whitney test is that the underlying distributions of the two samples are the same. We are treating as unit of observation a single group.

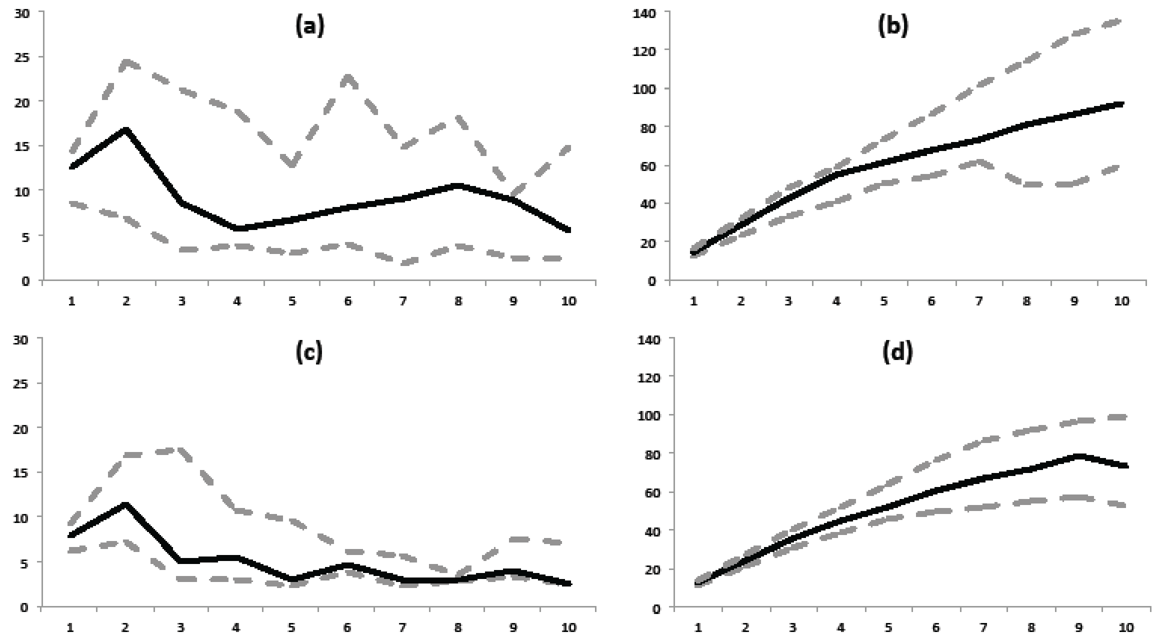

Figure 3: Quartiles of Time Paths of the Stock of g, All Treatments

according to t-tests[22], statistically significant at conventional levels (p<0.05) only for the first two periods in RIE, and the first four periods in IIE. This is in line with the Markovian equilibria discussed in the previous section, which predict small differences between the two group sizes. In RIE, the stock is predicted to converge quickly to similar steady state levels (1.77 and 1.38). In IIE, while the steady state levels are predicted to be exactly the same (16), the equilibrium investment trajectory is somewhat slower with larger groups. However, the differences induced by the different group sizes are small, with the predicted stock after 10 periods equal to 10.57 with five members groups and 10.91 with three members groups.

Because of the possibility of non-stationary equilibria it is natural to expect a fair amount of variation across groups. Figure 2, by showing the median time path of the stock of public good, masks some of this heterogeneity. Do some groups reach full efficiency? Are some groups at or below the equilibrium? We turn next to these questions.

Figure 3 illustrates the variation across groups by representing, for each period, the first, second and third quartile of investment levels for RIE with $n = 5$ (panel (a)), IIE with $n = 5$ (panel (b)), RIE with $n = 3$ (panel (c)), and IIE with $n = 3$ (panel (d)) games. The continuous line represents the median, while the dashed lines represent the range interquartile.

[22]Similar results are obtained using Wilcoxon-Mann-Whitney tests.

There was remarkable consistency across groups, especially considering this was a complicated infinitely repeated game with many non-Markov equilibria. With irreversible investment, many groups invested significantly more heavily than predicted by the Markov perfect equilibrium, but this was not enough to achieve efficient levels of the public good in the long-run, as nearly always such cooperation fell apart in later periods. The most efficient group in IIE with $n = 5$ invested 98% of W in the first period and W in periods 2-6, resulting in a public good level of 119.5. In the remaining four periods, group investment slowed down because of the contagious defection of some of his members: in period 7, one member invests zero; in period 8, two members invest zero, and in periods 9-10, three members invest zero (with average investment in the last three periods at 49% of W and final stock at period 10 of 157.8). Even this very successful group, started consuming resources for private investment well short of the efficient level (400). In RIE with $n = 5$, the most efficient committee in the early periods invested W in each of the first 2 periods, resulting in a public good level of 40. In the following period, one member disinvested the maximum allowed (that is, 1/5 of the stock). The same investment behavior was followed by two members in the fourth period and, finally, in the fifth and final period, every member disinvested his share of the public good, bringing the stock to zero. We observe similar patters for the most efficient groups with $n = 3$.

These findings are perhaps surprising since, from Proposition 3, we know that, for the parameters of the experiment, almost efficient levels of the public good can be supported as the outcome of the RIE game using non-stationary strategies.[23] In the IIE games, on the other hand, the optimal solution cannot be supported by any subgame perfect equilibrium with non-stationary strategies when there is discounting. This is in stark contrast with the unique Markov perfect equilibria derived in Sections 2.2 and 2.3, which predict the opposite comparative static: the long run level of the public good is predicted to be 10 times as large with irreversible investment than with reversible investment.

Figures 2 and 3, therefore, make clear that the predictions of the Markov perfect equilibrium are substantially more accurate than the prediction of the "best" subgame perfect equilibrium (that is the Pareto superior equilibrium from the point of view of the agents). This observation may undermine the rationale for using the "best equilibrium" as a solution

[23] With the parameters of the experiment, the public good stock sustainable with the non-stationary strategies proposed in the proof of Proposition 3 is 130 (vs. an efficient level of 150) for three members groups, and 333 (vs. an efficient level of 400) for five members groups. This steady state is reached in 9 periods for n=3 (with investment equal to W=15 in the first 8 periods), and in 17 periods for n=5 (with investment equal to W=20 in the first 16 periods).

concept.

4.2 Investing Behavior

So far, we have presented results for the public good stock accumulated by each group. In this section, we analyze the data at a finer level, using the investment decisions of each single individual in each group.

How much do individual agents invest in the public good? Figure 4 shows the time series of the median investment in the public good by treatment. The horizontal axis is the time period and the vertical axis is the investment in the public good. The maximum amount each agent can allocate to investment is the same in each period, and it is given by W/n, which is equal to 5 for the three-members groups and to 4 for the five-members groups. The minimum amount each agent can invest is always zero in the irreversible investment treatment, but it depends on the stock at the beginning of the period in the reversible investment treatment (since each agent can disinvest up to g/n units of the public good). For each period, we use the median level of individual investment from all subjects in a given treatment. Similar results hold if we use the mean or other measures of central tendency.

Figure 4 shows a series of interesting patterns. First, the median individual investment is always higher with irreversible investment than with reversible investment in periods 1-10. Second, the level of investment is decreasing, with median investment converging quickly to values around zero for the reversible investment economies and steadily decreasing towards zero for the irreversible economies.

How do these levels of individual investment compare to the theoretical predictions? The median time paths from Figure 4 are qualitatively in line with the predicted time paths: with reversible investment, the theory predicts positive investment only in the first period (when the equilibrium steady state is reached) and zero investment from the second period on; with irreversible investment, the theory predicts positive investment in each period, but at a monotonically decreasing pace (with convergence to the equilibrium steady state only asymptotically). There are, however, some differences between the finer details of the theoretical predictions and the data. We observe over-investment in the early periods: while individual investment is predicted to be less than 1 unit in the first period for all treatments, we observe medians between 2.5 and 4.5. In the reversible economies, this over-investment is corrected in the later periods: the median investment falls sharply to zero and a large fraction of individuals disinvests, with higher early over-investment followed by higher disinvestment. We discuss these observations in detail below.

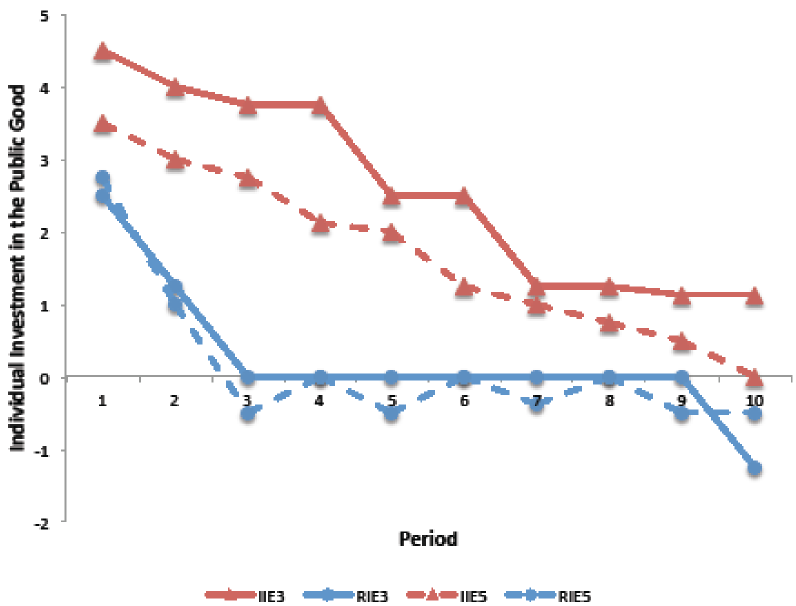

Figure 4: Median Time Paths of Individual Investment

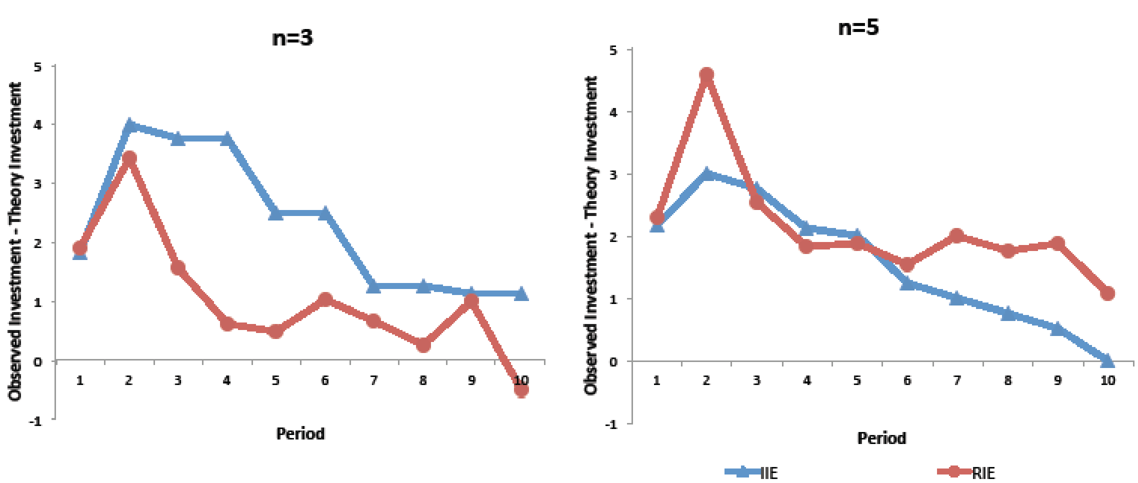

Figure 5: Time Paths of Median Difference between Observed and Theoretical Investment

The game we study is a dynamic game with an evolving state variable. In this game, the strategic incentives in each period are possibly different and determined by the level of the state variable and by subjects' expectations on other subjects' behavior. It follows that, to better compare the observed level of investment with the theoretical predictions, we need to take into account the state variable faced by each agent when making an allocation decision, that is the stock of the public good at the beginning of a period. For each subject in each period, we calculate the difference between his observed behavior and the investment predicted by the theory given the public good stock in his group in that period. Figure 5 shows the time series of the median of this difference. This series starts out significantly above zero for all treatments but decreases as more periods of the same match are played, suggesting that subjects' decisions respond to the evolution of the state variable, with their investment behavior closely matching the predictions of the unique concave Markovian equilibrium for later periods. Notice that this pattern leads to public good outcomes that are in line with the equilibrium steady states for reversible economies, but not for irreversible economies: in the former, subjects can correct the initial over-investment with negative investment, while in the latter the equilibrium investment for any level above the steady state (16) is bound to be zero and the initial over-investment persists. We summarize these findings below.

FINDING 4. In both treatments, there is over-investment relative to the equilibrium in the early periods. This is followed by negative investment approaching the theoretical predictions in RIE, while the over-investment decreases but persists in IIE. In RIE, the median group investment in the first two periods are (7.88=0.53W, 4.13=0.28W) for three members groups, and (12.63=0.63W, 4.88=0.24W) for five members groups. As a result, the median public good stock by the end of period 2 equals, respectively, 11.38, and 16.75. This compares with equilibrium investment policies in the first two periods equal to (1.77, 0) for $n = 3$, and (1.38, 0) for $n = 5$, and a predicted stock equal to, respectively, 1.38 and 1.77. In IIE, on the other hand, the median investment in the first two periods are (12.5=0.63W, 11.63=0.58W) for three members groups, and (14.25=0.71W, 14.25=0.71W) for five members groups. As a result, the median public good stock by the end of period 2 equals, respectively, 24.25, and 28.5. This compares with equilibrium investment policies in the first two periods equal to (5.33, 1.43) for $n = 3$ and (4.72, 1.42) for $n = 5$, and a predicted stock equal, respectively, to 6.76 and for 6.14. Thus, in all treatments, committees overshoot the equilibrium in early periods by a factor of ten (R3 and R5), four (IR3), and five (IR5).[24]

[24] The difference between the average investment in the early periods and the predicted investment in these

In RIE, this overshooting is largely corrected in later periods via disinvestment. When investment is reversible, convergence of the public good stock is close to equilibrium, with the difference between the median public good levels and the equilibrium public good levels in the last 4 periods of data measuring less than 3 units of the public good for three members groups (4.34 vs. 1.77) and less than 4 for five members groups (5.50 vs 1.38). With irreversible investment, investment remains positive but is monotonically decreasing with periods of play (in the same match): the median investment in periods 7-10 is 3.8 (=0.25W) with $n = 3$ and 6.63 (=0.33W), with the minimum median investment reached in period 10 (4.75=0.24W). Given the public good stock by the end of period 2 is already above the predicted stead state level (16), the positive - albeit slower - investment flow in the following periods brings the long-run level of public good to be six (91.75 vs. 16 for IR5) and five times (73.38 vs. 16 for IR3) larger than predicted.

We now turn to a descriptive analysis of the individual allocations between investment in the public good and current consumption good, as a function of period of play (within a match) and accumulation mechanism.

We break down the investment decisions into 3 canonical types: (1) *Positive Investment*; (2) *Zero Investment*; and, for RIE, (3) *Negative Investment*. The first category is further broken down by whether investment in the public good accounts for the whole budget, most of the budget (but less than W), or a minority of the budget. Similarly, the third category is further broken down by whether the negative investment is the maximum allowed by the mechanism ($= g/n$) or a smaller amount.

	RIE 3					**IIE 3**				
Investment Type	**ALL**	**R1**	**R2-4**	**R5-7**	**R8-10**	**ALL**	**R1**	**R2-4**	**R5-7**	**R8-10**
INV > 0	67.0	100	57.8	44.7	39.2	82.8	100	96.4	77.9	59.5
* $I = W$	5.1	12.4	2.6	-	-	27.7	47.9	35.7	21.4	9.5
* $I \in (.5W, W)$	16.2	31.4	13.9	3.5	2.0	24.2	33.3	34.4	20.3	12.3
* $I \in (0, .5W]$	45.7	56.2	41.3	41.2	37.3	30.9	18.8	26.4	36.3	37.7
INV $= 0$	6.7	-	5.9	14.9	17.7	17.2	-	3.6	22.1	40.5
INV < 0	26.4	-	36.3	40.4	43.1	-	-	-	-	-
* $I \in (0, -g/n)$	21.6	-	29.4	34.2	31.4	-	-	-	-	-
* $I = -g/n$	4.8	-	6.9	6.1	11.8	-	-	-	-	-

same periods is statistically significant at the 1% level for all treatments.

Table 4: Individual Investment Types, $n = 3$, # Observations: 705 for RIE, 1716 for IIE.

	RIE 5					IIE 5				
Investment Type	**ALL**	**R1**	**R2-4**	**R5-7**	**R8-10**	**ALL**	**R1**	**R2-4**	**R5-7**	**R8-10**
INV > 0	51.5	82.3	45.3	35.0	35.8	83.1	96	89.8	76.7	60.9
* $I = W$	26.4	47.7	23.8	14.2	15.8	33.1	46.3	37.5	25.0	18.2
* $I \in (.5W, W)$	3.5	4.7	4.6	1.3	0.8	16.5	19.0	20.0	13.3	9.3
* $I \in (0, .5W]$	21.5	30.0	17.0	19.6	19.2	33.5	30.7	32.3	38.3	33.3
INV $= 0$	12.9	17.7	8.8	13.3	14.2	17.0	4.0	10.2	23.3	39.1
INV < 0	35.7	-	45.9	51.7	50.0	-	-	-	-	-
* $I \in (0, -g/n)$	29.0	-	37.9	39.6	40.8	-	-	-	-	-
* $I = -g/n$	6.7	-	8.0	12.1	9.2	-	-	-	-	-

Table 5: Individual Investment Types, $n = 5$, # Observations: 1230 for RIE, 1740 for IIE.

Tables 4 and 5 show the breakdown of investment decisions for the four treatments. In each table, the first column lists the various investment types. The second (for RIE) and sixth (for IIE) columns lists the proportion of each allocation type when we lump together all observations. Columns 3 through 5 (for RIE), and 7 through 9 (for IIE) show how these proportions evolve within a match.

FINDING 5. In all treatments, most allocations involve positive investment in the public good. The proportion of decisions that belong to this category decreases with the period of play (within a single match). In RIE, a significant proportion of allocations involve negative investment. The proportion of decisions that belong to this category increases with the period of play (within a single match). In all treatments, most allocations had a positive amount of investment. In RIE3 and RIE5 this investment type accounts, respectively, for 67%, and 51.5% of all decisions; in IIE3 and IIE5, this type accounts, respectively, for 82.8% and 83.1%.

Allocations with zero or negative investment occurred 33.1% of the time in RIE3 groups, 48.6% of the time in RIE5 groups, but only 17.2% of the time in IIE3 groups and 17.0% of the time in IIE5 groups. The difference is mostly due to the negative investment allocations (which are not allowed in IIE). In contrast to the data, the Markov perfect equilibrium allocations should have been concentrated in the category "$I = 0$" for RIE and "$I \in (0, .5W)$" for IIE. However, notice that, in IIE, "$I \in (0, .5W)$" is the most common proposal type and accounts for around 1/3 of all allocations.

4.3 The Effect of Experience

Within the same match, subjects' investing behavior gets closer to the predictions as more periods are played. It is therefore natural to ask whether we observe a similar pattern across matches. Do subjects choose allocations closer to the predictions of the Markov equilibria when they are more experienced? Or do they still over-invest in early periods and reduce investment in later periods, even after many matches of the same (multi-period) game?

To answer these questions, we compare the average investment in each period in matches 1–5 and in matches 6–10, using t-tests. For the reversible investment treatments, we find no significant difference in investment levels between early and late matches. The difference is not statistically significant for any period of the three members group treatment; for the five members group treatment, the difference is weakly significant in three rounds (at the 10% level in rounds 2, 7, and 8; and at the 5% level in round 2) but there is no clear pattern: investment is higher in early matches in rounds 6 and 7, and higher in late matches in rounds 2 and 8). For the irreversible investment treatments: average investment is higher in late matches for all rounds of the three members group and this difference is statistically significant at the 1% level in rounds 1-5 and at the 5% level in rounds 7 and 8. For the five members groups, investment is higher in later matches only in rounds 1-3 with this difference being statistically significant (at the 1% level for round 1; at the 5% level for rounds 2 and 3).

FINDING 6. There is no evidence of an impact of experience in the reversible investment treatments. In the irreversible investment treatments, investment levels in the early periods are higher in later matches. According to t-tests on the equality of averages, the investment decisions are not affected by experience, at least not in the sense of playing closer to the theoretical predictions. In RIE, as subjects play more matches within the same session their endowment allocation is not significantly altered. In IIE, as subjects play more matches within the same session, they slightly increase their investment in the public good in early periods. This change slightly increases the difference between the observed investment and the investment predicted by the unique Markov equilibrium of the game.

4.4 Testing for Markovian Behavior

The final questions we attempt to address are: To what extent are the models we use adequate to study this problem? What equilibrium concepts should be used? This is a particularly important question since, depending on the equilibrium concept, we can have very different predictions for the same model. While it is difficult to identify the equilibrium adopted by players, the analysis of public good outcomes and investing behavior provides some interesting insights. As discussed above, we observe a consistent pattern of behavior across groups, despite the fact that we have multiplicity of potential equilibria; the investing behavior is correlated to the evolution of the stock in a way predicted by the theory; and, at least for RIE, the long term public good outcomes are close to the equilibrium steady states.

To further pursue this question, we construct a more direct test of the Markovian restriction, that is, of the assumption that players are forward-looking and condition their strategy only on the stock of the public good at the beginning of the period, irrespective of the histories. In particular, we conduct a one-period version of the reversible investment game, where the payoffs from the public good stock are complemented by the equilibrium value functions of the unique concave Markov perfect equilibrium of the game. In each one-period game, agent j receives the following payoff:

$$U^j(x^j, y) = x^j + 2\sqrt{y} + \delta v_R(y),$$

where x^j is the private consumption of agent j, y is the end-of-period public good stock, and $\delta v_R(y)$ is the discounted equilibrium value function from the dynamic game with reversible investment.

In each experimental session, subjects play for 40 matches. Contrary to the dynamic game, the length of each match is known and equal to one period. At the end of each one-period match, subjects are reshuffled into new groups and the public good stock starts out at a (potentially different) exogenous level. We use eight different g_0, to elicit an investment strategy (as a function of the state variable) comparable to the one observed in the fully dynamic game.[25] Table 6 below summarizes the experimental design.

n	W	δ	# Groups	# Subj	# Matches	g_0
3	15	0.75	80	24	40	0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75
5	20	0.75	60	30	40	0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75

Table 6: Experimental design, one-period reduced form treatments.

[25] The beginning-of-period public good stocks we use in this one-period reduced form treatment are 0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75. In each experimental session, each of these beginning-of-period stock is used in five different matches, in random order, for a total of forty matches. The range $[0 - 8.75]$ covers around 75% of observations in the dynamic game with three-members groups and around 55% of observations in the dynamic game with five-members groups.

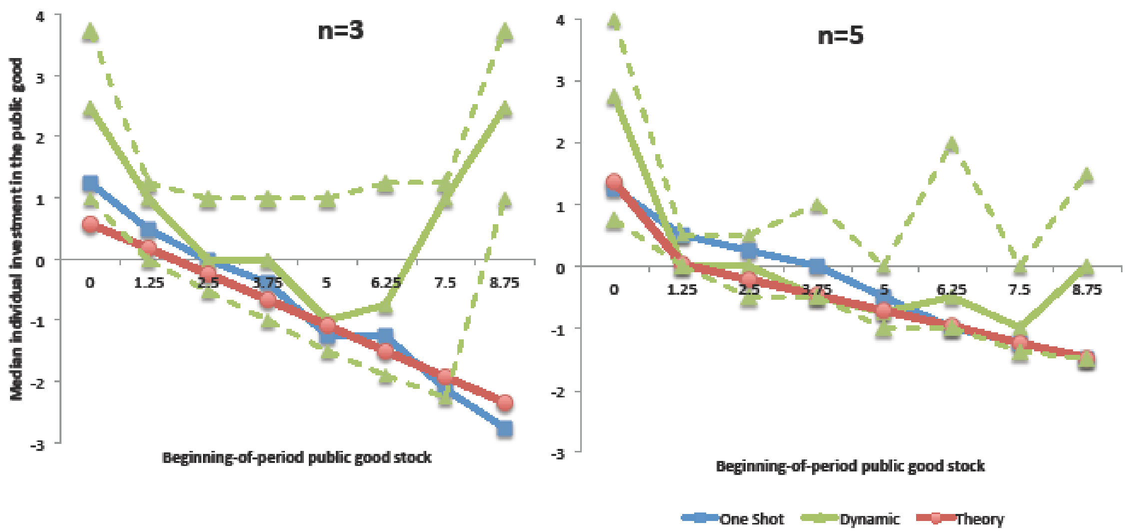

Figure 6: Investment as a Function of Beginning-of-Round Stocks, Reduced Form Treatment vs. Dynamic Treatment. Note: the bold lines represent median investments; the green dotted lines give the interquartile range for the dynamic treatment.

In the one-period reduced form treatments, the unique equilibrium of the game prescribes the same investment level predicted for the fully dynamic game under the Markovian assumption that subjects condition their strategies only on the public good stock. While there is no other equilibrium in this one-period game, in the fully dynamic game there is a plethora of different subgame perfect equilibria that can sustain higher level of investment with non-stationary strategies. Therefore, if we observe an identical behavior in the two treatments, we can consider this as evidence of Markovian strategies in the fully dynamic game. On the other hand, we can attribute any difference in behavior to the non-stationary strategies that can arise in a repeated game.

Figure 6 illustrates the median individual investment as a function of the initial stock for the one-period reduced form games described above and for the fully dynamic games.[26]

FINDING 7. In RIE, there is evidence of Markovian, forward-looking behavior. For three-members groups, investment is significantly higher in the dynamic treatment

[26] Since the beginning-of-round stock in the dynamic games is endogenous and does not necessarily match the values used in the one-round games, for these games we use a weighted median of investment levels for all periods-groups that started with a public good stock in a 6 experimental units interval around the starting size used in the one-round games. For example, the median investment corresponding to a beginning-of-round stock of 5 is computed as the weighted median investment from all periods-groups starting at a stock between 4.25 and 5.75. This allows us to have a comparable number of observations between one-round and dynamic games. The results are the same when we use intervals of 8 or 10 experimental units.

for initial stocks of 0, 6.25, 7.5, and 8.75, and statistically indistinguishable for the remaining initial stocks. For five-members groups, investment is significantly higher in the dynamic treatment for initial stocks of 0, 6.25, and 8.75; it is significantly higher in the reduced form treatment for initial stocks of 1.25, 2.5, and 5; and it is statistically indistinguishable for the remaining initial stocks (3.75 and 7.50). While there is some significant difference, these differences are small in magnitude (with the exception of initial stocks greater than 6.25 for $n = 3$), and we cannot conclude that investment is higher in the fully dynamic game than in the reduced form game (as a consequence of non-stationary strategies). As shown by Figure 6, the median investment in the one-shot treatment is in the interquartile range of the investment observed in the dynamic treatment for all initial stocks in both treatments, with the exception of an initial stock of 8.25 for the three-members groups. Regarding the high investment in the dynamic treatment for three-member groups and stocks greater than 6.25, this is due to a few groups who invested significantly more heavily than predicted by the Markov perfect equilibrium, but this only happened rarely and most of the observations from the dynamic treatment (where the initial public good stock is endogenous) have a beginning-of-period stock smaller than 6.25.[27]

We, thus, conclude that observed behavior is well approximated by the predictions of a purely forward looking Markov equilibrium, rather than by an equilibrium in which agents look back at the past to punish uncooperative behavior (or reciprocate cooperative behavior) by other members of the group.

5 Static versus Dynamic: What Have We Learned?

While this is the first experimental study of the dynamic accumulation process of a durable public good, a vast experimental literature has addressed the provision of public goods in static environments. This begs the following questions: How do the results from our dynamic public good experiments compare to the results from the repeated static public good games? And what new insights can we learn breaking out from the static framework?

Comparing directly our dynamic game with the static framework used by the previous experimental literature is generally a difficult task, for a number of reasons. Even when static

[27]Since the beginning-of-period stock in the dynamic treatment is endogenous we have a reduced number of observations for these high values: we use only 10 groups to compute the median investment for a starting stock of 8.75. The remaining 60 groups never accumulated these levels of public good. The beginning-of-period stock is smaller than 6.25 in 60% of observations (regardless of period number). The average beginning-of-period public good stock in rounds 8-10 (that is, the long run level of public good) is 4.6.

public good games are repeated a fixed number of times, the strategic environment is the same in every period, and there is a unique equilibrium prediction, that does not change over time. The most common example is the voluntary contribution game with linear payoffs, in which the individually optimal investment level is zero, while the socially optimal one is the whole budget. The equilibrium is in dominant strategies, so contributions in past and future periods do not matter for equilibrium behavior, and agents' expectations about other agents' current or future contributions are irrelevant from the standpoint of equilibrium.[28] The game we study, on the other hand, is not only a repeated game, but a stochastic game with an evolving state variable, and a strategic environment that changes in every period (as the durable public good is accumulated over time). Our theory makes predictions that are path dependent and change over time (equilibrium investments are sometimes positive, sometimes negative, and sometimes zero, depending on the current stock of public good), and, while we restrict attention to the unique Markov perfect equilibrium, the infinite horizon of the game generates a plethora of non-stationary equilibria that have much different properties. In this dynamic setting, not contributing is socially optimal in some continuation games (when the stock of the public good has reached the optimal level). More importantly, at any point in time, even in a Markov equilibrium, the individually optimal decision depends on past contributions through their effect on the current state, as well as on the expectations on current and future contributions of other agents. Moreover, a fundamental question of our paper, the impact of investment reversibility on the dynamic free rider problem, cannot be studied in a static framework where the public good starts out at zero in every round (and, thus, contributions can only be non-negative).

In spite of these clear difficulties in comparing the two frameworks, we can still draw some connections between the behavior observed in repeated static public good games and behavior in our dynamic durable public good experiments. In the reminder of this section we discuss a few of the most significant similarities and differences.

Over-investment, efficiency and irreversibility. The first has to do with the general issue of whether contributions tend to be above, below, or approximately equal to the theoretically predicted levels. In static environments, with few exceptions, actual contributions are generally above the equilibrium levels suggesting that theoretical predictions tend to overstate the seriousness of free riding.[29] Still, contributions not only fail to reach efficient

[28] This applies as well to some of the dynamic games based on the linear voluntary contributions model, including the variation with no completion benefit in Duffy et al. 2008.

[29] There are some exceptions. See Laury and Holt (2008) and Palfrey and Rosenthal (1991).

levels (as Ledyard 1995 reports in his survey), but are generally very much below. Average contributions in initial plays of the game typically fall in a range between 40% and 60% of the optimal level, with a systematic decline to very low levels with repetition (between 10% and 20% of the optimal level after 10 rounds of play). Similarly, in our dynamic environment, there is significant over-contribution with respect to the predictions of the unique MPE in the early periods of play while the public good stock is beginning to accumulate, but this over-contribution mostly disappears in later periods (especially with reversible investment, where the long run stock of the public good is very close to the MPE steady state). These declines over time in both the reversible and irreversible cases lead to significantly inefficient long-run public good levels (see Figure 2, and Findings 1 and 2). How serious is the inefficiency with a durable public good, as compared to one-shot public goods problems? Interestingly, the answer depends critically on whether contributions are reversible. With reversibility, the median public good stock converges to approximately 2% of the efficient steady state. In contrast, with irreversibility, the median period 10 stock of public good is approximately 50% of the efficient level. Thus, with reversible investments we see contribution levels that are less than is typically observed in static voluntary contribution games, but the opposite is the case with irreversible investment. The effect of irreversibility, clearly important in real world applications of the theory, can clearly not be observed in static models or in repeated models without a state variable.

Investment pattern and dynamics. Second, there are some similarities in terms of the investment pattern we observe over time: as in the static literature, in our dynamic experiments, there is a tendency for initial over-investment in the early periods, followed by investment levels approaching the theoretical predictions (see Figures 4 and 5). Moreover, a similar pattern is observed when subjects are re-matched into new groups and the public good stock starts out at zero, a phenomenon similar to the "re-start effect" from the static literature (see Andreoni and Croson 2008 for a survey).

Contrary to the static literature (where the predictions are no contributions in every period), the time paths we observe in the dynamic games are qualitatively in line with the predicted time paths: with reversible investment, theory predicts positive investment only in the first period (when the equilibrium steady state is reached) and zero investment from the second period on; with irreversible investment, the theory predicts positive investment in each period, but at a monotonically decreasing pace (with convergence to the equilibrium steady state only asymptotically). These general patterns are found in our data. Moreover, the convergence to the equilibrium predictions follows a different pattern from static experiments and the equilibrium predictions themselves are path dependent and endogenous: in the treatment with reversibility, we observe significant levels of negative investment, with subjects reacting to above-equilibrium accumulation levels and the stock of public good gradually declining in the direction of the equilibrium steady state.

Heterogeneity in behavior. Finally, another finding of the more recent experimental literature on static public good games is the existence of distinct types of behavior. This was first considered by Isaac, Walker, and Thomas (1984), who classify each investment decision as being "Strong Free-Riding", "Weak Free-Riding" or "Lindahl/Altruistic" depending on whether the investment is less than 33%, between 33% and 66%, and more than 66% of the individual budget, respectively. [30] According to this classification, they report 44% of investment decisions in their experiment are Strong Free-Riding, 27% are Weak Free-Riding, and 29% are Lindahl/Altruistic. We applied a similar analysis to our data (adjusting for the fact that the individual budget in the reversible investment treatments is state-dependent) and we found a rather similar distribution of decision types (pooling all treatments together): 42% of investment decisions in our experiments are Strong Free-Riding, 18% are decisions are Weak Free-Riding, and 41% are Lindahl/Altruistic.

A different approach traces these aggregate patterns of contribution behavior to heterogeneity at the individual level. For example, there is some evidence from static public good experiments that some individuals behave as "conditional cooperators", whose contribution to the public good is positively correlated with their beliefs about the contributions made by their group members (Keser and Van Winden 2000, Fischbacher et al. 2001, Burlando and Guala 2005, Fischbacher and Gaechter 2010).[31] While subjects in our experiment are not explicitly asked to make decisions contingent on the other group members' contributions, over the course of the experiment they experience a wide range of (endogenous) past group decisions and we can use this data to measure the extent to which individual contributions respond positively to other group members' past contributions. There is also evidence for the existence of other behavioral types who are either altruistic or completely selfish.

To identify these different behavioral types, for each subject we estimate a Tobit regression of the deviation from predicted investment (notice that, in the static experiment, this is simply equal to the investment level) on the average investment of the other group members in the previous period (with a constant). We then classify each subject as a conditional

[30] They do not actually classify individuals into behavioral types based on their behavior in the 10 rounds of play.

[31] There are different possible interpretations for these behavioral types, such as imitation, conformity, reciprocity, repeated game strategies, etc.

cooperator if the slope is significantly different than 0 at the 1% level; as an "unconditional cooperator" if the slope is not significantly different than 0 and his average investment deviation is in the third quantile; as "free rider" if the slope is not significantly different than 0 and his average investment deviation is in the first quantile. The results are reported in Table 7. Overall, we measure 68% conditional cooperators, 10% unconditional cooperators, 13% free riders, and 9% unclassified. This distribution of behavioral types is roughly in line with the static literature, although there is considerable variation. [32].

	IIE, 3	*IIE, 5*	*RIE, 3*	*RIE, 5*	*Overall*
Unconditional Cooperator	-	13% (4)	19% (4)	10% (3)	10%(11)
Conditional Cooperator	100% (24)	80% (24)	52% (11)	40% (12)	68% (71)
Free Rider	-	3% (1)	24% (5)	27% (8)	13% (14)
Other	-	3% (1)	5% (1)	23% (7)	9% (9)

Table 7: Classification of Subjects' Strategies. The number of subjects is in parentheses.

6 Discussion and Conclusions

This paper investigated the dynamic accumulation process of a durable public good in a voluntary contribution setup. Despite the fact that most, if not all, public goods are durable and have an important dynamic component, very little is known on this subject, both from a theoretical and empirical point of view. We attempt to provide some initial empirical findings about voluntary contribution behavior with durable public goods.

We have considered two possible cases: economies with reversible investments (RIE), in which in every period individual investments can either be positive or negative; and economies with irreversible investments (IIE), in which the public good cannot be reduced. Reversibility is an important feature of many public goods problems (for example, common pool problems), which is completely missed by static analysis. We also have a secondary treatment dimension: we compare three-members and five-members groups. For all treatments, we have characterized the steady states and the accumulation paths that can be supported by the optimal solution and by the unique symmetric concave Markov equilibrium.

[32] The fraction of conditional cooperators in those studies is usually around 50-60%, but ranges from 35% (Burlando and Guala 2005) to 80% (Keser and Van Winden 2000)

We have highlighted three main results. First, the dynamic free riding problem exists and it is severe, with the long run public good stock levels falling shorts of efficiency in all treatments. The additional free riding component that emerges in this dynamic game is most obviously seen in reversible investment economies: in line with the comparative static predictions, irreversible investment leads to significantly higher public good production than reversible investment. With reversibility, the dynamic dimension exacerbates the free rider problem present in static public good provision: if an agent contributes above the equilibrium levels, not only this reduces the future contributions by all agents, but it triggers negative investment by other agents that transform part of the public good stock in private consumption. In this treatments, the median public good stock converges to approximately 2% of the efficient steady states, versus long run contributions between 10% and 20% of the optimal level in repeated static public good games. On the other hand, the irreversibility constraint dampens the dynamic free riding problem, by creating a *commitment device* and reducing the strategic substitutability of contributions. Notice that this has nothing to do with history dependent trigger strategies made possible by the infinite horizon: a similar dynamic would arise in a model with a finite horizon (but losing stationarity of equilibrium strategies). In the treatments where the public good cannot be scaled back to consumption, the median period 10 stock of the public good is approximately 50% of the efficient level.

Second, we have shown that, in both treatments, there is over-investment in the early periods, compared to the equilibrium investment levels. In the treatment with reversibility, this is followed by a significant reversal, with the stock of public good gradually declining in the direction of the equilibrium steady state. When disinvestment is not feasible, investment steadily decreases but the initial over-investment cannot be corrected and the long run level of the public good remains above the equilibrium steady state.

Third, we have proposed a novel experimental methodology to test the assumption that subjects' strategies in this complex infinite-horizon game depend only on the state variable, that is, the accumulated level of the public good. We have shown that, for the reversible investment treatment, there is evidence of Markovian, forward-looking behavior.

This is the first experimental study of the dynamic accumulation process of a durable public good. Our design was intentionally very simple and used a limited set of treatments. As a consequence, there are many possible directions for the next steps in this research. The theory has interesting comparative static predictions about the effect of other parameters that we have not explored in this work, such as: the discount factor; the depreciation level; preferences; and endowments. For example, a higher discount factor increases both the optimal steady state and the equilibrium steady state of the durable public good for all values of n and for both reversible and irreversible economies. For similar reasons, positive depreciation in the public good technology leads to a decrease in the steady state of the Markov equilibrium studied here. Among these extensions, it would be particularly interesting to run experiments that allow a closer comparison with the results from the static literature. This can be done in a number of different ways: for example, experiments with a finite and known horizon of one period (that is, $\delta = 0$), or experiments with full depreciation of the stock at the end of each period and an infinite horizon (that is, $\delta > 0$).

Moreover, our model and experimental design does not consider different rules for negative investment (for example, allowing subjects to disinvest unilaterally up to the whole stock and adopting a rationing rule to keep a nonnegative level of public good), or the effect of a completion benefit at a specified accumulation threshold. We have also limited the analysis to voluntary contribution mechanisms that turn out to be highly inefficient, both in theory and in practice. Battaglini, Nunnari, and Palfrey (2012b) study how centralized mechanisms fare in providing durable public goods and show that efficiency increases with the majority rule required to approve an allocation decision. It would be interesting to consider different decentralized mechanisms and explore which ones are more efficient for the provision of durable public goods.

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Appendix A - Proofs of Propositions

Proof of Proposition 1

The fact that a strictly concave equilibrium has the property stated in the proposition follows from the discussion in the text. Here we prove existence and uniqueness.

Existence. Battaglini, Nunnari and Palfrey [2012a] show that there is a weakly concave Markov equilibrium with steady state equal to y_R^* for any $y_R^* \in \left[[u^{-1}]'(1-\delta/n), [u']^{-1}(1-\delta)\right]$. To prove that the equilibrium corresponding to the steady state $y_R^* = [u^{-1}]'(1-\delta/n)$ is strictly concave we provide an alternative (self contained) existence proof.

Let $y_R^* = [u^{-1}]'(1-\delta/n)$, and $g_R^1 = \max\{0, y_R^* - W\}$. For any $g > g_R^1$ define a value function $v_R^1(g) = \frac{1}{1-\delta}\left[\frac{W-(y_R^*-g)}{n} + u(y_R^*)\right]$. Note that this function is continuous, non decreasing, strictly concave and differentiable with respect to g, with derivative $[v_R^1]'(g) = \frac{1}{n}$. Let $g_R^2 = \max\{0, g_R^1 - W\}$, and define:

$$v_R^2(g) = \begin{cases} v_R^1(g) & g \geq g_R^1 \\ u(g+W) + \delta v_R^1(g+W) & g \in [g_R^2, g_R^1) \end{cases}$$

Note that $v_R(g)$ is continuous and differentiable in $g \geq g_R^2$, except at most at g_R^1 To see that it is also concave in this interval, note that it is strictly concave for $g \geq g_R^1$. Moreover, for any $g \in [g_R^2, g_R^1)$ and $g' \geq g_R^1$ we have:

$$\begin{aligned} [v_R^2]'(g) &= u'(g+W) + \delta [v_R^1]'(g+W) \\ &> u'(y_R^*) + \delta [v_R^1]'(y_R^*) = 1 > \frac{1}{n} = [v_R^2]'(g') \end{aligned}$$

The first inequality derives from $y_R^* > g + W$ (which is true, by definition of g_R^1 and g_R^2, for all $g \in [g_R^2, g_R^1)$), and strict concavity of $u(g)$ and $v_R^1(g)$. It follows that $v_R^2(g)$ strictly concave in $g \geq g_R^2$. Assume that for all $g \geq g_R^n$, with $g_R^n \geq 0$ and either $g_R^n < g_R^2$ or $g_R^n = 0$, we have defined a value function $v_R^n(g)$ that is concave and continuous, and that is differentiable in $g > g_R^1$. Define $g_R^{n+1} = \max\{0, g_R^n - W\}$, and

$$v_R^{n+1}(g) = \begin{cases} v_R^n(g) & g \geq g_R^n \\ u(g+W) + \delta v_R^n(g+W) & g \in \left[g_R^{n+1}, g_R^n\right) \end{cases}$$

Using the same steps as above, we can easily show that this function is strictly concave, continuous in $g \geq g_R^{n+1}$, and differentiable for $g > g_R^1$. Moreover, either $g_R^{n+1} = 0$ or $g_R^{n+1} < g_R^n$. We can therefore define inductively a value function $v_R(g)$ for any $g \geq 0$

that is continuous, strictly concave, and that is differentiable at least for $g > g_R^1$ and so, in particular, at y_R^*. Define now the following strategies:

$$y_R(g) = \min\{W + g, y_R^*\}, \text{ and } x_A(g) = [W + g - y_R(g)]/n. \tag{5}$$

We will argue that $v_R(g), y_R(g), x_A(g)$ is an equilibrium. To see this note that by construction, if the agent uses strategies $y_R(g), x_A(g)$, then $v_R(g)$ describe the expected continuation value function of an agent. To see that $y_R(g), x_A(g)$, are optimal given $v_R(g)$ note that for $g \geq g_R^1$, $\left\{y_R^*, \frac{W+g-y_R^*}{n}\right\}$ maximizes (2) when all the constraints except the second are considered; and for $g \geq g_R^1$, $W + g > y_R^*$, so the second constraint is satisfied as well. For $g < g_R^1$, we must have $y_R(g) = W + g$, $x_A(g) = 0$. We conclude that $y_R(g)$, $x_A(g)$ is an optimal reaction function given $v_R(g)$. ■

Uniqueness. Consider a strictly concave equilibrium with value function $v_R(g)$. Because $v_R(g)$ is strictly concave, there is a unique maximum y_R^* of the objective function of (2). It follows that we must have $y_R(g) = \min\{W + g, y_R^*\}$, implying that $y_R(g) = y_R^*$ for any $g \geq y_R^* - W$ and $y_R(y_R^*) = y_R^*$. It is straightforward to show that the derivative of the value function in $g \geq y_R^* - W$ exists and it is equal to $v_R'(g) = 1/n$. Using the first order condition that defines y_R^*, we must have $u'(y_R^*) + \delta v_R'(y_R^*) = 1$. This implies that in any strictly concave Markov equilibrium we must have a steady state $y_R^* = [u']^{-1}(1 - \delta/n)$. ■

Proof of Proposition 2

Since the equilibrium is weakly concave, we must have that $v_{IR}(g)$ admits a right and left derivative at any point g. Let us call $y_{IR}^+(y_{IR}^*)$ and $y_{IR}^-(y_{IR}^*))$ the, respectively, right and left derivatives. Since at y_R^* we must have $y_{IR}(y_{IR}^*) = y_{IR}^*$, it is easy to see that $y_{IR}^+(y_{IR}^*) = 1$, since $y_{IR}(y_{IR}^* + \Delta) = y_{IR}^* + \Delta$.

Consider now the left derivative. In a left neighborhood of y_{IR}^*, we must have $y_{IR}(g) \in (0, W + g)$, so $x_{IR}(g) > 0$ and

$$y_{IR}(g) \in \arg\max_y \{u(y) + \delta v_{IR}(y) - y\} \tag{6}$$

. We can write:

$$\begin{aligned} v_{IR}(g) &= \frac{W + g - y_{IR}(g)}{n} + u(y_{IR}(g)) + \delta v_{IR}(g)(y_{IR}(g)) \\ &= u(y_{IR}(g)) + \delta v_{IR}(g)(y_{IR}(g)) - y_{IR}(g) + \frac{W + g - (n-1)y_{IR}(g)}{n} \end{aligned}$$

By the theorem of the maximum we therefore have: $v'_{IR}(g) = \frac{1}{n} + \frac{n-1}{n} y'_{IR}(g)$. Combining this expression with the first order condition of (6) we obtain:

$$y'_{IR}(g) = \frac{1 - n\left(1 - u'(g)\right)/\delta}{1 - n}$$

for any $g < y^*_{IR}$. At y^*_{IR} the left derivative must therefore be $y^-_{IR}(y^*_{IR}) = \frac{1-n\left(1-u'(y^*_{IR})\right)/\delta}{1-n}$. Imposing $y^-_{IR}(y^*_{IR}) = y^+_{IR}(y^*_{IR}) = 1$, and recalling that $u(g) = 2\sqrt{g}$, we obtain that in any concave Markov equilibrium with irreversibility we must have $y^*_{IR} = (1-\delta)^{-2}$ as claimed. ■

Proof of Proposition 3

The efficient outcome (the social planner solution characterized in Section 2.1) can be sustained in the voluntary contribution game with reversible investment, when agents use non-stationary strategies entailing reversal to the unique concave Markov equilibrium characterized in Section 2.2. To show this, we construct strategies whose outcome is the efficient level of public good and we show that there is no profitable deviation from the equilibrium path. The symmetric strategy for each committee member is to invest $i^*_P(g) = \min\left\{\frac{W}{n}, \frac{y^*_P - g}{n}\right\}$ if $g_t = y^*(g_{t-1})$ (i.e. if the observed level of the public good at the beginning of the period is consistent with equilibrium strategies, or, in other words, it is the efficient level of public good given the stock of g at the beginning of the previous period) and to invest $i^*_R(g) = \min\left\{\frac{W}{n}, \frac{y^*_R - g}{n}\right\}$ where $y^*_R < y^*_P$ (i.e. the investment associated with the Markov equilibrium characterized in Proposition 1) if $g_t \neq y^*(g_{t-1})$ (i.e. if a deviation from equilibrium has occurred in the previous period). To prove that this strategy profile is an equilibrium we show that agents have no profitable deviation.

An agent's payoff if she follows the equilibrium strategy is:

$$\frac{W}{n} - i^*_P(g) + 2\sqrt{g + n i^*_P(g)} + \delta V_{EQ}(g + n i^*_P(g))$$

An agent's payoff if she deviates (according to her most profitable deviation) is:

$$\frac{W}{n} + \frac{g}{n} + 2\sqrt{g - \frac{g}{n} + (n-1) i^*_P(g)} + \delta V_{DEV}\left(g - \frac{g}{n} + (n-1) i^*_P(g)\right)$$

An agent's most profitable deviation is to invest $-g/n$ (i.e. to subtract from the public good her share and to consume it). The gains from this deviation are greater the closer g is to y^*_P. Therefore, we will check whether an agent has an incentive to deviate when $g \in [g_P, y^*_P]$, or whether:

$$\frac{W}{n} - \frac{y_P^* - g}{n} + 2\sqrt{y_P^*} + \delta V_{EQ}(y_P^*) \geq \frac{W}{n} + \frac{g}{n} + 2\sqrt{g - \frac{g}{n} + (n-1)\frac{y_P^* - g}{n}} + \delta V_{DEV}\left(g - \frac{g}{n} + (n-1)\frac{y_P^* - g}{n}\right)$$

where:

$$V_{EQ}(y_P^*) = \frac{1}{1-\delta}\left[\frac{W}{n} + 2\sqrt{y_P^*}\right]$$

and:

$$\begin{aligned} V_{DEV}\left(\frac{n-1}{n}y_P^*\right) &= \frac{W}{n} - \frac{y_R^* - \frac{n-1}{n}y_P^*}{n} + 2\sqrt{y_R^*} + \delta V_{DEV}\left(y_R^*\right) \\ &= \frac{W}{n} - \frac{y_R^* - \frac{n-1}{n}y_P^*}{n} + +2\sqrt{y_R^*} + \frac{\delta}{1-\delta}\left(\frac{W}{n} + 2\sqrt{y_R^*}\right) \end{aligned}$$

After we plug in $V_{EQ}(y_P^*)$ and $V_{DEV}\left(\frac{n-1}{n}y_P^*\right)$, the inequality above becomes:

$$\begin{aligned} \frac{W}{n} - \frac{y_P^* - g}{n} + 2\sqrt{y_P^*} + \frac{\delta}{1-\delta}\left[\frac{W}{n} + 2\sqrt{y_P^*}\right] \;&\geq\; \frac{W}{n} + \frac{g}{n} + 2\sqrt{\frac{n-1}{n}y_P^*} + \delta \\ &\left[\frac{W}{n} - \frac{y_R^* - \left(\frac{n-1}{n}y_P^*\right)}{n} + 2\sqrt{y_R^*} + \frac{\delta}{1-\delta}\left(\frac{W}{n} + 2\sqrt{y_R^*}\right)\right] \\ \frac{1}{1-\delta}\left[2\sqrt{y_P^*} - \delta 2\sqrt{y_R^*}\right] - \frac{\delta}{n}\left[\frac{(n-1)}{n}y_P^* - y_R^*\right] \;&\geq\; 2\sqrt{\frac{n-1}{n}y_P^*} + \frac{y_P^*}{n} \end{aligned}$$

Replacing y_P^* and y_R^* (who both depend on δ), the inequality we want to prove becomes:

$$\frac{1}{1-\delta}\left[\frac{2n}{1-\delta} - \frac{\delta 2n}{n-\delta}\right] - \frac{\delta(n-1)}{(1-\delta)^2} + \frac{\delta}{n}\left(\frac{n}{n-\delta}\right)^2 \;\geq\; \sqrt{\frac{n-1}{n}n^2} + \frac{n}{(1-\delta)}$$

Multiplying both sides by $(1-\delta)^2$ and rearranging, we have:

$$n - (n-1)\delta \geq \frac{(1-\delta)^2\delta 2n}{n-\delta} + \frac{\delta}{n}\left(\frac{n}{n-\delta}\right)^2(1-\delta)^2 + \sqrt{\frac{n-1}{n}n^2(1-\delta)}$$

There is $\widehat{\delta}_R$ such that $\forall \delta > \widehat{\delta}_R$ the inequality above holds. To see this note that as δ approaches 1 the RHS approaches zero, while the LHS is positive for any $\delta \in [0,1]$. ■

Using the parameters and the utility function of our experiments, $\widehat{\delta}_R = 0.80$ for $n = 3$ and $\widehat{\delta}_R = 0.86$ for $n = 5$. We use $\delta = 0.75$, which means that, in the experimental setting, the efficient level of the public good cannot be sustained in equilibrium. However, it can be shown that non-stationary strategies of the type proposed above can sustain an almost efficient level of the public good, y^*. In this case, the inequality we want to prove is:

$$\frac{1}{1-0.75}\left[2\sqrt{y^*} - 0.752\sqrt{y_R^*}\right] - \frac{0.75}{n}\left[\frac{(n-1)}{n}(y^*) - y_R^*\right] \geq 2\sqrt{\frac{n-1}{n}(y^*)} + \frac{y^*}{n}$$

This inequality holds for $y^* = 130$ in the treatment with 3 agents (where $y_P^* = 144$) and for $y^* = 333$ in the treatment with 5 agents (where $y_P^* = 400$).

Appendix B - Additional Tables [Not For Publication]

Period	RIE 3					RIE 5				
	Theory	Obs	Avg	Mdn	SE	Theory	Obs	Avg	Mdn	SE
1	1.77	70	7.93	7.88	2.26	1.38	60	11.85	12.63	4.84
2	1.77	46	11.89	11.38	5.41	1.38	48	17.55	16.75	11.43
3	1.77	31	9.72	5.00	7.94	1.38	33	14.99	8.50	15.19
4	1.77	24	8.59	5.50	9.15	1.38	24	12.77	5.63	14.00
5	1.77	21	7.29	3.00	9.04	1.38	21	10.57	6.75	13.08
6	1.77	10	6.83	4.63	7.58	1.38	15	12.65	8.00	11.25
7	1.77	7	4.18	3.00	3.26	1.38	12	9.15	9.13	7.98
8	1.77	7	4.32	3.00	4.10	1.38	12	11.13	10.63	7.89
9	1.77	7	4.68	4.00	2.11	1.38	6	8.00	8.88	4.99
10	1.77	3	4.00	2.50	2.60	1.38	6	7.20	5.50	6.46
11	1.77	3	5.42	7.25	3.62	1.38	3	6.00	6.00	0.50
12	1.77	3	6.67	7.75	2.79	1.38	3	7.67	10.75	6.00
13	1.77	3	7.58	7.75	4.00	1.38	3	7.92	9.25	6.36

Table 8: Summary statistics of public good stock per period, reversible investment treatments. Observations are groups.

Period	**IIE 3**					**IIE 5**				
	Theory	Obs	Avg	Mdn	SE	Theory	Obs	Avg	Mdn	SE
1	5.33	80	12.17	12.50	2.22	4.72	60	14.03	14.25	3.39
2	6.76	72	23.58	24.25	4.92	6.14	57	27.46	28.50	7.10
3	7.74	64	34.35	36.63	7.92	7.13	54	39.69	42.13	11.72
4	8.48	60	43.83	45.00	11.29	7.91	42	50.01	54.50	15.70
5	9.07	56	52.08	52.25	15.08	8.53	33	59.79	61.00	22.16
6	9.56	52	60.74	60.25	18.20	9.06	30	67.67	67.50	27.94
7	9.97	40	68.45	67.25	21.48	9.51	21	75.43	73.00	33.55
8	10.33	36	72.93	71.88	24.37	9.91	18	80.99	81.00	38.50
9	10.64	28	77.13	78.63	28.14	10.26	15	86.32	86.50	45.74
10	10.91	20	74.6	73.38	29.85	10.57	12	96.52	91.75	47.00
11	11.15	20	78.85	79.50	32.70	10.85	6	91.58	77.13	61.23
12	11.37	16	75.94	72.75	30.71	11.11	–	–	–	–
13	11.57	8	95.56	96.25	25.51	11.35	–	–	–	–
14	11.75	8	100.09	97.88	29.20	11.57	–	–	–	–
15	11.91	4	108.75	106.25	47.89	11.77	–	–	–	–
16	12.06	4	116.25	114.88	51.77	11.96	–	–	–	–
17	12.20	4	123.94	124.00	55.87	12.13	–	–	–	–

Table 9: Summary statistics of public good stock per period, irreversible investment treatments. Observations are groups.

Round	IIE 3				IIE 5			
	Obs	Avg	Mdn	SE	Obs	Avg	Mdn	SE
1	210	2.64	2.50	1.46	300	2.37	2.75	1.69
2	138	1.36	1.25	2.22	240	0.95	1.00	2.62
3	93	−0.64	0.00	2.91	165	−0.45	−0.50	2.90
4	72	−0.39	0.00	2.65	120	−0.44	0.00	3.07
5	63	−0.23	0.00	2.12	105	−0.70	−0.50	2.82
6	30	0.28	0.00	1.46	75	0.19	0.00	2.91
7	21	0.00	0.00	1.31	60	−0.59	−0.38	3.22
8	21	0.05	0.00	1.14	60	0.40	0.00	2.36
9	21	0.12	0.00	1.80	30	−0.48	−0.50	2.27
10	9	−0.81	−1.25	1.58	30	−0.16	−0.50	2.01
11	9	0.47	1.00	1.54	15	0.83	0.50	1.34
12	9	0.42	0.75	2.48	15	0.33	−0.25	1.59
13	9	0.31	0.00	2.27	15	0.05	0.00	1.67

Table 10: Summary statistics of individual investment per period, reversible investment treatments. Observations are individual investment decisions.

Round	IIE 3				IIE 5			
	Obs	Avg	Mdn	SE	Obs	Avg	Mdn	SE
1	240	4.06	4.50	1.15	300	2.81	3.50	1.34
2	216	3.83	4.00	1.15	285	2.67	3.00	1.41
3	192	3.60	3.75	1.42	270	2.47	2.75	1.49
4	180	3.26	3.75	1.54	210	2.21	2.13	1.56
5	168	2.70	2.50	1.81	165	1.93	2.00	1.57
6	156	2.46	2.50	1.86	150	1.68	1.25	1.57
7	120	2.11	1.25	1.90	105	1.61	1.00	1.63
8	108	1.60	1.25	1.72	90	1.48	0.75	1.58
9	84	1.44	1.13	1.71	75	1.32	0.50	1.58
10	60	1.47	1.13	1.67	60	1.00	0.00	1.52
11	60	1.00	0.38	1.82	30	0.68	0.00	1.27
12	48	1.04	0.25	1.54	–	–	–	–
13	24	1.55	1.13	1.85	–	–	–	–
14	24	1.51	0.75	1.90	–	–	–	–
15	12	2.71	2.50	2.19	–	–	–	–
16	12	2.50	2.00	1.97	–	–	–	–
17	12	2.56	2.25	1.92	–	–	–	–

Table 11: Summary statistics of individual investment per period, irreversible investment treatments. Observations are individual investment decisions.

Round	RIE3 vs. IIE3	RIE5 vs. IIE5	RIE3 vs. RIE5	IIE3 vs. IIE5
1	0.0000	0.0050	0.0000	0.0001
2	0.0000	0.0000	0.0030	0.0004
3	0.0000	0.0000	0.0897	0.0040
4	0.0000	0.0000	0.2700	0.0211
5	0.0000	0.0000	0.3493	0.0543
6	0.0000	0.0000	0.1661	0.1810
7	0.0000	0.0000	0.1375	0.3270
8	0.0000	0.0000	0.0506	0.3253
9	0.0000	0.0006	0.1358	0.4186
10	0.0006	0.0003	0.4473	0.1158

Table 12: P-values of t-tests on the equality of public good stock averages.

	RIE 3			RIE 5		
g_0	inv^D	inv^{OS}	p-value	inv^D	inv^{OS}	p-value
0.00	2.64	1.62	0.0000	2.37	1.81	0.0009
1.25	1.21	1.02	0.5972	0.53	1.22	0.0022
2.50	0.25	0.52	0.2287	0.18	0.93	0.0003
3.75	0.11	0.16	0.8538	0.47	0.69	0.3939
5.00	-0.24	-0.52	0.3119	-0.38	0.06	0.0338
6.25	-0.19	-0.93	0.0033	0.54	-0.03	0.0473
7.50	0.35	-1.38	0.0000	-0.33	-0.36	0.9320
8.75	1.82	-1.88	0.0000	0.35	-0.57	0.0105

Table 13: Average individual investment as a function of beginning-of-the-period public good stock in dynamic experiments vs reduced form one shot experiments, RIE with n=3 and n=5.