ON THE NATURE OF COMPETITION IN ALTERNATIVE ELECTORAL SYSTEMS (FORMALLY "IDEOLOGY AND COMPETENCE IN ALTERNATIVE ELECTORAL SYSTEMS)

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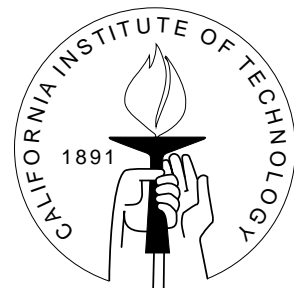
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Abstract

In this paper we argue that the number of candidates running for public office, their ideological differentiation, and the intensity of campaign competition are all naturally intertwined, and jointly determined in response to the incentives provided by the electoral system. We propose a simple general equilibrium model that integrates these elements in a unitary framework, and provide a comparison between majoritarian and proportional electoral systems.

On the Nature of Competition in Alternative Electoral Systems[*]

Matias Iaryczower and Andrea Mattozzi[†]

1 Introduction

In all elections for major public office positions, candidates invest a considerable amount of time, effort, and financial resources in persuasive campaigning: a varied source of activities "intended to convince an individual to vote for a candidate regardless of the candidate's position on issues" (Mueller and Stratmann (1994)). From broadcasting TV ads highlighting desirable characteristics of the candidate, to publishing and disseminating information aimed at reducing uncertainty about the candidate's platform, or communicating readiness to voters by hiring expert staff and formulating appropriate responses to current events.[1]

In spite of its relevance in modern elections, campaigning has not been systematically integrated in a theory of elections, together with the number and ideological position of candidates running for office. This omission could be of no major consequence if the nature of campaign competition were unrelated to other characteristics of the menu of alternatives available to voters. On the contrary, however, the number of candidates running for office, their ideological differentiation, and the intensity of campaign competition are all naturally intertwined. On the one hand, the more diverse are the policy positions represented by candidates running for office, the larger is the incentive for a new candidate to run representing an intermediate ideological alternative. On the other hand, the less diverse the ideological positions represented by candidates running for office, the

[1]In Section 2 we discuss the foundations of persuasive campaigning, and provide several examples of campaign activities through which candidates can influence rational voters' candidate selection.

larger is the number of voters that will be swayed by persuasive campaigning. These features, moreover, are all jointly determined in response to the rules shaping the nature of competition among candidates, and in particular by the electoral system. By affecting how votes cast in elections translate to representation in government - and ultimately how voters' preferences are mapped into policy outcomes - electoral systems shape the characteristics of the alternatives available to voters through the responses they induce in voters and politicians.[2]

In this paper, we tackle jointly the effect of alternative electoral systems on the number of candidates running for office, the ideological diversity of their platforms, and the intensity of competition in persuasive campaigning. We focus on a comparison between Proportional (PR) and Majoritarian (FPTP) electoral systems. This is a natural starting point for both practical and theoretical purposes. First, PR and FPTP are two of the most commonly used electoral systems in modern democracies around the world: about one fourth of all countries use FPTP electoral systems, and about one third use PR systems. Second, proportional and majoritarian systems represent *ideal* entities at the opposite side of the spectrum of what is possibly the main attribute of electoral systems: how they map votes into seats. While in its purest form PR translates the share of votes obtained by each party in the election to an equal share of seats in the legislature, FPTP gives a disproportionate representation to the candidate obtaining a plurality of votes.[3]

Our model integrates three different approaches in formal models of elections, allowing free entry of candidates, differentiation in a private value dimension, or *ideology*, and in a common value dimension, through *persuasive campaigning*. In our model, each potential candidate is endowed with an ideological position that she can credibly represent if she chooses to run and gets elected. With the field of competitors given, candidates running for office then invest resources in persuasive campaigning, developing (the perception of) an attribute that is valued by all voters alike. We assume that in deciding whether to run for office or not, each potential candidate cares about the spoils she can appropriate from being in office, and that voters are fully rational and vote strategically.

The incentives of voters and politicians are shaped by the electoral system under consideration. We assume that in FPTP the candidate who wins a plurality of votes appropriates all rents from office and implements the policy she represents, while in PR systems the policy outcome is the result of a *probabilistic compromise* between the elected candidates, where the likelihood that the policy represented by a candidate emerges as

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the policy outcome is increasing in the candidate's vote share.[4] The expected share of rents captured by each candidate is also assumed to be proportional to her vote share in the election.

The central result of the paper captures the interaction between strategic candidacy, endogenous ideological differentiation, and the intensity of campaign competition. First, we show that FPTP elections induce candidates to campaign more aggressively than PR elections. In particular, we show that all PR candidates invest (weakly) less in campaigning than any FPTP candidate, and that under mild conditions, the ranking is strict. Second, we show that in all equilibria in which candidates are ideologically differentiated, the number of candidates running for office is (weakly) larger in PR (strictly larger under mild conditions) than in FPTP, where exactly two candidates run. Third, we show that the ideological differentiation between candidates running for office can in general be larger or smaller in PR than in FPTP. While electoral equilibrium in PR restricts the minimum and maximum degree of differentiation between candidates, this is not the case in FPTP, where both full centrism and complete polarization are possible.

To prove our main result, we begin by characterizing equilibria in FPTP elections. The steep incentives provided by the winner-takes-all nature of FPTP induce candidates to invest all available resources in persuasive campaigning. Since candidates running for office must anticipate winning with positive probability, and voters must therefore vote sincerely between candidates on the equilibrium path, equilibrium candidates must be symmetrically located around the median ideological position in the electorate (they do not need however to be centrist - although this is possible - and in fact can be fully polarized). We also show that in equilibrium only two candidates compete for office. On the contrary, PR elections admit multi-candidate equilibria in which no candidates fully invest in persuasive campaigning. The number of candidates running for office and the degree of ideological differentiation among candidates are determined in equilibrium by two opposing forces. First, in any electoral equilibrium in PR, candidates must be sufficiently differentiated in the ideological spectrum. This is due to the basic tension that emerges in our model between campaign competition and policy differentiation: the closer candidates are in terms of their ideological position, the larger is the number of voters that can be attracted with a given increase in campaigning by one of the candidates. This implies in turn that candidates will campaign more aggressively the closer they are to one another, eventually competing away their rents. Second, the maximum degree of horizontal differentiation among candidates is bounded by entry: candidates cannot be too differentiated in PR elections without triggering the entry of an additional candidate, who would be able to attain the support of a sufficiently large niche of voters.

Our analysis demonstrates that while a FPTP system perfectly decouples ideological differentiation, number of candidates, and the incentives to invest in persuasive campaigning, this will typically not be true in other electoral systems. This shows the importance of using a systemic approach in the comparative analysis of electoral institutions. An

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[4]In this we follow Grossman and Helpman (1996) and Persico and Sahuguet (2006). In Section 5.3 we show that our main results do not hinge on the assumption of a probabilistic compromise.

additional advantage of our methodology is that the model we propose uses a fairly traditional framework that turns out to be relatively simple to handle, and which can be extended in several directions.

In Section 5.1 we show that our main results are qualitatively unchanged if we allow candidates to be both policy and office motivated, as long as the office motivation is sufficiently important. In essence, we can think of the benchmark model as a simplified version of a more general model, where office motivation dominates but does not preclude, policy motivation. In Section 5.2 we consider a variant of the main model in which candidates are perceived by voters as heterogeneous in non-ideological attributes even in the absence of any investments in persuasive campaigning. We show that if these attributes cannot be affected during the campaign, then for some parameters it is possible to find equilibria in which the non-ideological appeal of candidates is larger in PR than in FPTP elections. However, if candidates can complement their innate attributes by campaigning, then the non-ideological appeal of candidates (inherited and/or acquired) will be higher in FPTP than in PR elections, as in the case of the benchmark model. In Section 5.3 we consider alternative specifications of the policy function mapping elected representatives to policy outcomes. We argue that while the probabilistic compromise that we adopt in the benchmark model simplifies considerably the analysis of electoral equilibria in PR elections - by producing vote share functions that are uniquely determined and well behaved on and off the equilibrium path - it does not bias the results towards lower levels of campaign spending. We show, in particular, that if the policy outcome is selected as the median policy of all elected representatives in the ideological space, PR elections also admit electoral equilibria with more than two candidates running for office in which no candidate fully invests in persuasive campaigning.[5] Finally, in Section 5.4 we introduce a modified version of PR elections, in which the candidate with a plurality of votes obtains a premium in both the likelihood with which her policy is implemented, and in the proportion of office rents she attains after the election (PR-Plus). We show that for a given plurality premium, but sufficiently large electorates, equilibrium behavior in PR-Plus resembles that in FPTP. This suggests that it is the discontinuity in payoffs implicit in both FPTP and PRP which induces a decoupling of the intensity of campaign competition from the number of candidates and their ideological differentiation. For a fixed size of the electorate, however, the size of this discontinuity is also relevant. In fact, if the plurality premium is sufficiently small (approximating PR), PR-Plus elections admit equilibria with more than two candidates not fully investing in persuasive campaigning, as in the case of pure PR.

The rest of the paper is organized as follows. We begin by discussing the foundations of persuasive campaigning in Section 2. We then introduce the model in Section 3, and present the results in Sections 4 and 5. We review related literature in Section 6, and conclude in Section 7. All proofs are in the appendix.

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[5]A result similar in spirit also holds in an environment in which the policy outcome obtains as a convex combination of the ideological position of the elected representatives. See Section 5.3.

2   Persuasive Campaining: Foundations

Candidates invest in persuasive campaigning for a good reason: it works (see Coleman and Manna (2000), Erikson and Palfrey (2000), and Green and Krasno (1988)). In most of this paper we take this relationship as-is, black-boxing the underlying mechanism by which voters' choices are affected by campaigning. In this section, we briefly discuss some of the campaign activities through which candidates can influence rational voters' candidate selection.

Reduced Uncertainty in Policy Positions. Persuasive campaign can be effective in reducing uncertainty about the policy that the candidate will implement once in office. If voters dislike uncertainty over the policies to be implemented by each candidate, then candidates' efforts aimed at informing voters about their policy goals - by publishing and disseminating informative material through TV, newspapers and other media - will be valued by *all* voters. This idea was first formalized by Austen-Smith (1987). Our model is fully consistent with this mechanism, interpreting the common value dimension in the model as reflecting the electorate's uncertainty about the true positions that candidates will champion once in office (we return to this after introducing the model in Section 3; see footnote 8). This informative view of persuasive campaigning finds support in the empirical literature. Focusing on data for US legislative elections, Coleman and Manna (2000) show that "Campaign spending increases knowledge of and affect toward the candidates, improves the public's ability to place candidates on ideology and issue scales, and encourages certainty about those placements."

Ready at Day One. By selecting high quality staff, researching appropriate responses to current events, and shaping drafts of future policies, candidates are - and are seen by voters as being - more likely to succeed in office. This is most clearly illustrated by the idea of *being ready to take office at day one*, which was ubiquitous in the recent US Presidential election: "voters want candidates to meet a threshold of readiness that makes them an acceptable risk to elect as president. 'What they seem to do is decide, 'Do you have enough?'"[6] This argument can be easily formalized within a standard moral-harzard framework: candidates choose effort today, which stochastically improves performance while and if in office. For example, getting prepared during the campaign can reduce the probability of making mistakes that could have serious consequences, or increase the probability of avoiding a crisis by reacting properly to unexpected challenges: "The question of readiness matters because presidents often face unexpected challenges in their first weeks and months in office, before there's been much time to install a staff or learn the ropes. Less than three months after taking office in 1961, Kennedy approved an invasion of Cuba by anti-Castro forces that had been planned during the Eisenhower administration. ... Less than four months after taking office, Harry Truman approved dropping atomic bombs on two Japanese cities - the culmination of a nuclear weapons

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[6]David Frum [White House speechwriter], in *USA TODAY*, Febraury 24, 2008. See also "Obama Ready on 'Day One,' Kennedy Says" *Washington Post*, January 29, 2008.

program he hadn't even been told about as vice president. " *USA TODAY*, Febraury 24, 2008.

Commitment. Candidates may use public announcements to commit to specific policy positions. In their study of 1998 US midterm elections, for example, Spillotes and Vavrek (2002) report that among all candidates, 32 percent made at least one add classified as committing to a position on an issue. But commitment does not have to be related to policy. Candidates can successfully use public announcements to commit to transparent policy-making processes, ameliorating possibilities for corruption or improving efficiency at the expense of rents.

In conclusion, we have argued that by reducing uncertainty about the ideological position that the candidate will represent if in office, by hiring high quality staff and researching, drafting and communicating appropriate policy responses to current events, or by reducing their own degrees of freedom in both policies or processes, candidates can influence rational voters' candidate selection. In the analysis, we will mostly refer to these activities as *persuasive campaigning*, and return to the underlying mechanisms only for interpretation purposes.

3   The Model

There are three stages in the game. In the first stage, a finite set of potential candidates simultaneously decide whether or not to run for office. In the second stage, all candidates running for office simultaneously choose a level of campaign investment. In the third stage, a finite set of strategic voters vote.

For given T, define the ideology space $X \equiv \{t/T : t = 0, 1, \ldots, T\} \subset [0, 1]$. In any $x \in X$ there are at least two potential candidates, each of whom will perfectly represent policy x if elected. In the first stage, all potential candidates simultaneously decide whether or not to run for office. Potential candidates only care about the spoils they can appropriate from being in office, and must pay a fixed cost F to participate in the election.[7] We denote the set of candidates running for office at the end of the first stage by $\mathcal{K} = \{1, \ldots, K\}$. In the second stage, all candidates running for office simultaneously choose a level of campaign investment $\theta_k \in [0, 1]$. Candidates can invest θ_k at a cost $C(\theta_k)$, $C(\cdot)$ increasing and convex. We let $C(1) \equiv \bar{c}$ and - to allow competitive elections in all electoral systems - we assume that $F + \bar{c} \leq \frac{1}{2}$. In the third stage, n fully strategic voters vote in an election, where we think as n being a large finite number. A voter i with ideal point $z^i \in X$ ranks candidates according to the utility function $u(\cdot; z^i)$, which assigns to

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[7]In Section 5.1 we show that our results are robust to the introduction of a policy motivation to run for office. Note that in the presence of policy motivation, candidates can only credibly commit to champion their policy position if elected. This would be the case even if policy motivation were infinitely less important than office motivation. See Prat (2002) for a similar assumption in a different context.

candidate k with characteristics (θ_k, x_k) the payoff $u(\theta_k, x_k; z^i) \equiv 2\alpha v(\theta_k) - (x_k - z^i)^2$, with v increasing and concave. The parameter α captures voters' responsiveness to persuasive campaigning. Voters' ideal points are uniformly distributed in X.[8]

The electoral system determines the mapping from voting profiles to policy outcomes and the allocation of rents. In FPTP the candidate with a plurality of votes appropriates all rents from office and implements the policy she represents. In PR, each candidate k obtains a share of the total seats in the legislature equal to her share of votes in the election, s_k. The policy outcome is the result of a *probabilistic compromise* between the elected candidates, where the likelihood of the policy represented by a candidate emerging as the policy outcome is increasing in the candidate's vote share, or seat share in the assembly (Grossman and Helpman (1996), Persico and Sahuguet (2006)). The (expected) share of rents captured by candidate k, denoted m_k, is proportional to her vote share in the election. Let $\theta_{\mathcal{K}} \equiv \{\theta_k\}_{k \in \mathcal{K}}$, and $x_{\mathcal{K}} \equiv \{x_k\}_{k \in \mathcal{K}}$ denote the level of persuasive campaigning and policy positions of the candidates running for office. Normalizing total political rents in both systems to one, the expected payoff of a candidate k running for office in electoral system j can then be written as

$$\Pi_k^j(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = m_k^j(\theta_{\mathcal{K}}, x_{\mathcal{K}}) - C(\theta_k) - F. \tag{1}$$

For simplicity, and without any real loss of generality, we assume that $m_k^{PR}(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = s_k(\theta_{\mathcal{K}}, x_{\mathcal{K}})$. We also assume that in FPTP ties are broken by the toss of a fair coin, so that letting $H \equiv \{h \in \mathcal{K} : s_k = s_h\}$,

$$m_k^{FP}(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = \begin{cases} \frac{1}{|H|} & \text{if } s_k \geq \max_{j \neq k}\{s_j\} \\ 0 & \text{o.w.} \end{cases}$$

A strategy for candidate k is a decision of whether to run for office or not $e_k \in \{0, 1\}$, and a campaign investment $\theta_k(\mathcal{K}, x_{\mathcal{K}}) \in [0, 1]$. A strategy for voter i is a function $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) \in \mathcal{K}$, where $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = k$ indicates the choice of voting for candidate k, and $\sigma = \{\sigma_1(\cdot), \ldots, \sigma_N(\cdot)\}$ denotes a voting strategy profile. An *electoral equilibrium* is a Subgame Perfect Nash Equilibrium in pure strategies of the game of electoral competition in which voters do not use weakly dominated strategies, i.e., a strategy profile such that *(i)* voters cannot obtain a better policy outcome by voting for a different candidate in any voting game (on and off the equilibrium path), *(ii)* given the location and campaign

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[8] We argued in Section 2 that if voters dislike uncertainty over the policies to be implemented by each candidate, candidates' efforts aimed at informing voters about their policy goals will be valued by *all* voters. This happens generically whenever voters are policy-risk-averse, as within our model given the assumption of a quadratic policy payoff function. In particular, when the policy payoff function is quadratic, we can recover the exact benchmark model starting from primitives. Suppose then that $U(x_k, z^i) = -\beta(x_k - z^i)^2$, and that the policy y_k of candidate k is perceived by voters to be distributed uniformly on $[x_k - \epsilon(\theta), x_k + \epsilon(\theta)]$, where $\epsilon(\theta)$ is a decreasing and convex function of the investement in persuasive campaign θ. Then it is immediate to show that the expected utility of a voter with ideal point z^i can be written as $E[U(x_k, z^i); \theta] = -\beta(x_k - z^i)^2 + v(\theta)$, where $v(\theta)$ is an increasing and concave function of θ.

decisions of other candidates, and given voters' voting strategy, candidates cannot increase their expected rents by modifying their campaign levels, *(iii)* candidates running for office obtain non-negative rents, and *(iv)* candidates not running for office prefer not to enter: would obtain negative rents in an equilibrium of the continuation game. Ruling out weakly dominated strategies restricts the behavior of non-pivotal voters, requiring that they do not vote for their least preferred alternative. An *outcome* of the game is a set of candidates running for office $\mathcal{K}$, policy positions $x_{\mathcal{K}}$, and campaign investments $\theta_{\mathcal{K}}$. A *polity* is a triplet $(\alpha, \bar{c}, F) \in \Re^3_+$. We say that the model *admits* an electoral equilibrium with outcome $(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}})$ if there exists a set of polities $P \subseteq \Re^3_+$ with positive measure such that if a polity $p \in P$, then there exists an electoral equilibrium with outcome $(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}})$.

4 The Basic Comparison of Electoral Systems

In this section we state our main result regarding the comparison between alternative electoral systems (Theorem 1). We begin our analysis by considering majoritarian/ FPTP electoral systems. We show that in our setting, Duverger's law holds in almost all electoral equilibria. Although many candidates can run for office, majoritarian elections trim down competition between differentiated candidates to two candidates, each of whom invest as much as possible in persuasive campaigning. The degree of ideological differentiation between candidates, however, is not pinned down by equilibrium: FPTP elections admit both an equilibrium with two centrist candidates, and one in which candidates are maximally polarized (as well as any symmetric configuration). For some parameter values, there also exists an equilibrium in which more than two perfectly centrist (and in all respects identical) candidates run for office. We summarize these results in Proposition 1.

Proposition 1 *Consider elections in FPTP electoral systems. An electoral equilibrium exists. In any equilibrium in which candidates represent different ideological positions: (i) exactly two candidates compete for office, (ii) candidates are symmetrically located around the median in the policy space, and (iii) both candidates fully invest in persuasive campaigning (i.e., $\theta_1^* = \theta_2^* = 1$).*

To see the intuition for the result, note first that given the winner-takes-all nature of FPTP elections, all candidates running for office must tie in equilibrium. From this it follows that *(a)* voters must vote sincerely, and that *(b)* candidates must fully invest in persuasive campaigning. These facts also imply that *(c)* in any equilibrium, the set of candidates running for office must be symmetrically located with respect to the median ideological position. To see that there cannot be an electoral equilibrium with $K > 2$ differentiated candidates running for office, note that if this were the case, *(a)* and *(b)* imply that by deviating and voting for any candidate j other than her preferred candidate, a voter could get candidate j elected with probability one. Revealed preference from equilibrium therefore implies that this voter must prefer the lottery among all $\mathcal{K}^*$

candidates running for office to having j elected for sure. However, strict concavity of voters' preferences imply, together with *(c)*, that any voter must prefer a centrist candidate (i.e., located at the median) to the equilibrium lottery. As a result, any voter must also prefer a centrist candidate to any other candidate that is not her most preferred choice, and in particular a candidate with an ideological position that is between the median and her most preferred ideological position. But this leads to a violation of single-peakedness, which is not consistent with the assumption of a strictly concave utility function. Therefore, in equilibrium, we must have exactly two symmetrically located candidates fully investing in persuasive campaigning.[9] In the proof we show that such an equilibrium exists, and that there is, in fact, a continuum of two-candidate symmetric equilibria, with candidates fully investing in persuasive campaigning.[10]

This result can be shown to hold even without strict concavity if the ideological distance between any two adjacent voters is sufficiently small (i.e., if T is sufficiently large). The intuition is the following: consider a proposed equilibrium profile with three candidates, and a voter i who is close to being indifferent between 2 and 3 but still strictly prefers 2 to any other candidate (as T gets larger, we can make this voter be closer and closer to indifference). In equilibrium, i must prefer the equilibrium lottery to have candidate 3 elected for sure. But this cannot happen, because in equilibrium all candidates make the same level of investment, and x_1 is worst than x_3 for voter i. The same argument can be used to rule out equilibria with multiple candidates in a multidimensional policy space.

Proposition 1 establishes that in equilibrium, FPTP perfectly decouples ideological differentiation, number of candidates, and the incentives to invest in persuasive campaining. To see why this is the case, recall the two basic channels linking strategic entry decisions, the degree of ideological differentiation, and the intensity of campaign competition. First, less diverse ideological positions represented in elections imply generically a larger impact of non-ideological related issues on voters' choices: as candidates represent more similar positions, becoming less differentiated to voters in the ideological dimension, the incentive to differentiate themselves in non-ideological dimensions is larger. The uniformly steep incentives provided by the winner-takes-all nature of FPTP break the first link between ideological differentiation and the incentive for candidates to differentiate in non-ideological attributes. Second, the more diverse the ideological positions represented in the election, the larger will be the niche of voters that would prefer an alternative candidate to run for office and win. This channel is also broken in equilibrium in FPTP because strategic voting off the equilibrium path in FPTP prevents entry, and thus the larger niche of unsatisfied voters does not translate to a larger incentive for alternative candidates to run for office. As we will see below, both of these channels are present in

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[9]Feddersen, Sened, and Wright (1990) use a similar argument in a pure private values model in which candidates decide both whether to enter or not and which policy position they will represent.

[10]This result can appear at first glance to be due to the fact that candidates are solely office motivated. This is not the case. If candidates were solely policy motivated, strategy profiles with no or very low polarization will not be equilibria. In Section 5.1 we show, however, that if candidates are both policy and office motivated and the weight of office motivation is sufficiently large, then all equilibria of the benchmark model remain.

equilibrium in PR elections. The next lemma reestablishes this second channel in PR.

Lemma 1 *In (any voting subgame of) any electoral equilibrium in PR elections, voters vote sincerely.*

Recall that in the PR model each candidate running for office is elected and implements her ideology with a probability proportional to the share of votes received in the election. As a consequence, when a voter i votes for a candidate k, voter i is affecting the probability distribution over outcomes by increasing the weight of candidate k's position. But this implies that voting for a candidate other than the most preferred one is always a strictly dominated strategy. In fact, by switching his vote to his most preferred candidate, a voter only affects the lottery's weights of exactly two candidates. But with two alternatives, sincere voting is rational (the sincere voting profile is a joint best response).[11] The fact that strategic or sophisticated voting boils down in PR to sincere voting greatly simplifies the characterization of electoral equilibria, assuring uniquely determined, smooth and well behaved vote share functions for all candidates on and off the equilibrium path. It should be clear, however, that sincere voting in PR does not tilt the balance in favor of the comparison in Theorem 1: if anything, sincere voting makes entry more accessible, inducing candidates to invest more heavily in persuasive campaigning. We return to this in Section 5.3.

Proposition 2 establishes the core result for PR elections. First, we show that for a *large* set of parameters there exists an electoral equilibrium in PR elections in which more than two candidates run for office without fully investing in persuasive campaigning. Moreover, we show that PR elections do not generically admit electoral equilibria in which different candidates represent the same policy.

Proposition 2 *PR elections (i) admit electoral equilibria in which more than two candidates run for office without fully investing in persuasive campaigning, and (ii) do not admit electoral equilibria in which two or more centrist candidates run for office.*

To prove this result we provide conditions for the existence of electoral equilibria of a simple class, which we call location symmetric (LS) equilibria. In equilibria of this class, all candidates running for office are located at the same distance to their closest neighbors in the ideological space; i.e., $x_{k+1} - x_k = \Delta$ for all $k = 1, \ldots, K-1$, $x_1 = 1 - x_K = \Delta_0$, and all interior candidates $k = 2, \ldots, K-1$ choose the same level of investment in persuasive campaigning.[12] Within this class, the relevant competitors for any candidate k's decision problem are k's neighbors, $k+1$ and $k-1$. This is enough to show that

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[11]Our modelling of PR elections abstracts from issues related to seats indivisibilities. Indeed, it can be shown that there are situations where the application of the D'Hondt formula to assign remainders would lead to strategic voting in equilibrium. See Morelli (2004) for a model that studies strategic voting in PR taking into account indivisibilities and formulas with residuals.

[12]In all our formal analysis of PR, we consider the limit of the discrete case as T goes to infinity, and treat both the policy space and the set of potential candidates, as an interval of $\mathcal{R}$. As it will become evident throughout the analysis, this simplification does not sacrifice anything of importance.

payoff functions are twice differentiable in the relevant set (non-differentiabiliies can only arise for campaigning choices that are not optimal), and that whenever rents cover variable costs, first order conditions in the investment subgame completely characterize best response correspondences.[13]

The number of candidates running for office and the degree of ideological differentiation between candidates are determined in equilibrium by two opposing forces. On the one hand, in any electoral equilibrium in PR, candidates must be sufficiently differentiated in the ideological spectrum, because of the basic tension that emerges in our model between persuasive campaigning and differentiation in policies: the closer candidates are in terms of their ideological position, the larger is the effect of persuasive campaigning by any of the candidates. This is the first channel linking strategic entry decisions, ideological differentiation and persuasive campaigning in PR. To see how this channel operates in our LS equilibrium, consider two candidates k and $j > k$ with policy positions x_k and $x_j > x_k$, and choosing persuasive campaign investment levels θ_k and θ_j, and let $\tilde{x}_{k,j} \in \mathcal{R}$ denote the (unique) value of x for which $u(\theta_k, x_k; x) = u(\theta_j, x_j; x)$, so that $u(\theta_k, x_k; z^i) > u(\theta_j, x_j; z^i)$ if and only if $z^i > \tilde{x}_{k,j}$,

$$\tilde{x}_{k,j} = \frac{x_k + x_j}{2} + \alpha \frac{[v(\theta_k) - v(\theta_j)]}{|x_j - x_k|}. \tag{2}$$

In an LS equilibrium, k's only relevant competitors are neighbors $k-1$ and $k+1$, k's vote share is $s_k(\theta_k; \theta_{-k}, x) = \tilde{x}_{k,k+1} - \tilde{x}_{k-1,k} = \Delta$, and therefore from (5.1) for PR, the payoff for an interior candidate $k = 2, \ldots, K-1$ is

$$\Pi_k(\theta_{\mathcal{K}}, x_{\mathcal{K}}, \mathcal{K}) = \Delta + \alpha \left[\frac{v(\theta_k) - v(\theta_{k+1})}{\Delta} + \frac{v(\theta_k) - v(\theta_{k-1})}{\Delta} \right] - C(\theta_k) - F.$$

Defining $\Psi(\theta) \equiv v'(\theta_k)/C'(\theta_k)$, k's best response is then[14]

$$\theta_k^* = \begin{cases} \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) & \text{if } \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \leq 1 \\ 1 & \text{if } \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) > 1. \end{cases} \tag{3}$$

Noting that $\Psi(\cdot)$ is a decreasing function, it follows that candidates will be more aggressive in campaigning the closer they are to one another, eventually competing away their rents. Candidates that are sufficiently differentiated in the ideological dimension, instead, are not close substitutes for voters. In this case, PR leads to low powered

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[13]Non-differentiabilities of the marginal vote share mapping can occur when a given increase in the level of investment by a candidate induces a switch in the identity of the relevant competitor faced by the candidate (towards a worst substitute in the ideological dimension). See Iaryczower and Mattozzi (2009) for more details.

[14] Similarly, for an extreme candidate (say $k = 1$), $k = 1$'s best response is $\theta_1^* = \Psi^{-1}(\Delta/\alpha)$ if $\Psi^{-1}(\Delta/\alpha) \leq 1$, and $\theta_1^* = 1$ otherwise.

incentives, non-ideological competition is relaxed, and candidates running for office can choose lower (less costly) levels of persuasive campaigning while still getting a positive share of office rents in equilibrium.

To sum up, the strategic effect of ideological differentiation (on the aggressiveness in campaigning) imposes a lower bound on differentiation in equilibrium. On the other hand, the limit to the degree of horizontal differentiation among candidates is given by the threat of entry: candidates cannot be too differentiated in PR elections without triggering entry of an additional candidate, who would be able - given sincere voting in the electorate - to attain the support of a sufficiently large niche of voters. The same logic implies in fact that PR elections do not admit an electoral equilibrium in which two or more perfectly centrist candidates run for office. If all candidates running for office were centrist, it would always be possible for a candidate representing a policy position close to the median to run for office, capturing almost half of the votes. Since the centrist candidates were making non-negative rents in the proposed equilibrium, the entrant's expected payoff from running must be positive as well, and there is no way to deter his entry. As a result, the fully centrist equilibrium in FPTP cannot be generically supported in PR.

In the proof we obtain an upper bound on differentiation among equilibrium candidates as a sufficient condition to guarantee that for any possible non-equilibrium entrant, there exists an equilibrium of the continuation game in which the entrant would make negative rents. We then show that there exists a non-trivial set of parameters for which all the previous conditions on Δ are simultaneously satisfied. In particular, we show that for a LS equilibrium with $K \geq 3$ candidates not fully investing in persuasive campaigning to exist it is sufficient that *(i)* the responsiveness of voters to campaigning is not too high (i.e., $\alpha < \overline{\alpha}(K) \equiv C'(1)/2Kv'(1)$), that *(ii)* the fixed cost of running for office is always larger than the cost of campaigning (i.e., $F > \overline{c}$), and that *(iii)* the fixed cost of running for office is not too low (to deter entry) or too high (for nonnegative rents); i.e., $1/2K < F < 1/K - \overline{c}$. Note in particular that we can support equilibria with an increasingly larger number of candidates given sufficiently lower costs of running for office and of campaigning, and a sufficiently smaller responsiveness of voters to persuasive campaign - equivalently, a sufficiently larger *ideological focus* of voters (Stokes (1963)).[15]

Combining the results of Proposition 2 together with our earlier results in Proposition 1, we obtain the main result of this section.

Theorem 1 *(1) In any admissible electoral equilibrium under PR, (a) all candidates running for office invest (weakly) less resources in persuasive campaigning than any can-*

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[15]Propositions 1 and 2 do not depend on the assumption that voters are uniformly distributed in X. This assumption plays no role in the proof of Proposition 1. In Proposition 2, it allows us to construct equilibria with no candidate fully investing in persuasive campaigning focusing on simple LS profiles. With an arbitrary distribution of voters, candidates would generically be more concentrated around popular ideological positions in order to attract enough votes to make running for office worthwhile. These candidates would then be forced to invest more in persuasive campaigning than candidates in less popular positions. The basic nature of the analysis, however, would remain unchanged.

didate does in any admissible equilibrium in FPTP, and (b) the number of candidates running for office is (weakly) larger than the number of candidates in any admissible equilibrium in FPTP in which candidates are differentiated. Moreover, (2) PR elections admit electoral equilibria for which the above comparisons are strict.

A natural question at this point is whether we can establish normative results within this framework: is either FPTP or PR generically better for voters? The answer is no, or more precisely, not without making further assumptions and imposing a particular criterion for selecting among equilibria. As we pointed above, all of our results so far hold without assuming (strict) concavity of the voters' policy payoff function (which is implicit in our quadratic representation of policy preferences). Without assuming concavity, however, not much can be said about the efficiency of alternative electoral systems within this framework. If one is willing to maintain that the assumption of concavity of voters' payoff function holds generically, then some limited welfare results follow. First, for any given parameter values, the best equilibrium in FPTP is better for voters than the best equilibrium in PR.[16] Second, the ranking of the worst equilibria for voters does depend on parameter values. The worst equilibrium in FPTP given any feasible parameter configuration has two extreme candidates exhausting all resources available for persuasive campaigning. On the other hand, the worst equilibrium that can be supported in PR for *some* parameter configuration has two extreme candidates slacking in persuasive campaigning effort. For other feasible parameter configurations, however, the worst equilibrium for voters in PR has $K > 2$ candidates exhausting all resources available for persuasive campaigning. All in all, the results in terms of welfare comparison are ambiguous.

We close this section with two remarks. First, while in the benchmark model we considered electoral systems in which all votes are aggregated in a single district, most systems currently used to select representatives to a national legislature admit several electoral districts.[17] However, the benchmark model can be readily extended to allow a comparison between a system of FPTP elections in D districts with a system of PR elections in D districts of possibly heterogeneous size. It can be shown that extending the probabilistic compromise to the multiple district setting, the result of Theorem 1 remains

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[16]Within the class of LS equilibria under PR, the welfare comparison comes as an immediate corollary of our previous results, for we know that it is not possible to have convergence in PR elections. Given the same level of investment in persuasive campaigning, concavity of voters' preferences implies that any voter strictly prefers the *expected candidate* with ideological position corresponding to the expected value of the equilibrium lottery to the lottery itself. The same result holds more generally for any electoral equilibrium in PR: for *any* equilibrium in PR, any voter prefers the expected candidate of the equilibrium lottery to the lottery itself. If this expected candidate is centrist, then as before, we are done. If not, then still the concavity of voters' preferences implies that a centrist candidate will be preferred by a majority of voters to the expected candidate.

[17]The US, for example, elects the 435 members of the House of Representatives by FPTP in 435 different electoral districts; Argentina, on the other hand, elects the 257 members of the Camara de Diputados by a PR electoral system composed of twenty four electoral districts, of which ten select five members each, twenty select ten or less members each, twenty three select twenty five or less, and one (Buenos Aires) selects seventy members.

unchanged, stated now as a district-to-district comparison.[18] It should be clear that while this result is a strong statement in terms of the level of persuasive campaigning across systems - it does not imply a national-level Duverger law. Considering this matter would require introducing elements of strategic party formation that are beyond the scope of this paper.

A second simplification of the benchmark model is that it does not incorporate minimum thresholds to achieve representation of the kind that are commonly used in many PR systems around the world.[19] While introducing a threshold for representation in FPTP would not affect the equilibrium behavior of voters or candidates, doing so in PR elections can have a large impact on electoral outcomes. Even with all voters voting sincerely as in our benchmark PR elections, the threshold for representation has a direct impact on the size of the smallest party allowed to be represented in parliament, and thus (possibly) on the number of candidates competing for office. Most notably, however, introducing a threshold for representation allows strategic voting along the lines of FPTP elections, and it is this feature which can most significantly affect behavior in PR elections. First, it is now possible to support in PR elections the centrist FPTP equilibrium. Since the two candidates are perfect substitutes for voters, the low powered incentives typical of PR play no role here, and candidates have an incentive to fully invest in persuasive campaigning. The threat of entry that breaks this equilibrium in the benchmark PR model with no thresholds is ruled out here by coordinating voters' behavior so that an entrant would receive no support if such a deviation were to come about. This is entirely due to the threshold for representation, which allows a *spoiler* effect much as in FPTP to be in play. Second, it is also possible to support the worst possible PR equilibrium: two extreme parties running for office, representing the most outward positions in the ideological space, investing as low as possible in persuasive campaigning as it is consistent with electoral competition in PR. As before, entry is ruled out by coordinating voters' behavior away from a possible out-of-equilibrium entrant. In this case, however, competition on the equilibrium path is restricted to a contest among highly differentiated candidates, and as a result the low powered incentives of PR competition induce low campaigning effort by candidates.

5   Beyond the Basic Model

In this section we explore the implications of relaxing key assumptions of the benchmark model.

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[18]Details are available from the authors upon request.

[19]This is a common feature of many PR electoral systems. This threshold is five percent in Poland's Sejm, Germany's Bundestag, and New Zealand's House of Representatives, two percent in Israel's Knesset, and as high as ten percent in the Turkish parliament. Portugal, South Africa, Finland, and the Netherlands, on the other hand, are examples of PR systems without a threshold.

5.1 Policy Motivation

We have argued above that our main results are qualitatively unchanged if we allow candidates to be both policy and office motivated, as long as the office motivation is sufficiently important. In essence, we can think of the benchmark model as a simplified version of a more general model, where office motivation dominates but does not preclude, policy motivation.[20] In this section, we make this argument more precise. We write the expected gross payoff of a candidate k running for office in electoral system j as

$$\Pi_k^j(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = \mu m_k^j(\theta_{\mathcal{K}}, x_{\mathcal{K}}) - (1-\mu) \sum_{l \in \mathcal{K}\backslash k} m_l^j(\theta_{\mathcal{K}}, x_{\mathcal{K}})(x_l - x_k)^2 - C(\theta_k) - F,$$

where $\mu \in (0,1)$ denotes the weight attached to office motivation, and as before, $m_k^{PR}(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = s_k(\theta_{\mathcal{K}}, x_{\mathcal{K}})$, and $m_k^{FP}(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = \frac{1}{|H|}$ if $s_k \geq \max_{j \neq k}\{s_j\}$, zero otherwise. Note that our benchmark model is nested in the above specification when $\mu = 1$.

Consider first FPTP elections. Introducing policy motivation in FPTP elections has one relevant effect in equilibrium: the payoff differential of running for office or not for any given candidate now depends on how she evaluates the policy position of the other candidates running for office. In particular, for any given μ, each candidate will have a smaller incentive to run for office the closer the other candidates are to her position in the policy space. Consider a proposed equilibrium candidate in which two candidates $j = 1, 2$ are symmetrically located in the policy space, at a distance Δ. Note that the payoff of candidate j in the proposed equilibrium is $\mu/2 - (1-\mu)\Delta^2/2 - \bar{c} - F$, while her payoff is $-(1-\mu)\Delta^2$ if she does not run for office (since in this case candidate 2 wins for sure). Thus candidate 1 prefers to run for office if and only if $(\mu + (1-\mu)\Delta^2)/2 - \bar{c} - F \geq 0$, or equivalently $\mu + (1-\mu)\Delta^2 \geq 2(\bar{c} + F)$. For a given μ not too large ($\mu < 2(\bar{c} + F)$) this introduces a bound on how close candidates can be in equilibrium. On the other hand, since $\bar{c} + F < 1/2$, it follows that for any $\Delta > 0$, candidate 1 will prefer to run for office rather than not if the office motivation μ is sufficiently large. The previous argument seems special in that it assumes two candidates symmetrically located in the policy space. However, it is easy to see that every other step in the proof of proposition 1 (for FPTP elections) remains unchanged. Thus in any equilibrium in competitive FPTP elections we must have two candidates running for office symmetrically located in the policy space. Formally, we have the following result.

Proposition 3 *Consider FPTP elections in which candidates have both office and policy motivations. There exists a weight on office motivation $\hat{\mu} \in (0,1)$ such that if $\mu > \hat{\mu}$, then (a) there exists an equilibrium in which elections are contested, and (b) in any equilibrium in which candidates represent different ideological positions: (i) exactly two candidates compete for office, (ii) candidates are symmetrically located around the median*

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[20]The classical reference for models in which candidates are office motivated is Hotelling (1929). Wittman (1977, 1983) and Calvert (1985) assume instead that candidates are policy motivated. See the "citizen-candidate" models of Osborne and Slivinski (1996) and Besley and Coate (1997), and more recently Callander (2008) for models with both policy and office motivation.

in the policy space (i.e., $x_1 + x_2 = 1$), and (iii) both candidates fully invest in persuasive campaigning.

Consider now PR elections. Introducing policy motivation in PR has two relevant effects in equilibrium. First, there is an effect on entry, similar in spirit to that in FPTP elections. In addition, there is now a second effect of policy motivation, that operates in the campaign competition stage, after the field of candidates is resolved. As candidates become better substitutes for voters, the marginal rent-related benefit of campaigning increases, just as in the benchmark model. But now there is also a marginal policy-related benefit of campaigning, which decreases as candidates get closer to each other. We show, however, that if the office motivation is sufficiently strong, the marginal rent-related benefit of campaigning dominates the marginal policy-related benefit of campaigning, and the analysis of the benchmark model is fundamentally unaltered. Fix $\mu \in (0, 1)$, and consider a LS equilibrium. The equilibrium payoff for an interior candidate $k = 2, \ldots, K - 1$ is

$$\Pi_k(\theta_\mathcal{K}, x_\mathcal{K}, \mathcal{K}) = \mu \left(\Delta + \alpha \left(\frac{v(\theta_k) - v(\theta_{k+1})}{\Delta} + \frac{v(\theta_k) - v(\theta_{k-1})}{\Delta} \right) \right) +$$
$$- (1 - \mu) \sum_{l \in \mathcal{K} \backslash k} s_l(\theta_\mathcal{K}, x_\mathcal{K})(x_l - x_k)^2 - C(\theta_k) - F.$$

Thus k's best response is

$$\theta_k^* = \begin{cases} \Psi^{-1} \left(\frac{\Delta}{2\alpha\tilde{\mu}_\Delta} \right) & \text{if } \Psi^{-1} \left(\frac{\Delta}{2\alpha\tilde{\mu}_\Delta} \right) \leq 1 \\ 1 & \text{if } \Psi^{-1} \left(\frac{\Delta}{2\alpha\tilde{\mu}_\Delta} \right) > 1, \end{cases}$$

where $\tilde{\mu}_\Delta \equiv \mu + (1 - \mu)\Delta^2$. Note that if the office motivation is sufficiently important relative to the policy motivation (here it is enough that $\mu > 1/2$) then the equilibrium level of campaigning θ^* is decreasing in the differentiation between candidates Δ. This suggests that when office motivation is sufficiently important, the analysis of PR elections with policy motivation is similar to that of the benchmark model. This intuition is in fact correct, and allows us to establish the following proposition

Proposition 4 *Consider PR elections in which candidates have both office and policy motivations. There exists a weight on office motivation $\tilde{\mu} \in (0, 1)$ such that if $\mu > \tilde{\mu}$, then PR elections (i) admit electoral equilibria in which more than two candidates run for office without fully investing in persuasive campaigning, and (ii) do not admit electoral equilibria in which two or more centrist candidates run for office.*

The proof of this proposition (which is available from the authors upon request) is very similar to showing the analogous result in the context of the benchmark model. The main difference is that the bounds on ideological differentiation will now also be a function of μ. Combining Proposition 4 together with Proposition 3 we can conclude that our main results also hold when office motivation dominates, but does not preclude, policy motivation.

5.2 Pre-Campaign Heterogeneity: A Model of Selection

In the benchmark model we assumed that candidates are endowed with an ideological position and choose first whether to run or not for office, and if so then how much to invest in persuasive campaign. Implicitly, candidates are assumed to be perceived by voters as homogeneous in non-ideological attributes before any effort is devoted to campaigning. In this section we consider a variant of the main model in which candidates are perceived by voters as heterogeneous in non-ideological attributes even in the absence of any investments in persuasive campaigning. We consider two possible variations of the benchmark model.

1. We assume that candidates are endowed with both an ideological position and a level θ_k of an attribute that *(i)* is known to voters even in the absence of any campaign action, and that *(ii)* captures, for given ideological position, their exogenous appeal to voters. Candidates cannot, however, invest resources to make the alternative they represent more appealing to voters. This common value attribute is now either completely exogenous as in the case of valence or it can be the outcome of a previous investment in human capital. We refer to this alternative framework as the *selection* model and to the benchmark framework as the *choice* model.

2. We consider a *selection+choice* model where candidates, heterogeneous with respect to their exogenous appeal to voters as in the selection model, can also choose to complement it by investing in persuasive campaign as in the choice model.

We begin with the analysis of the selection model. We assume that there is a candidate representing each point in the attribute-ideology space, and that candidates with higher level of θ have a higher opportunity cost of running for office $c(\theta)$. To make the results comparable to the benchmark choice model, we represent the opportunity cost of types in the selection model with the same cost function $C(\cdot)$ of the benchmark model, so that $c(\theta) \equiv C(\theta)$. The action space of candidates is therefore restricted to a decision of whether or not to run for office.

To see how the alternative electoral systems operate in the selection model, consider first FPTP elections. As in the choice model, the winner-takes-all nature of FPTP elections implies that potential candidates will run for office only if they have a strictly positive expected probability of winning. Furthermore, in any equilibrium in which candidates are differentiated, only two candidates will run for office (the argument used in the proof of Proposition 1 builds on deviations by voters for a given set of candidates, and can therefore be applied in this case as well). These properties imply that voters must vote sincerely between the two candidates running for office on the equilibrium path, and therefore that these candidates must be symmetrically located around the median voter. As a consequence, any configuration of candidates' characteristics that can be supported as an equilibrium of the choice model in FPTP elections can also be supported as an equilibrium of the selection model. Contrary to the choice model, however, *every* symmetric configuration of candidates (in both location and level of θ) can

be supported as an equilibrium of the selection model. In fact, in this alternative timing specification - a simultaneous game of entry - strategic voting is effective in deterring entry of any third candidate irrespectively of his characteristics (the analogy with the intuition behind Myerson (1993a)'s result is immediately apparent).

Consider now LS equilibria in PR. First, note that Lemma 1 still applies, so that voting is sincere on and off the equilibrium path in all equilibria. Second, note that if there is a candidate running for office with policy position x_k and $\theta_k < 1$, and he earns strictly positive rents, i.e., $\Pi_k^{PR}(\theta_k, \theta_{-k}, x_{\mathcal{K}}) > 0$, then also $\Pi_k^{PR}(\theta_k', \theta_{-k}, x_{\mathcal{K}}) > 0$ for an alternative candidate with identical ideological position x_k and $\theta_k' > \theta_k$. Therefore in any LS equilibrium of the selection model, either $\theta_k = 1$ or $\Pi_k^{PR}(\theta_k, \theta_{-k}, x_{\mathcal{K}}) = 0$ for all $k \in \mathcal{K}$. This implies that, if $x_k - x_{k-1} \equiv \Delta > \bar{c} + F$ in a LS equilibrium of the choice model with $K \geq 3$ (i.e., candidates are sufficiently differentiated so that interior candidates would earn positive rents even choosing $\theta_k = 1$), then in the selection model it must be that $\theta_k = 1$ for all interior candidates.[21] If, on the other hand, $K \geq 3$ and $\Delta < \bar{c} + F$ in a LS equilibrium of the choice model (i.e., candidates choosing $\theta_k = 1$ would earn negative rents), then the equilibrium level of θ of both the choice and the selection model would be interior. Summarizing, if candidates earn no rents at the equilibrium $\theta^* < 1$ in the choice model, then $\theta^* < 1$ will also be the equilibrium of the selection model; if instead candidates earn positive rents in the choice model, then the equilibrium of the selection model will be characterized by a higher level of θ.[22] We conclude that

Remark 1 *The selection model allows "mediocre" candidates to run for office in FPTP, and leads to higher level of θ than the choice model in PR. Hence there exists a selection of equilibria such that the non-ideological appeal of candidates is larger in PR than in FPTP elections. The conclusions regarding the number of candidates do not change throughout.*

The driving force behind this result is that the selection model introduces more competition among candidates in PR elections: if a mediocre candidate is not completely dissipating his rents in an equilibrium of the selection model in PR, a candidate with higher θ would find it profitable to run for office as well. This is not the case in the choice model, where competition in persuasive campaign takes place among a given set of candidates running for office. On the other hand, in the selection model under FPTP, strategic voting can prevent the entry of any third candidate irrespectively of his characteristics, as in Myerson (1993a).

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[21]We have two possible scenarios: either *(i)* in the equilibrium of the choice model candidates choose $\theta^* = 1$, in which case the same thing must be true in the selection model, or *(ii)* in the choice model candidates choose $\theta^* < 1$, in which case for any ideological position x_k, rents must be positive for $\theta_k \in [\theta^*, 1)$, which implies that in the selection model $\theta_k = 1$ for all interior candidates. If also $\Delta < 1/K$, extreme candidates must choose $\theta^* = 1$ too, for $\Delta < 1/K$ implies that extreme candidates obtain higher rents than interior candidates.

[22]Note that in this case the level of θ in the equilibrium of the selection model will be still not maximal since rents are decreasing in θ_k for $\theta_k \in [\theta^*, 1)$, and negative at $\theta_k = 1$, so must be zero at some $\bar{\theta}_k < 1$. In Iaryczower and Mattozzi (2009) we show that PR elections admit equilibria such that $K \geq 3$ and $\Delta < \bar{c} + F$.

As the examples in Section 2 illustrate, however, the assumption that candidates cannot complement their initial appeal (if any) with campaign actions seems unwarranted. Candidates are largely *defined* for voters during campaigns. Interestingly, when candidates can complement their initial perceived differences through campaign actions, as in the *selection+choice* model, the tension between the choice and the selection models is resolved in favor of the benchmark choice model: the non-ideological appeal of candidates (inherited and/or acquired) is larger in FPTP than in PR elections (as before, the conclusions regarding the number of candidates do not change throughout).

Within the selection+choice model, denote by θ_k^s the exogenous component of the overall appealing to voter of candidate k's alternative, and by θ_k^c the endogenous component due to persuasive campaign, where $\theta_k^s + \theta_k^c \in [0, 1]$. Note that in equilibrium θ_k^c will be a function of θ_k^s. As in the selection model, candidates with higher level of θ^s have a higher opportunity cost of running for office $c(\theta^s)$. Furthermore, as in the choice model, candidates running for office can also choose to increase their exogenous appeal to voters by investing in persuasive campaign at a cost $\hat{c}(\theta^c)$. In order to make the results comparable to the benchmark choice model, we also assume that $c(\theta^s) + \hat{c}(\theta^c) \equiv C(\theta^s + \theta^c)$, where $C(\cdot)$ is the same cost function of the benchmark model.

Consider first FPTP elections. It is immediate to verify that strategy profiles such that $\theta_k^s + \theta_k^c < 1$ for some k can not be electoral equilibria, for - as in the benchmark choice model- this would give k a profitable deviation. We conclude that the difference between the results of the choice and the selection models entirely relies on the somewhat knife-hedge assumption that, during the campaign, candidates cannot render the alternative they represent more appealing to voters. When instead candidates can complement their exogenous appeal to voters investing in persuasive campaigning, the uniformly steep incentives provided by the winner-takes-all nature of FPTP elections can fully take effect as in the benchmark choice model. This result rules out already any possible reversals in the conclusions of Theorem 1. Moreover, we can also show that in this setting PR admits electoral equilibria with an interior equilibrium in campaign spending. Thus the result of Theorem 1 holds unchanged. To show this, we exploit the fact that the continuation games of the selection+choice model are a generalization of the choice model, allowing heterogeneous initial conditions θ_k^s. Consider then a LS profile in the benchmark choice model such that $\theta^* < 1$ in which all interior candidates earn zero rents (this can be supported as an equilibrium, see Iaryczower and Mattozzi (2008, 2009)). Fixing parameters, consider a strategy profile in the selection+choice model such that $\theta_k^s = 0$ and $\theta_k^c = \theta^*$ for all interior candidates k. It can be shown that for any given θ_{-k}, $\theta_k^s + \theta_k^{c*}(\theta_k^s)$ is increasing in θ_k^s (the higher initial "valence" acts as a subsidy in the continuation game). Together with the fact that $c(\theta^s) + \hat{c}(\theta^c) = C(\theta^s + \theta^c)$, the zero profit condition implies that no candidate k' such that $\theta_{k'}^s + \theta_{k'}^c = 1$ has an incentive to run for office. Entry of candidates with different ideologies are ruled out by the same arguments as in the benchmark choice model. We can summarize our findings in the following remark:

Remark 2 *In the selection+choice model, the non-ideological appeal of candidates (in-*

herited or acquired) is larger in FPTP than in PR elections (strictly larger for a non-trivial set of parameters). The conclusions regarding the number of candidates do not change throughout.

In conclusion, if candidates are perceived by voters as heterogeneous in non-ideological attributes even in the absence of any investments in persuasive campaigning, *and* these attributes cannot be affected during the campaign, then for some parameters it is possible to find equilibria in which the non-ideological appeal of candidates is larger in PR than in FPTP elections. However, if candidates need to campaign in order to differentiate themselves in non-ideological attributes or if candidates can complement their innate attributes by campaigning, then the non-ideological appeal of candidates (inherited and/or acquired) will be higher in FPTP than in PR elections.

5.3   From Representation to Policy Outcomes

A central element of any model of elections is the mapping from votes in the electorate to a set of elected representatives. With fully rational and strategic voters, however, a second element of the model becomes equally important. In order for rational voters to be able to link their vote choices to payoffs, they need to be endowed with a mapping from the characteristics of the set of elected representatives to final policy outcomes. In this paper we have maintained the simplifying assumption that the policy outcome in PR comes about as the realization of a probabilistic compromise among the policies represented by the candidates participating in the election, with weights equal to their vote shares (or seat share in the assembly).

The assumption of a probabilistic compromise simplifies considerably the analysis of electoral equilibria in PR elections: given probabilistic compromise in the elected legislature, all voters find voting for their most preferred candidate to be a dominant strategy, and thus sincere voting is rational on and off the equilibrium path; this, in turn, produces vote share functions that are uniquely determined, continuous, and well behaved, on and off the equilibrium path. It should be clear, however, that the assumption of a probabilistic compromise does not bias the results towards lower levels of campaign spending than what would obtain under alternative protocols for determination of policy: if anything, sincere voting facilitates entry, and therefore leads to less ideological differentiation and higher levels of investment in persuasive campaign in equilibrium. In this section we complement this logic by showing that our main results hold under alternative specifications of the policy function mapping elected representatives to policy outcomes.

First of all it is immediate to see that within the probabilistic compromise framework weights need not be equal to election shares. Indeed, any probabilistic compromise such that the weights are a nondecreasing, anonymous/symmetric function of the election shares would leave all results unchanged. Furthermore, *any* alternative mechanism inducing sincere voting will lead to the same results. More interestingly perhaps, we show that under two simple alternative non-stochastic protocols for the determination

of policy in the elected assembly, which do encourage voters to vote strategically under some conditions, our results hold unchanged. We consider first the *median protocol*:

Definition 1 ***The Median Protocol*** *For given profile $(x_{\mathcal{K}}, \theta_{\mathcal{K}})$, and vote shares $\{s_k\}$, the outcome is $(x_{\tilde{k}}, \theta_{\tilde{k}})$, where $\tilde{k} \equiv \min k : \sum_{j \in \mathcal{K}} s_j \geq 1/2$ is the (seat-weighted) median representative.*

In the median protocol, the policy outcome is determined by the characteristics of the median representative in the assembly. By seat-weighted representative we mean that for the purposes of computing the median, candidate k with vote share s_k is assumed to be equivalent to a mass s_k of individuals representing policy x_k. Proposition 5 shows that the conclusions of Proposition 2 and Theorem 1 hold unchanged under the median protocol.

Proposition 5 *Suppose the policy outcome is determined according to the median protocol. Then PR elections admit an electoral equilibrium in which more than two candidates run for office without fully investing in persuasive campaign. Furthermore, any candidate strategy profile that can be supported in a LS equilibrium in PR elections under a probabilistic compromise can be supported as an equilibrium with the median protocol.*

To see why the result obtains, note first that on the equilibrium path of a LS equilibrium, sincere voting is a rational voting strategy profile. In fact, in a LS equilibrium with $K \geq 3$ candidates, extreme candidates can never become the median legislator, and all non-extreme candidates choose to invest equally in persuasive campaign θ^*. Since voters have single-peaked preferences in the ideological dimension, this implies that voters have single-peaked preferences among all relevant options. As a result, any voter i can never gain by not voting for her preferred candidate: either her deviation produces no change in the median (e.g., when i votes for any candidate on the same side of the median in the ideological space) or it produces a detrimental change in the outcome (e.g, when i votes for a candidate on the opposite side of the median in the ideological space). If the profile of candidates' campaign expenditures is not symmetric, however, as would occur off the equilibrium path following deviations by an equilibrium candidate in the campaign stage (or in the continuation game after entry of a non-equilibrium candidate), then strategic voting can become rational.[23] In the proof of Proposition 5 in the appendix we show, however, that *(i)* sincere voting is rational in any voting subgame of a LS equilibrium following a deviation in the campaign stage by an equilibrium candidate, and that *(ii)*

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[23]To see this, consider three candidates, 1, 2 and 3, such that $x_1 < x_2 < x_3$, and suppose that $\theta_1 > \theta_2 = \theta_3$. Then some voter i who would rank candidates $3 \succsim_i 2 \succsim_i 1$ on a purely ideological dimension, could possibly rank candidates $1 \succsim_i 3 \succsim_i 2$ when taking into consideration both their ideology and the level of persuasive campaigning, leading to a non-single-peaked preference profile (this requires of course the investment differential to be sufficiently high given the responsiveness of voters to persuasive campaigning, α). In this circumstance, our previous analysis of the rationality of a sincere voting profile would not necessarily apply: if i is decisive for the median between 1 and 2 she would prefer to select 1, so sincere voting is rational for i. But if i were decisive for the median among candidates 2 and 3, then i would find it optimal to deviate from sincere voting and vote strategically for 3.

for every deviation at the entry stage, there is an equilibrium in the continuation voting subgame in which either all or all but a small number of voters vote sincerely, and for which out of equilibrium entry is not sequentially rational.

A result similar in spirit to what we obtained under the median protocol can be shown to hold in an environment in which the policy outcome obtains as a convex combination of the ideological position of the elected representatives. We call this the *bargaining protocol* of policy determination.

Definition 2 *The Bargaining Protocol For given profile $(x_{\mathcal{K}}, \theta_{\mathcal{K}})$, the policy outcome is $(\sum_{k \in \mathcal{K}} s_j x_j, \theta_{\tilde{k}})$, where $\tilde{k}$ is the identity of the candidate obtaining a plurality of the votes.*

While a full characterization of electoral equilibria under the bargaining protocol is beyond the scope of this paper, here we provide a simple example in which candidates running for office do not fully invest in persuasive campaigning.

Example 4. Let $\alpha < 1/\Psi(1)$ and consider a two-candidate on-the-equilibrium-path action profile $(x_1 = 0, x_2 = 1, \theta_1 = \theta_2 = \theta^* \equiv \Psi^{-1}(1/\alpha) < 1)$. Given this action profile, sincere voting is rational and therefore θ^* is optimal (this follows from the best response correspondence for extreme candidates, see footnote 14). Suppose that upon entry of a non-equlibrium candidate, all voters would still vote for their preferred candidate among the equilibrium candidates 1 and 2. Note that no voter would find it optimal to deviate from this voting strategy profile and vote for the entrant, for this deviation could only move the policy outcome away from the voter's ideal point. It follows that this is an electoral equilibrium.

To sum up, we have shown that our main results hold under alternative specifications of the policy function mapping elected representatives to policy outcomes, and therefore are not driven by our assumption that policy outcomes are determined as a probabilistic compromise among elected representatives. In particular, any alternative mechanism inducing sincere voting will leave Proposition 2 and Theorem 1 unchanged. As the previous analysis shows, even alternative protocols for the determination of policy that do not lead to sincere voting being rational in all continuation games are consistent with our conclusions. The logic of entry deterrence in proportional representation works easily with sincere voting but does not require it.

5.4   A Plurality Premium in PR

We have assumed that in PR elections each candidate running for office captures a proportion of office rents equal to her share of votes in the election. In various political systems, however, the party with a plurality of votes obtains an additional reward over

and above its share of votes in the election. In several parliamentary democracies adopting some form of PR, for instance, the *formateur* is typically the head of the majority party. To gain insight about this problem, we consider an abstract electoral system that incorporates the key feature of FPTP elections into our model of PR elections. In this modified version of the model - which we call PR-plus (PRP)- the candidate with a plurality of votes obtains a premium $\gamma \in (0, 1)$ in both the likelihood with which her policy is implemented and in the proportion of office rents she attains after the election. PRP can then be thought of as an intermediate electoral system between PR ($\gamma = 0$), and FPTP ($\gamma = 1$). Letting as before $H \equiv \{h \in \mathcal{K} : s_k = s_h\}$, k's proportion of office's rents after the election is given by

$$
m_k = \begin{cases} s_k(1 - \gamma) + \frac{\gamma}{|H|} & \text{if } s_k \geq \max_{j \neq k}\{s_j\} \\ s_k(1 - \gamma) & \text{o.w.} \end{cases} \tag{4}
$$

The next proposition characterizes PRP elections in large electorates. We show that in large electorates there exists an electoral equilibrium with two candidates symmetrically located around the median voter fully investing in persuasive campaigning, provided that the candidates are not too polarized. We also show that for any plurality premium γ, electoral equilibria in large elections are either of this kind, or such that a single candidate appropriates the plurality premium with certainty.

Proposition 6

(1) *There exists $\overline{n}$ such that for all $n \geq \overline{n}$, there is an electoral equilibrium in which two candidates symmetrically located around the median voter, run for office fully investing in persuasive campaigning.*

(2) *Fix any sequence of equilibria $\{\tilde{\Gamma}_n\}_{n_0}^{\infty}$. There exists $\overline{n}$ such that if $n \geq \overline{n}$, then in $\tilde{\Gamma}_n$, either two symmetrically located candidates run for office fully investing in persuasive campaigning, or a single candidate appropriates the pluarality premium with certainty.*

The main intuition for existence of equilibria with two candidates fully investing in persuasive campaigning is that for any plurality premium γ, and sufficiently large electorates, the strategic problem of individual voters in PRP resembles the analogous problem in FPTP. As a result, we can support an equilibrium with two candidates, 1 and 2, by having voters coordinate on voting for their preferred choice among these candidates, even after entry of a third candidate ℓ. To see this, consider without loss of generality a voter i with preferences $\ell \succ_i 1 \succ_i 2$ (note that we only need strategic voting among voters whose preferred candidate in $\{1, 2, \ell\}$ is the entrant, ℓ). Voter i faces the following tradeoff. On the one hand, by switching to vote sincerely in favor of the entrant, the voter is transfering $1/n$ probability mass from his second best candidate ($k = 1$) to his most preferred candidate (ℓ). On the other hand, he is also inducing a jump of $\gamma/2$ in the probability that the policy of his least favorite candidate in $\{1, 2, \ell\}$ emerges as the

policy outcome, to be "financed" by a similar decrease in the probability of his second best candidate's policy being chosen. For large n, the second effect dominates, and i has incentives to vote strategically. The intuition for the second part of the proposition follows along the same lines, and is only slightly more involved.

The previous result should not be interpreted as implying a complete discontinuity with the PR environment. Note that for fixed n, and given a strategy profile for all other voters, the incentive to vote strategically increases monotonically in the plurality premium γ, and in the polarization of candidates 1 and 2: for any strategy profile of the remaining voters, if i has an incentive to vote strategically given some γ, then i also has an incentive to vote strategically given $\gamma' > \gamma$. Similarly, if i has an incentive to vote strategically for some given degree of ideological differentiation between candidates 1 and 2, then i also has an incentive to vote strategically for a larger payoff differential among equilibrium candidates. In fact, it is easy to see that if candidates running for office are not differentiated at all, then there cannot be strategic voting of this type, as in this case supporting the preferred candidate ℓ comes at not cost. But this implies that there cannot be electoral equilibria with perfect convergence in PRP. On the other hand, in general candidates cannot be too polarized either, for otherwise a deviation by one of the candidates to less effort in persuasive campaigning, forgoing the plurality premium, can be profitable for sufficiently small γ. All in all, while equilibrium behavior of voters and candidates in PRP can resemble behavior in FPTP, the set of equilibria of this class has to be pruned to rule out complete convergence and under some conditions also extreme polarization.

At this point, a natural question to ask is whether equilibria with more than two candidates running for office all not fully investing in persuasive campaigning - which we have shown can be supported in equilibrium in PR - can survive in the case of PRP elections. The answer is yes, provided that the size of the plurality premium is not too large. To see this, note first that whenever a candidate is ahead by at least two votes in a PRP election, strategic voting must be sincere, since in this case any individual deviation in the voting strategy cannot affect the identity of the majority candidate. With this result in mind, consider a location symmetric equilibrium in PR ($\gamma = 0$) such that three candidates run for office without fully investing in persuasive campaigning, and the centrist candidate obtains the sincere vote of slightly more than a third of the electorate. Consider now the case of a positive but small premium γ, fixing all other parameters of the model. From our previous remark, sincere voting remains a best response when other voters vote sincerely. Moreover, with small enough γ, winning a plurality of the vote is not worth a deviation from the optimal campaign investment in the pure PR environment. Finally, note that if the entry of a fourth candidate was not profitable in the case of $\gamma = 0$, this has to be true also in the case of a small plurality premium. In fact, it is enough for this that when $\gamma = 0$, the equilibrium candidates' rents in the continuation game following entry are strictly positive, but we know that this will be the case generically.

To sum up, we have shown that for a given plurality premium, but sufficiently large

24

electorates, equilibrium behavior in PR-Plus resembles that in FPTP. This suggests that it is the discontinuity in payoffs implicit in both FPTP and PRP which induces a decoupling of the intensity of campaign competition from the number of candidates and their ideological differentiation. For a fixed size of the electorate, however, the size of this discontinuity is also relevant. In fact, if the plurality premium is sufficiently small (approximating PR), PR-Plus elections admit equilibria with more than two candidates not fully investing in persuasive campaigning, as in the case of pure PR.

6 Related Literature

Our paper is related to three strands of literature. A first group of papers focuses on the effect of different electoral systems on the number of candidates running for office. This literature provides several formalizations of the well-known *Duvergerian* predictions, namely that majoritarian elections leads to a two-party system (Duverger's law), and that PR tends to favor a larger number of parties than FPTP (Duverger's hypothesis). A relatively large literature focuses on Duverger's law, studying the equilibrium number of candidates in FPTP elections.[24] Among these papers, the closest to our work are Feddersen (1992) and Feddersen, Sened, and Wright (1990) (FSW). Our model of FPTP elections differs from these papers on two accounts. First, while in our set up candidates are endowed with an ideological position that they can credibly implement if elected, in FSW candidates can adjust their ideological positions after entry without costly consequences. Second, while in FSW candidates can only differ in an ideological dimension, in our model candidates can also differentiate themselves by investing in persuasive campaigning. Finally, two papers compare the effect of alternative electoral systems on the number of candidates competing for office. Osborne and Slivinski (1996) compare plurality and plurality with runoff under sincere voting, and Morelli (2004) compares majoritarian and proportional electoral systems under strategic voting. Differently than in our paper, Morelli focuses on how different electoral systems influence the incentives of politicians to coordinate their candidacies, addressing more directly the issue of party formation. See also Cox (1997) for an empirical discussion of the Duvergerian predictions.

A second group of papers analyzes how variations in the electoral system affect policy outcomes. Myerson (1993b) focuses on how the nature of electoral competition affects promises of redistribution made by candidates in the election. Building on this work, Lizzeri and Persico (2001) consider redistribution and provision of public goods in PR and FPTP electoral systems. In both papers, the emphasis is not on differentiation (in ideological or non-ideological dimensions) but rather on the vote-buying strategies of the candidates. Austen-Smith and Banks (1988) and Baron and Diermeier (2001) consider models of elections and legislative outcomes in PR, were rational voters anticipate the

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[24]For papers that study entry in FPTP under the assumption of sincere voting see, e.g., Palfrey (1984), and Greenberg and Shepsle (1987). For papers that study entry in FPTP under strategic voting see, e.g., Palfrey (1989), Besley and Coate (1997), and Patty (2006).

effect of their vote on the bargaining game between parties in the elected legislature. In these papers, however, the number of parties is exogenously given. Finally, several recent papers consider the effects of alternative electoral systems and strategic voting when the relevant policy outcome is not bargaining over a fixed prize, but instead taxation and redistribution (e.g., Austen-Smith (2000) and Persson, Roland, and Tabellini (2003)), or corruption (e.g., Myerson (1993a) and Persson, Tabellini, and Trebbi (2006)). In particular, Myerson (1993a) considers a model where potential candidates are known to differ in their level of corruption (which all voters dislike) but also in a second policy dimension, over which there is disagreement among voters. Myerson (1993a) concludes that a PR electoral system is more effective in reducing the probability of selecting a corrupt candidate than a FPTP system. It is interesting to note that - interpreting the persuasive campaigning as investments that reduce the probability of corruption in government - our model yields the opposite result. The reason is that in Myerson (1993a) the level of corruption is an exogenous characteristic of electoral candidates. Together with strategic voting, this assumption is enough to guarantee the existence of an equilibrium in FPTP where exactly two corrupt candidates tie, even if non corrupt alternatives are available to voters. This cannot occur in a PR system, where voting sincerely for non corrupt candidates is a dominant strategy. In our model, candidates' level of corruption in office is endogenous. As a result the winner-takes-all nature of FPTP elections provides the strongest incentive to invest in actions that discourage corruption in office as compared to PR electoral systems (see Section 5.2).

Our paper is also related to the large literature that, following Stokes (1963)'s original critique to the Downsian model, incorporates competition in *valence* issues, typically within FPTP, and with a given number of candidates (two). For recent papers see Ashworth and Bueno de Mesquita (2007), Carrillo and Castanheira (2006), Eyster and Kittsteiner (2007), Herrera, Levine, and Martinelli (2008), and Meirowitz (2007).[25] Of these, the closest paper to ours is Ashworth and Bueno de Mesquita (2007). They show that in FPTP elections with two candidates, candidates have an incentive to "diverge" in order to soften valence competition. Although this effect is also present in our model for PR elections, this does not occur in our set up in FPTP elections, since here the set of candidates is endogenous, candidates are endowed with fixed policy positions, and voters are strategic.

7   Conclusion

In spite of its relevance in modern elections, campaigning has not been systematically integrated in a theory of elections together with the number and ideological position of candidates running for office. This omission could be of no major consequence if the intensity of campaign competition were unrelated to other characteristics of the menu of alternatives available to voters. On the contrary, however, the number of candidates

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[25]See also Groseclose (2001), Aragones and Palfrey (2002), Schofield (2004), and Kartik and McAfee (2007) for models where one candidate has an exogenous valence advantage.

running for office, their ideological differentiation, and the intensity of campaign competition are all naturally intertwined, and jointly determined in response to the incentives provided by the electoral system.

In this paper, we tackle jointly the effect of alternative electoral systems on the number of candidates running for office, the ideological diversity of their platforms, and the intensity of competition in persuasive campaigning. The central result of the paper is to establish a comparison between PR and FPTP electoral systems. First, we show that FPTP elections induce candidates to campaign more aggressively than PR elections. In particular, we expect candidates in FPTP elections to invest more than their counterparts in PR systems to reduce voters' uncertainty about the policy they will implement once in office, to hire higher quality staff and generically to invest more in researching, drafting and communicating appropriate policy responses to current events. Second, we show that in all equilibria in which candidates are ideologically differentiated, the number of candidates running for office is larger in PR than in FPTP, where exactly two candidates run. Third, we show that the ideological differentiation between candidates running for office can in general be larger or smaller in PR than in FPTP: while electoral equilibrium in PR bounds the minimum and maximum degree of differentiation between candidates, this is not the case in FPTP, where both full centrism and complete polarization are possible.

We show that our main comparison also holds under alternative specifications of the policy function mapping elected representatives to policy outcomes, and in electoral systems with multiple electoral districts. We also consider a variant of the main model in which candidates are perceived by voters as heterogeneous in non-ideological attributes even in the absence of any investments in persuasive campaigning. We show that if these attributes cannot be affected during the campaign, then for some parameters it is possible to find equilibria in which the non-ideological appeal of candidates is larger in PR than in FPTP elections. However, if candidates can complement their innate attributes by campaigning, then the non-ideological appeal of candidates (inherited and/or acquired) will be higher in FPTP than in PR elections, as in the case of the benchmark model. Finally, we show that introducing a threshold of representation in PR can have a large impact on electoral outcomes, both directly, restricting the number and characteristics of candidates competing for office, and indirectly, through strategic voting.

Many interesting aspects remain to be addressed, and are left for future research. We believe that the simplicity and flexibility of the framework introduced in this paper will facilitate this progress.

8 Appendix

Proof of Proposition 1.   Note first that in any equilibrium all candidates that are running for office must tie, since otherwise there would be at least one candidate who would lose for sure and - given the fixed cost of running for office $F > 0$ - would prefer not to run. Since candidates are tying, in equilibrium voters must vote sincerely. If this were not the case, there would exist some voter who is not voting for her most preferred candidate in equilibrium but who could have this candidate winning with probability one by deviating to voting sincerely. Third, note that in any equilibrium it must be that $\theta_k^* = 1$ for all $k \in \mathcal{K}^*$. In fact, since all candidates that are running for office must tie in equilibrium, if $\theta_h^* < 1$ for some $h \in \mathcal{K}^*$, candidate h can profitably deviate by choosing $\tilde{\theta}_h = \theta_h^* + \nu$, for some sufficiently small $\nu > 0$ (winning the election with probability one). The previous results and the assumption that voters' preferences are uniformly distributed in X imply that in any equilibrium the set of candidates running for office must be located symmetrically with respect to $\frac{1}{2}$. We have then established that in any equilibrium *(i)* candidates running for office must tie, *(ii)* voting is sincere, and *(iii)* $\theta_k^* = 1$ for all $k \in \mathcal{K}^*$, and that *(iv)* candidates must be symmetrically located.

We show next that there cannot be an electoral equilibrium with $K > 2$ candidates running for office representing different ideological positions. If this were the case, *(i)* and *(iii)* imply that by deviating and voting for any candidate j other than her preferred candidate, a voter could get candidate j elected with probability one. But then equilibrium implies that this voter must prefer the lottery among all $\mathcal{K}^*$ running candidates to having j elected for sure. This implies, in particular, that

$$\frac{1}{|\mathcal{K}^*|} \sum_{k \in \mathcal{K}^*} u(1, x_k^*; z^i) \geq u(1, x_{K-1}^*; z^i) \tag{5}$$

for all voters such that $z^i > \frac{x_{K-1}^* + x_K^*}{2}$, i.e., all voters whose most preferred winning candidate is $k = K$ and next most preferred winning candidate is $k = K - 1$. On the other hand, strict concavity of $u(\cdot; z^i)$ with respect to policy and *(i)*, *(iii)*, and *(iv)* imply that for all z^i

$$u(1, \frac{1}{2}; z^i) > \frac{1}{|\mathcal{K}^*|} \sum_{k \in \mathcal{K}^*} u(1, x_k^*; z^i). \tag{6}$$

Combining (5) and (6), we obtain

$$u(1, \frac{1}{2}; z^i) > u(1, x_{K-1}^*; z^i)$$

for all voters such that $z^i > \frac{x_{K-1}^* + x_K^*}{2}$. But $K > 2$ and *(iv)* imply that $\frac{1}{2} \leq x_{K-1}^*$. Hence $u(\cdot; z^i)$ cannot be single-peaked for all voters such that $z^i > \frac{x_{K-1}^* + x_K^*}{2}$, contradicting the strict concavity of $u(\cdot; z^i)$.[26]

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[26]It should be noted that property *(iv)*, which follows from the assumption that voters' preferences are uniformly distributed, is in fact not needed to show that an equilibrium with more than two candidates cannot exist. Indeed, the argument can be slightly modified in order to account for a general continuous distribution of voters' preferences.

Finally, note that $\bar{c} + F \leq \frac{1}{2}$ implies that a unique candidate equilibrium cannot be supported, since otherwise a second candidate, symmetrically located with respect to the median, will always find it profitable to run. As a result, the only possible equilibrium must have exactly two symmetrically located candidates fully investing in campaign. We are only left to show that such an equilibrium exists. So consider a strategy profile with two candidates fully investing in persuasive campaigning, 1 and 2, symmetrically located around the median voter (i.e., $x_1 = 1-x_2 < 1/2$), compete for office. Voters vote sincerely among these two candidates on the equilibrium path. If, off the equilibrium path, a third candidate ℓ enters the electoral competition, then we require that voters vote sincerely among candidates in $\{1,2\}$ for all $(\theta_1, \theta_2, \theta_3)$ for which $\max\{\theta_1, \theta_2\} = 1$.[27] We show that this strategy profile is an electoral equilibrium. First note that on the equilibrium path, voters are best responding, since with two candidates strategic voting is sincere. Next note that given that $\bar{c} + F \leq \frac{1}{2}$, equilibrium rents of the two candidates running for office are always non-negative. Since candidates are choosing maximal investment in equilibrium, $\theta_1^* = \theta_2^* = 1$, the only possible deviation in the campaing game is downwards. But any such deviation would entail sure loss, and is thus not profitable. Suppose now that a third candidate ℓ such that $x_\ell \in [0,1]$ decides to enter. Recall that voters vote sincerely among candidates in $\{1,2\}$ for all $(\theta_1, \theta_2, \theta_3)$ for which $\max\{\theta_1, \theta_2\} = 1$. But given these strategies, there is no voter which can benefit from a deviation. In fact, since candidates 1 and 2 are tying, any deviation from sincere voting between candidate 1 and candidate 2 in order to support the entrant will determine a victory of the least preferred candidate instead of having a lottery between $k = 1$ and $k = 2$. But then the strategy profile $(x_1^*, \theta_1 = 1)$, $(x_2^* = 1 - x_1^*, \theta_2 = 1)$, $(x_3, \theta_3 = 0)$, together with the same strategy profile for voters is an equilibrium in the continuation, and entry is not profitable. ∎

Proof of Lemma 1. Suppose voter i's preferred candidate is $k^*(i) \in \mathcal{K}$, and that $\tilde{k} \in \mathcal{K}$ and $\tilde{k} \neq k^*(i)$. Let $t_k(\sigma_{-i}^v)$ denote the number of votes for candidate k given a voting strategy profile σ_{-i}^v for all voters other than i. The payoff for i of voting for $\tilde{k}$ given σ_{-i}^v, $U(\tilde{k}; \sigma_{-i}^v)$, is

$$\sum_{k \neq \tilde{k}, k^*(i) \in \mathcal{K}} \frac{t_k(\sigma_{-i}^v)}{N} u(x_k; z^i) + \frac{[t_{\tilde{k}}(\sigma_{-i}^v) + 1]}{N} u(x_{\tilde{k}}; z^i) + \frac{t_{k^*(i)}(\sigma_{-i}^v)}{N} u(x_{k^*(i)}; z^i).$$

Similarly, the payoff for i of voting for $k^*(i)$ given σ_{-i}^v, $U(k^*(i); \sigma_{-i}^v)$, is

$$\sum_{k \neq \tilde{k}, k^*(i) \in \mathcal{K}} \frac{t_k(\sigma_{-i}^v)}{N} u(x_k; z^i) + \frac{t_{\tilde{k}}(\sigma_{-i}^v)}{N} u(x_{\tilde{k}}; z^i) + \frac{[t_{k^*(i)}(\sigma_{-i}^v) + 1]}{N} u(x_{k^*(i)}; z^i).$$

Thus

$$U(k^*(i); \sigma_{-i}^v) - U(\tilde{k}; \sigma_{-i}^v) = \frac{1}{N}[u(x_{k^*(i)}; z^i) - u(x_{\tilde{k}}; z^i)],$$

which is positive by definition of $k^*(i)$. Since σ_{-i}^v was arbitrary, this shows that voting sincerely strictly dominates voting for any other available candidate and is thus a domi-

nant strategy for voter i. It follows that in all Nash equilibria in the voting stage voters vote sincerely among candidates running for office. ∎

Proof of Proposition 2.

Proof of Part 1. Take $K \geq 3$ given. We will show that if the inequalities (7), (8), and (9) are satisfied,

$$\bar{c} < F, \tag{7}$$

$$\frac{1}{2K} \leq F \leq \frac{1}{K} - \bar{c}, \tag{8}$$

and

$$\alpha < \frac{1}{\Psi(1)K}, \tag{9}$$

then there exists a LS equilibrium in which K candidates run for office without fully investing in persuasive campaigning. These conditions define a non-trivial set of parameters: if $\bar{c} < \frac{1}{2K}$, there exists an interval $[\underline{F}(K), \overline{F}(K)]$ such that $F \in [\underline{F}(K), \overline{F}(K)]$ satisfies (7), and (8). Finally, any $\alpha < \frac{1}{\Psi(1)K}$ satisfies (9).

So, define $L \equiv \max\{2\bar{c}, \bar{c} + F, \frac{1-2F}{K-1}\}$ and $U \equiv \min\{2(\bar{c} + F), \frac{1}{K}\}$. We show below that if $\max\{2\alpha\Psi(1), L(K)\} < U(K)$, then there exists a LS equilibrium in which $K \geq 3$ run for office without fully investing in persuasive campaigning. But this is enough to prove the first part of the proposition, since these conditions are implied by the inequalities (7), (8), and (9).[28]

Consider first the interior candidates $k = 2, \ldots, K - 1$. If $\theta_j^* = \theta_r^* < 1$ for all $j, r \neq k$, then k's marginal vote share is differentiable, and k's FOC is given by $\frac{2\alpha}{\Delta}v'(\theta_k^*) = C'(\theta_k^*)$. Therefore,

$$\theta_k^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \quad \text{for all } k = 2, \ldots, K - 1.$$

Moreover, since $\theta^* < 1$, it must be that $\Delta > 2\alpha\Psi(1)$. Non-negative rents for interior candidates requires that $\Pi_k^* = \Delta - C(\theta^*) - F \geq 0$, or equivalently $\theta^* \leq C^{-1}(\Delta - F)$. Substituting θ^* we get $\Delta \geq 2\alpha\Psi(C^{-1}(\Delta - F))$. Note that $2\alpha\Psi(1) \geq 2\alpha\Psi(C^{-1}(\Delta - F))$ if and only if $\Delta \geq \bar{c} + F$. Then, as long as in equilibrium $\Delta \geq \bar{c} + F$ (i.e., $\Pi_k(1) \geq 0$ for $k = 2, \ldots, K - 1$), $\Delta \geq 2\alpha\Psi(1)$ implies $\Delta \geq 2\alpha\Psi(C^{-1}(\Delta - F))$; i.e., if interior candidates are choosing (the same) non-maximal campaign investment, they obtain non-negative rents. It will be sufficient for our result to look for equilibria in which $\Delta \geq \bar{c} + F$, and therefore we require that

$$\max\{\bar{c} + F, 2\alpha\Psi(1)\} < \Delta. \tag{10}$$

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[28]The condition $\max\{2\alpha\Psi(1), L(K)\} < U(K)$ embodies six relevant inequalities: *(a)* $\alpha\Psi(1) < \bar{c} + F$, *(b)* $2\alpha\Psi(1) < 1/K$, *(c)* $2\bar{c} < 1/K$, *(d)* $(\bar{c} + F) < 1/K$, *(e)* $\frac{1-2F}{K-1} < 1/K$ and *(f)* $\frac{1-2F}{K-1} < 2[\bar{c} + F]$. Note that *(e)* can be written as $F > \frac{1}{2K}$, and *(f)* as $F > \frac{1}{2K} - \frac{K-1}{K}\bar{c}$. Thus *(e)* implies *(f)*. Moreover, from this it follows that $\frac{1}{K} < 2[\bar{c} + F]$, and that therefore *(b)* implies *(a)*. Finally, given (7), *(d)* implies *(c)*. Inequalities *(d)* and *(f)* give (8).

Next, we consider the possibility of entry. First, we require that all equilibrium candidates have an incentive not to drop from the competition in any continuation game. For this it is sufficient that $\min\{\Delta_0, \frac{\Delta}{2}\} \geq \bar{c}$. Since $2\Delta_0 + (K-1)\Delta = 1$, then $\Delta_0 = \frac{1-(K-1)\Delta}{2}$, and the previous condition can be written as

$$2\bar{c} \leq \Delta \leq \frac{1-2\bar{c}}{K-1}. \tag{11}$$

Suppose now that j enters at $x_j \in (x_k, x_{k+1})$ for $k = 1, \ldots, K-1$, and define $\delta_j^r \equiv \frac{x_{k+1}-x_j}{\Delta}$. Suppose first that in the continuation $\hat{\theta}_k = \hat{\theta}_{k+1} = \hat{\theta}_j = 1$. Then it must be that

$$\alpha v'(1) \left[\frac{1}{\delta_j^r \Delta} + \frac{1}{\Delta} \right] \geq C'(1),$$

$$\alpha v'(1) \left[\frac{1}{(1-\delta_j^r)\Delta} + \frac{1}{\Delta} \right] \geq C'(1).$$

Then if $\delta_j^r \geq \frac{1}{2}$ (j enters in (x_k, x_{k+1}) closer to x_k than to x_{k+1}) the first two inequalities above hold if and only if $\Delta \leq \alpha\Psi(1)\left[1 + \frac{1}{\delta_j^r}\right]$, or $\delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta-\alpha\Psi(1)}$. Thus, the continuation strategy profile is a Nash equilibrium for $\frac{1}{2} \leq \delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta-\alpha\Psi(1)}$, which is feasible if and only if $\Delta \leq 3\alpha\Psi(1)$. When instead $\delta_j^r \leq \frac{1}{2}$ (j enters closer to x_k) then we need $\Delta \leq \alpha\Psi(1)\left[1 + \frac{1}{(1-\delta_j^r)}\right]$, or $\delta_j^r \geq \frac{\Delta-2\alpha\Psi(1)}{\Delta-\alpha\Psi(1)}$. Thus, the continuation strategy profile is a Nash equilibrium for $\frac{\Delta-2\alpha\Psi(1)}{\Delta-\alpha\Psi(1)} \leq \delta_j^r \leq \frac{1}{2}$, which is feasible if and only if $\Delta \leq 3\alpha\Psi(1)$. Therefore, the strategy profile $\hat{\theta}_k = \hat{\theta}_{k+1} = \hat{\theta}_j = 1$ is a Nash equilibrium in the continuation for entrants such that

$$\frac{\Delta - 2\alpha\Psi(1)}{\Delta - \alpha\Psi(1)} \leq \delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}, \tag{12}$$

where $2\alpha\Psi(1) < \Delta \leq 3\alpha\Psi(1)$. Since the entrant in this case obtains $\hat{\Pi}_j = \frac{\Delta}{2} - [\bar{c} + F]$, then as long as in equilibrium

$$\Delta < 2[\bar{c} + F], \tag{13}$$

entry in an "interior" region as in (12) is not profitable. It should be clear that this rules out "interior" entrants only, since $2\alpha\Psi(1) < \Delta$ from (10) implies with (12) that $\delta_j^r \in (0,1)$.

Consider then $\delta_j^r > \frac{\alpha\Psi(1)}{\Delta-\alpha\Psi(1)}$ (j enters close to x_k; the other case is symmetric). Consider the continuation $\hat{\theta}_k = \hat{\theta}_j = 1$, $\hat{\theta}_{k+1} = \Psi^{-1}(\frac{\delta_j^r}{1+\delta_j^r}\frac{\Delta}{\alpha}) < 1$. This is clearly an equilibrium in the continuation (j and k have even a greater incentive to choose 1 than in the previous case since they are now closer substitutes). For entry not to be profitable, we need

$$\hat{\Pi}_j = \frac{\Delta}{2} + \frac{\alpha}{\delta_j^r \Delta}[v(1) - v(\hat{\theta}_{k+1})] - [\bar{c} + F] < 0,$$

and a sufficient condition for the above inequality to be true is

$$\Delta \le 2F. \tag{14}$$

To see this, suppose that the division of the electorate between k and j were fixed, with cutpoint $\tilde{x}_{kj} = \frac{x_k + x_j}{2}$. Then j would optimally choose $\tilde{\theta}_j = \Psi^{-1}(\frac{\delta_j^r \Delta}{\alpha}) < \hat{\theta}_{k+1}$, and we have that

$$\hat{\Pi}_j \le \frac{\Delta}{2} - \frac{\alpha}{\delta_j^r \Delta}[v(\hat{\theta}_{k+1}) - v(\tilde{\theta}_j)] - [C(\tilde{\theta}_j) + F] < \frac{\Delta}{2} - [C(\tilde{\theta}_j) + F].$$

Consider next optimality and non-negative rents for extreme candidates, and no-entry conditions at the extremes. Note first that given that interior candidates are choosing non-maximal campaign investment, then optimal campaign investment by extreme candidates must be non-maximal as well. In particular, it must be that $\theta_1^* = \theta_K^* = \Psi^{-1}(\frac{\Delta}{\alpha})$. For no entry at the extremes it is sufficient as before that $\Delta_0 < F$, and since $\Delta_0 = \frac{1 - (K-1)\Delta}{2}$ this can be written as

$$\frac{1 - 2F}{K - 1} < \Delta. \tag{15}$$

For non-negative rents we need $\Pi_1^* = \Delta_0 + \frac{\Delta}{2} - \frac{\alpha}{\Delta}[v(\theta^*) - v(\theta_1^*)] - C(\theta_1^*) - F \ge 0$. Since Π_1^* is maximized at θ_1^*, then $\Pi_1(\theta_1^*) \ge \Pi_1(\theta_1)$ for all $\theta_1 \ne \theta_1^*$ and, as a result, it suffices to show that $\Pi_1(\theta^*) > 0$, or equivalently, $\frac{(K-2)}{2}\Delta + [C(\theta^*) + F] \le \frac{1}{2}$. But since in equilibrium $\Delta \ge C(\theta^*) + F$, then it is sufficient that

$$\Delta \le \frac{1}{K}. \tag{16}$$

We have then shown that the strategy profile specified above is an electoral equilibrium (in which all candidates choose non-maximal campaign investment) if Δ satisfies conditions (10) - (16). Now, (10) and (14) imply that for this to be feasible it is necessary that $\bar{c} < F$ (∗). From (∗), $\bar{c} + F < \Delta$ in (10) and (16) imply (11), and (13) implies (14). The relevant conditions on the degree of policy differentiation, Δ, can then be written as $\max\{2\alpha\Psi(1), L\} \le \Delta < U$, as we wanted to show.

<u>Proof of Part 2</u>. Consider first the case of $K = 2$. Note that since identically located candidates are perfect substitutes, in equilibrium campaign investment must be maximal. Otherwise candidate k can increase rents discretely (in fact capturing all votes) by increasing θ_k (and costs) only marginally. The rents of candidates are non-negative if and only if $\frac{1}{2} - \bar{c} \ge F$. To show that an equilibrium cannot exist it is enough to show that there exists a small positive ν such that entry of a third candidate at $x' = \frac{1}{2} - \nu$ is always profitable. Note that if a third candidate j enters at x' with $\theta_j = 1$ either $\hat{\theta}_k = 1$ for $k = 1, 2$, or $\hat{\theta}_k = 1$ and $\hat{\theta}_{-k} = 0$, $k = 1, 2$ ($\frac{1}{2} - \bar{c} \ge F$ implies that the case $\hat{\theta}_k = 0$, $k = 1, 2$ can never happen). If $\frac{1}{2}(1 - \frac{x' + 1/2}{2}) - \bar{c} = \frac{3 - 2x'}{8} - \bar{c} \ge 0$, we have that in the

continuation game $\hat{\theta}_k = 1$, $k = 1, 2$, and to deter entry at $\tilde{x}$ we need $\frac{x'+\frac{1}{2}}{2} - \overline{c} < F$. When $\nu \to 0$ the two last inequalities become $\frac{1}{2} - \overline{c} \in \left[\frac{1}{4}, F\right]$. Together with the above condition for non-negative rents for candidates, the last expression implies that a two candidate equilibrium with perfectly centrist candidates exists if and only if $F \geq \frac{1}{4}$ and $\frac{1}{2} - \overline{c} = F$. If instead $\frac{3-2\tilde{x}}{8} - \overline{c} < 0$, we have that in the continuation game one of the two running candidates will drop, i.e., $\hat{\theta}_k = 1$, and $\hat{\theta}_{-k} = 0$, $k = 1, 2$. Since to deter entry at $\tilde{x}$ it must be that $\frac{x'+\frac{1}{2}}{2} - \overline{c} < F$, in this case when $\nu \to 0$ we need $\frac{1}{2} - \overline{c} \leq \min\left\{\frac{1}{4}, F\right\}$. Once again combining the last expression with the above condition for non-negative rents for candidates we get that a two candidate equilibrium with perfectly centrist candidates exists if and only if $F \leq \frac{1}{4}$ and $\frac{1}{2} - \overline{c} = F$. If $K > 2$ we need $\frac{x'+\frac{1}{2}}{2} - \overline{c} < F$ and $\frac{1}{K} - \overline{c} \geq F$, which leads to a contradiction when $\nu \to 0$. ∎

Proof of Proposition 6.

(1) For given n, consider a strategy profile in which two candidates fully investing in persuasive campaigning, 1 and 2, symmetrically located around the median voter (i.e., $x_1 = 1 - x_2 < 1/2$), compete for office. Voters vote sincerely among these two candidates on the equilibrium path. If, off the equilibrium path, a third candidate ℓ enters the electoral competition, then we require that voters vote sincerely among candidates in $\{1, 2\}$ for all $(\theta_1, \theta_2, \theta_3)$ for which $\max\{\theta_1, \theta_2\} = 1$.[29] We show that a strategy profile of this class, with $\Delta \equiv x_2 - x_1$ sufficiently small, is an electoral equilibrium for large n. First note that on the equilibrium path, voters are best responding, since with two candidates strategic voting is sincere. Next note that given $\overline{c} + F \leq \frac{1}{2}$, equilibrium rents of the two candidates running for office are always non-negative. Since candidates are choosing maximal campaign investment in equilibrium, $\theta_1^* = \theta_2^* = 1$, the only possible deviation in the campaign game is downwards. So suppose that candidate 1 deviates to some $\theta_1 < 1$. Note that since candidates were tying in equilibrium, and that voters must vote sincerely, this deviation entails the loss of the majority premium γ for sure. Given $\theta_2^* = 1$, and $\theta_1 < 1$, the payoff of candidate 1, $\Pi_1 = (1 - \gamma)\tilde{x}_{12}(\theta_1, 1) - C(\theta_1)$ is continuous and differentiable (as before, $\tilde{x}_{12}(\theta_1, \theta_2)$ represents the voter who is indifferent between candidates 1 and 2 given θ_1, θ_2). Extending the choice set to include $\theta_1 = 1$, but assuming away the possibility of obtaining the majority premium γ, the most profitable "deviation" is then to play

$$\hat{\theta}_1 = \begin{cases} \Psi^{-1}\left(\frac{\Delta}{\alpha(1-\gamma)}\right) & \text{if } \Delta > \alpha(1-\gamma)\Psi(1), \\ 1 & \text{if } \Delta \leq \alpha(1-\gamma)\Psi(1). \end{cases} \tag{17}$$

It follows that if $\Delta \leq \alpha(1-\gamma)\Psi(1)$, 1 prefers not to deviate. To deter this deviation, therefore, it suffices to consider strategy profiles such that $\Delta \leq \alpha(1-\gamma)\Psi(1)$. Suppose now that a third candidate ℓ such that $x_\ell \in [0, 1]$ decides to enter. Recall that voters vote sincerely among candidates in $\{1, 2\}$ for all $(\theta_1, \theta_2, \theta_3)$ for which $\max\{\theta_1, \theta_2\} = 1$. But given these strategies, no voter can benefit from a deviation, provided that n is large

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[29]It is not necessary to specify the strategy profile any further.

enough. To see this, suppose without loss of generality that voter i prefers candidate 1 to candidate 2, and note that i's equilibrium payoff, voting for $k = 1$, is

$$U(1; \sigma^v_{-i}) = \left(\frac{1}{2}(1 - \gamma) + \frac{\gamma}{2} \right) [u(x_1; z_i) + u(x_2; z_i)].$$

Deviating and voting for an entrant ℓ, i obtains

$$U(\ell; \sigma^v_{-i}) = \frac{n-2}{2n}(1 - \gamma)u(x_1; z_i) + \left(\frac{1}{2}(1 - \gamma) + \gamma \right) u(x_2; z_i) + \frac{1}{n}(1 - \gamma)u(x_\ell; z_i).$$

For equilibrium, it is necessary that $U(\ell; \sigma^v_{-i}) - U(1; \sigma^v_{-i}) < 0$, which is always true if $u(x_\ell; z_i) < u(x_1; z_i)$. If instead $u(x_\ell; z_i) > u(x_1; z_i)$, this occurs if and only if

$$\frac{1 - \gamma}{\gamma} < \frac{n}{2} \frac{[u(x_1; z_i) - u(x_2; z_i)]}{[u(x_\ell; z_i) - u(x_1; z_i)]},$$

but this is satisfied for large enough n, since $x_1 \neq x_2$. This concludes the proof of part (i).

(2) Suppose, contrary to the statement of the proposition, that there does not exist such $\bar{n}$. Then for any n there exists $n' > n$ such that $K \geq 3$ candidates tie for the win in $\tilde{\Gamma}_n$. We show that this is not possible. First, note that if a set of candidates $W \subseteq \mathcal{K}$ tie for the win, then all voters voting for candidates in $W \subseteq \mathcal{K}$ vote for their preferred candidate within W (for otherwise a voter could induce a strictly preferred lottery over outcomes by voting for her preferred candidate in W). But then $\theta_k = 1$ for all $k \in W$, for otherwise there exists a candidate $\ell \in W$ with $\theta_\ell < 1$, who would gain from deviating to $\theta'_\ell = \theta_\ell + \eta$ for sufficiently small $\eta > 0$. So suppose first that in equilibrium all $K > 2$ candidates in $\mathcal{K}$ tie, with $\theta_k = 1$ for all k, and let $k^*(i)$ denote i's preferred candidate in $\mathcal{K}$. It is immediate here that all voting is sincere, for otherwise any voter not voting sincerely would induce a strictly preferred lottery over outcomes by voting for their preferred candidate $k^*(i)$. Since all candidates are tying choosing maximal campaign investment and voting is sincere, candidates must be equally spaced. Next, note that equilibrium implies that all voters $i \in N$ must prefer the equal probability lottery among all $k \in \mathcal{K}$ induced in equilibrium to the lottery that is implied after a deviation to any candidate $\ell \neq k^*(i)$. Now, if for any n there exists $n' > n$ such that this strategy profile is an equilibrium, it must be that all voters $i \in N$ must prefer the equal probability lottery among all $k \in W$ induced in equilibrium to the degenerate lottery in which they get any candidate $\ell \neq k^*(i)$ for sure. To see this, note that i's equilibrium payoff, voting for $k^*(i)$, is

$$U(k^*(i); \sigma^v_{-i}) = \sum_{k \in \mathcal{K}} \left[\frac{1}{K} \frac{n-1}{n}(1 - \gamma) + \frac{\gamma}{K} \right] u(x_k; z_i) + \frac{1}{n}(1 - \gamma)u(x_{k^*(i)}; z_i).$$

34

Deviating and voting for $\ell \neq k^*(i)$, i obtains

$$U(\ell; \sigma^v_{-i}) = \sum_{k \in \mathcal{K}} \left[\frac{1}{K} \frac{n-1}{n} (1 - \gamma) \right] u(x_k; z_i) + \left[\frac{1}{N}(1-\gamma) + \gamma \right] u(x_\ell; z_i).$$

The deviation gain $U(\ell; \sigma^v_{-i}) - U(k^*(i); \sigma^v_{-i}) < 0$ implies then that

$$u(x_\ell; z_i) - \frac{1}{K} \sum_{k \in \mathcal{K}} u(x_k; z_i) < \frac{1}{n} \frac{(1-\gamma)}{\gamma} \left[u(x_{k^*(i)}; z_i) - u(x_\ell; z_i) \right],$$

but since for any n there exists $n' > n$ such that this strategy profile is an equilibrium, it must be that $u(x_\ell; z_i) < \frac{1}{K} \sum_{k \in \mathcal{K}} u(x_k; z_i)$, for otherwise, we can always find an n' that would reverse this inequality. Thus, if there does not exist a largest finite n for which all $K > 2$ candidates in $\mathcal{K}$ can tie in equilibrium, it must be that all voters $i \in N$ must prefer the equal probability lottery among all $k \in W$ induced in equilibrium to the degenerate lottery in which they get any candidate $\ell \neq k^*(i)$ for sure. But then the same argument as in Theorem 1 shows that this can not be an equilibrium.

Next suppose that $2 \leq |W| < K$ candidates tie for the win in equilibrium, where again W denotes the set of winning candidates and L the set of losing candidates. This cannot be an equilibrium either for sufficiently large n, since otherwise a voter i voting for one of the losing candidates $\ell_0 \in L$ could gain by breaking the tie among the candidates in W in favor of her favorite candidate among W, w_0. To see this, denote the fraction of votes obtained by candidate in W by ω, and note that i's equilibrium payoff, voting for $\ell_0 \in L$, is

$$U(\ell_0; \sigma^v_{-i}) = \sum_{w \in W} \left[\omega(1 - \gamma) + \frac{\gamma}{|W|} \right] u(x_w; z_i) + \sum_{\ell \in L} \frac{t_\ell}{n}(1-\gamma)u(x_\ell; z^i).$$

The expected payoff of deviating and voting for $w_0 \in W$ is instead

$$U(w_0; \sigma^v_{-i}) = \sum_{w \in W} \omega(1-\gamma)u(x_w; z^i) + \left[\frac{1}{n}(1-\gamma) + \gamma \right] u(x_{w_0}; z^i) +$$
$$\sum_{\ell \neq \ell_0 \in L} \frac{t_\ell}{n}(1-\gamma)u(x_\ell; z^i) + \frac{(t_{\ell_0} - 1)}{n}(1-\gamma)u(x_{\ell_0}; z^i).$$

But then $U(w_0; \sigma^v_{-i}) - U(\ell_0; \sigma^v_{-i}) > 0$ if and only if

$$\frac{\gamma}{1-\gamma} > \frac{1}{n} \frac{[u(x_{\ell_0}; z^i) - u(x_{w_0}; z^i)]}{\left[u(x_{w_0}; z^i) - \frac{1}{|W|} \sum_{w \in W} u(x_w; z_i)\right]},$$

which holds for sufficiently large n. ∎

Proof of Proposition 5.   It remains to show that sincere voting is rational in any voting subgame following a deviation in campaign investment by an equilibrium candidate in a LS equilibrium, and that for every deviation at the entry stage, there is an equilibrium in the continuation voting subgame in which either all or all but a small number of voters vote sincerely, and such that out of equilibrium entry is not sequentially rational.

Consider first voting subgames following a deviation in campaign investment by an equilibrium candidate in a LS equilibrium. Suppose that candidate k deviates to $\theta_k \neq \theta^*$. We know from the proof of Proposition 2 that this cannot be a profitable deviation for k if voters vote sincerely. Moreover, given that candidates care exclusively about vote shares this cannot be a profitable deviation if all but a small number of voters vote sincerely either. As a result, a sufficiently large number of voters must be voting strategically for this to be a profitable deviation. On the other hand, if any voter is to vote strategically, it must be that k is either tying or contending for the median position by at most one vote. But this implies that if all voters vote sincerely, k can't be close to contending for the median, and therefore no voter can have an incentive to vote strategically for candidate k. Since all other relevant candidates choose the same level of campaigning, then there cannot be strategic voting for any other candidate either, and sincere voting is rational.[30] Thus choosing θ^* is a best response for k in the campaign competition stage.

Similarly, we can show that for every deviation at the entry stage, there is an equilibrium in the continuation voting subgame in which either all or all but a small number of voters vote sincerely, and for which out of equilibrium entry is not sequentially rational. Consider then a deviation at the entry stage. Note that if voters vote sincerely after every continuation, or if all but a small number of voters vote sincerely after every continuation, then entry is not profitable, in the sense that for every possible entry there exists an equilibrium in the continuation game such that the entrant obtains a negative payoff. Now suppose that after a deviation at the entry stage, candidates play the continuation strategy profile that deters entry in the proof of Proposition 2, and suppose that all voters vote sincerely. Then the event in which two candidates contend for the median position by a one vote difference given sincere voting and given this particular strategy profile by candidates has probability zero. But if no two candidates are contending for the median position by a one vote difference, sincere voting is rational. Now consider a deviation from this profile by one of the candidates. By our previous argument, this can only be

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[30]Moreover, voting sincerely is not a weakly dominated strategy for any voter i, as it is always possible to find a voting profile for the remaining voters for which i's vote can be decisive between i's favorite candidate and some other candidate running for office.

a profitable deviation if a sufficiently large number of voters is voting strategically in the voting subgame following this deviation. But then we can always choose a voting strategy profile in which all but a small number of voters vote sincerely. Then no voter can be decisive for the median, and no voter will have an incentive to deviate. All voters, moreover, are using undominated strategies (we know that voters voting sincerely are not using weakly dominated strategies, but neither are the voters who continue to vote as in the strategic voting profile, since in fact this was a best response against this strategy profile by the other voters). Since candidates only care about voting shares, and since with a large electorate the impact of a small number of votes on payoffs is negligible, this cannot be a profitable deviation. This concludes the argument. ∎

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