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# IS THE STATUS QUO RELEVANT IN A REPRESENTATIVE DEMOCRACY?

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# Is the Status Quo Relevant in a Representative Democracy? *

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**Abstract**

This work studies the effect of the value of the status quo in the candidates' decisions and policy outcomes in a representative democracy with endogenous candidates. Following the citizen-candidate model due to Besley and Coate (1997) we show, for a unidimensional policy issue and for both an odd and even number of citizens, that some equilibria only hold for certain values of the status quo policy. In particular we find that a moderate status quo rules out equilibrium outcomes in which there is an uncontested candidate and that two-candidate equilibria exist more generally when the number of citizens is even.

**Keywords:** Status quo, endogenous candidates.

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# 1 Introduction

In a representative democracy, voters (or "citizens") elect some representatives (or "politicians") who in turn, choose policies. It is generally assumed in this context that one or several citizens (or parties) will always propose themselves as candidates for the representation job. Nevertheless, every political system based on representative democracy should provide a rule to determine the political action to be taken in the case when no member of the society is willing to work as representative. This paper focuses on the influence that the existing rule for such a case has over the electoral decisions of citizens and candidates, and over the policies that are ultimately implemented in a representative democracy.

The first model of representative democracy is due to Downs (1957). He studies the policy outcomes when there are only two candidates, which appear exogenously.

Feddersen, Sened and Wright (1990) consider endogenous candidates, so that each citizen decides whether or not to present his candidacy in order to be elected as representative. They suppose that candidates can commit to fulfill their campaign promises and they show that, if the policy issue is unidimensional, all running candidates propose the median voter´s ideal policy.

More recently, there are two papers studying in depth what happens if these endogenous candidates cannot commit themselves to implement any other policy but their favorite one:

Osborne and Slivinski (1996) restrict attention to the case of sincere voting, a unidimensional policy issue and agents with single-peaked preferences. They consider that winning candidates obtain a direct benefit just for holding office. They compare outcomes under plurality rule and a run-off system.

Besley and Coate (1997) present a model of representative democracy with endogenous candidates and strategic voting. Their model is based on an abstract policy space, but they study in more detail the specific case in which the policy has a single dimension, the number of citizens is odd and the status quo policy takes an extreme value of zero. Following their approach, we will analyze the unidimensional case in which agents have Euclidean preferences, but we let the status quo take any value and we consider societies with either an odd or even number of citizens.

The status quo is a default policy, implemented when no agent is willing to run for office. We think that the default policy does not necessarily take a value of zero. For instance, if there is no candidate, and no proposed policy, the actual implemented policy could be the policy implemented in the past. Or it could be an exogenously given "minimum standard". In any case, it might be realistic in many scenarios to consider that the status quo policy takes a positive value. We wish to study whether the equilibrium outcomes obtained in Besley and Coate (1997) are robust to different values of the status quo policy or not and whether they are affected by the number of citizens being odd or even.

We will show that some equilibria are dependant on the value of the status quo and, in particular, we will see how uncontested elections are ruled out by a moderate value of the default policy. Besley and Coate (1997) consider that single candidate equilibria are rare in practice since they parallel the existence of a Condorcet winner. Building on the same model to study the influence of lobbies, Felli and Merlo (2001) disregard uncontested elections and restrict their attention to two-candidate equilibria, justifying their choice on grounds of realism. We offer an alternative explanation to support two-candidate equilibria: For a moderate status quo, we find that not even a Condorcet winner may stand as lone candidate so long as the benefit of holding office is not high.

Moreover, we find that when the number of citizens is even, two-candidate equilibria exist more generally than when the number of citizens is odd whereas conditions for existence of a single candidate equilibrium become more restrictive. We also find that the value of the status quo policy is similarly relevant to the existence and characterization of single candidate equilibria when the number of citizens is even. Regardless of the number of citizens, a moderate status quo will guarantee competition in the electoral process.

The rest of the paper is structured as follows: Section 2 illustrates our case with an example, Section 3 presents the model, Section 4 analyses the influence of the status quo location in the existence and characterization of equilibria with an odd number of voters, Section 5 extends the previous results to a society with an even number of citizens and Section 6 concludes and proposes other extensions to Besley and Coate (1997).

# 2 An Example

Imagine a group of students, a class, which are to elect a representative. Students will vote for any of those who wish to be a candidate and the one with the most votes will be their representative. If there is no candidate, there will be no representative and therefore no defense of the rights of students. Candidates are required to present a written inform of their intentions and objectives to the Dean, and they have to make a speech to the class; therefore, to be candidate is costly. The representative´s task will be to defend the rights of the students. He can be very mild in this defense and submissive with faculty, or on the contrary he can be aggressive and organize protests about everything. The only significant difference in electing one candidate over another will be the attitude that he will show towards faculty, specifically the intensity of his animosity, which depends on his character.

Students attitudes range from a positive disposition with, and no intention whatsoever to defend any rights against authority, to open hostility towards faculty. Most of them are moderate, do not want trouble or protests, but would like their representative to politely but firmly fight for some minimums such as knowing the dates of exams in advance, or teachers being available in their office hours. If there is no representative, none of these minimums will be guaranteed.

In this setting, although nobody likes being a candidate, someone will volunteer in order to have a representative and prevent the undesired outcome of no representation. In fact, we have got two possible outcomes in equilibrium:

The first possibility is a single candidate who becomes the representative with no election. It has to be a moderate. If it was an extremist, moderates will be as unhappy as not having representative. Moderate students dislike protests and strikes as much as not having any defense of their rights. They prevent this outcome presenting a moderate as candidate.

The second possibility is two candidates, one hostile towards faculty, the other one submissive. Both of them have the same support and the class splits into two excited camps. No moderate would stand a chance in this scenario. Voting, and most likely disorder, will follow. Somehow, due to uncertain reasons (miscounting of votes being one of the main ones), one of the two will win.

This class corresponds to the specific case studied in detail by Besley and Coate in which the status quo takes an undesired extreme value. Now let us introduce a subtle change in the setting:

We still have the same students, the same class (we will call it class A) and the same decision to be made.

Now, there is another class, class B, which has previously elected a representative. We do not care about how or who has been elected, or about preferences in this class. What matters is that class B already has a representative when class A is to elect its own.

The Dean rules that it is undesirable for a class to have no representative, so if class A presents no candidate, the representative of class B will also be representing class A. It seems reasonable.

It means that class A's political conditions have changed. Now there is an exogenous status quo: The representative in class B. If he is an extremist for class A standards, the change is unimportant, because the status quo is as undesired as it was in the previous situation, and a moderate will be willing to be the lone candidate and representative.

However... What happens if the representative of class B is considered a moderate in class A? In other words, what happens if the status quo for class A takes a moderate value?

Then moderates in class A will be happy enough to delegate everything to the exogenous representative. They will not be willing to assume any cost of candidacy, since representative B would conveniently do the job for them. Extremists in class A will think differently. They do not like the representative of class B. As long as there is no moderate candidate in class A, submissive extremists would like to present a candidate and win unopposed. So would aggressive extremists. But they cannot. They both know that as soon as an extremist candidate appears, a moderate one would come to defeat it.

Equilibrium with one student as the lone candidate will not occur. An extremist can not run unopposed. Moderate students can, but are unwilling to do so.

The class is bound to the din and disorder which precedes and follows the voting when there are two alternatives with equal support: A submissive extremist candidate, and an aggressive extremist candidate. If one of each type presents his candidacy, the students no longer bother about the exogenous status quo. They simply support the side they prefer.

If the status quo takes a moderate value, we will see in the following sections how some equilibria are ruled out, as has happened in our class.

Other examples could be an employees committee electing a speaker to talk to the employer, or a social club electing a president.

# 3 The Model

We follow Besley and Coate (1997), and consider the specific case in which the policy space is unidimensional. It is an endogenous candidates model of representative democracy with the following characteristics:

Let N be a finite set of citizens. Initially we will suppose that the number of citizens is odd, in order to easily identify the median.

The set of policy alternatives is the unit interval [0,1]. The implemented policy will be denoted by $p$.

Each citizen $i \in N$ has Euclidean preferences with an ideal policy $\pi_i \in [0,1]$ and cares only about the policy outcome $p$, not the identity of the representative (we will briefly introduce a benefit of holding office at the end of sections 4 and 5). Preferences over lotteries (denoted by $L$) will be measured by means of a Von Neumann - Morgenstern utility function.

Any citizen can be a candidate, but doing so is costly. Let $c_i$ denote the cost that citizen $i$ assumes: $c_i = 0$ if citizen $i$ does not run for office and $c_i = c > 0$ if citizen $i$ is a candidate. Therefore, the utility function of citizen $i$, where $p$ is the policy outcome, is as follows:

$$U_i(p, c_i) = -|p - \pi_i| - c_i.$$

Let $\pi_m$ denote the median voter's ideal policy and let $q$ denote the status quo policy, implemented when the set of candidates is empty (no citizen decides to run for office).

Candidates cannot commit themselves to implement any other policy but their favorite one if they come to power.

The political process has three stages. In the first stage, citizens decide whether to enter or not the race for office; in a second stage each citizen casts his vote for one of the self-declared candidates; and in the third stage, the winner candidate implements his favorite policy. Agents vote strategically, thus not necessarily supporting their favorite candidate in the contest, and the voting rule is plurality: The candidate with the most votes wins. If several candidates tie for the most votes, one of them will be selected at random.

At the first stage, citizens face only two alternatives: To enter the race for office, or not to enter. A vector of entry decisions contains the decision by every citizen $i \in N$ .

Equilibrium is defined as a vector of entry decisions and a voting behavior such that:

a) The vector of entry decisions is a Nash equilibrium at the entry decision stage, given the existing voting behavior.

b) The voting behavior results in an undominated Nash equilibrium at the voting stage for any non-empty set of candidates.

In an equilibrium no citizen can obtain any benefit from individual changes in either his entry decision or his vote.

Under the assumption that the status quo policy is $q = 0$, and considering only pure strategies, the following result is obtained:

**Proposition 1** *(Besley and Coate, 1997) A single candidate equilibrium in which $i$ is the unopposed candidate exists if and only if:*
*1.i) $\pi_i \geq c$, and*
*1.ii) there is no citizen $k$ such that $2\pi_m - \pi_i < \pi_k < \pi_i - c$ or $\pi_i + c < \pi_k < 2\pi_m - \pi_i$.*

Condition 1.i) guarantees that citizen $i$ wants to run against the default outcome. Condition 1.ii) guarantees that the unopposed candidate is close enough to the median ideal policy so that whoever could beat him has no incentive to present himself.

**Proposition 2** *(Besley and Coate, 1997) A two-candidate equilibrium in which $i$ and $j$ run against each other exists if and only if:*
*2.i) $\frac{(\pi_i + \pi_j)}{2} = \pi_m$, and*
*2.ii) $|\pi_i - \pi_j| \geq 2c$.*

Condition 2.i) says that the two candidates' favorite policies are at the same distance away from the median voter's ideal policy (and on opposite sides). As a result, both candidates will obtain half of the votes (an implicit assumption being that the median is unique, thus the same number of citizens lie to its left and to its right).

Condition 2.ii) guarantees that in such circumstances (one half chance of winning), both candidates have an incentive to present themselves, since the distance between the two proposed policies is sufficiently big.

Under very mild conditions, no equilibrium with three candidates hold. Those conditions are:

1. A "non-clumping" assumption: Given any interval which contains the ideal points of at least a third of the citizens, any other bigger interval is not empty.

2. Abstinence of Indifferent Voters: Citizens will abstain if they are indifferent between all candidates.

Equilibria with four or more candidates occur only under very extraordinary conditions. Besley and Coate do not study them in detail, only indicating that no more than two of the candidates can be winning; the rest being mere spoilers, and that voters for the spoilers must be indifferent between all winning candidates.

# 4 Status Quo and Equilibria with an Odd Number of Citizens

In this section we analyze how the value of the status quo policy affects the results obtained by Besley and Coate (1997) to see whether their equilibria hold if $q \neq 0$ or, on the contrary, if significant differences arise in policy outcome, depending on the status quo.

We will suppose that the cost of entry $c$ is not too large. If the cost of entry was sufficiently large, no citizen would be a candidate. We will only consider the cases in which the cost of running for office is reasonably low:

Assumption: $c \leq \min\left\{\frac{2}{3}\pi_m, \frac{2}{3}(1-\pi_m)\right\}$ . Note that $\pi_m$ is not necessarily equal to $(1-\pi_m)$.

If the status quo can take different values, Proposition 1 has to be restated:

**Proposition 3** *Given $q \in [0,1]$, a single candidate equilibrium in which $i$ is the unopposed candidate exists if and only if:*
*(i) $|\pi_i - q| \geq c$, and*
*(ii) there is no citizen $k$ such that $2\pi_m - \pi_i < \pi_k < \pi_i - c$ or $\pi_i + c < \pi_k < 2\pi_m - \pi_i$.*

Proposition 3 coincides with Proposition 1 in the particular case of $q = 0$.

But if $q$ has a positive value, the first condition for the case of an unopposed candidate has to be modified. It now says that in order to have an incentive for assuming the cost of running for office, citizen $i$ must have a favorite policy sufficiently far from the status quo. If this condition was not met, agent $i$ would drop his candidacy, accept $q$ as the implemented policy, and avoid the cost $c$ of running for office.

The second condition remains unchanged. It describes the citizens who are suitable for running unopposed. If any of them were to be a candidate, no other citizen who can beat the candidate has the incentive to do so. Each one of these citizens knows that he could find no opposition if he decided to be a candidate.

Let $X_{con}$ denote the interval $[\underline{x}, \overline{x}] \subset [0,1]$ consisting of points $\pi_x$ such that for all $\pi_y \in [0,1]$, if there were a citizen $x$ with an ideal preference $\pi_x$ and a citizen $y$ with an ideal preference $\pi_y$, and $x$ was running alone for office, then $y$ would not be willing to run against $x$ because the chances of winning do not compensate the cost of entry in the race (either $y$ would lose, or would tie but find it not worthwhile to run for a tied race, or would win but find it not worthwhile to run not even for a victory). For most $\pi_x \in X_{con}$ and $\pi_y \in [0,1]$ there will not exist such citizens $x$ and $y$ with these particular preferences, for the interval of preferences is continuous and the number of citizens finite, but still we can argue that such theoretical citizen $x$, if he existed and ran for office, he would surely find no opposition from any single other citizen in equilibrium. We will call $X_{con}$ "strict consensus interval", for if a candidacy belonging to it was presented, then it could raise "consensus" in the sense that it could face no competition in equilibrium.

The extension of the "strict consensus interval" will be a function of the location of the median's ideal point $\pi_m$ and the entry cost $c$. From condition *(ii)* in Proposition 3, it follows that for the purpose of this section, $X_{con} = [\pi_m - \frac{1}{2}c, \pi_m + \frac{1}{2}c]$. For any given $\pi_l$ candidacy, only those citizens with an ideal preference closer to that of the median would defeat $l$. But if $\pi_l \in X_{con}$, then those who could beat $l$ are not willing to pay the cost of running in order to do so, since $\pi_l$ is sufficiently close to their ideal outcome (the distance separating them is less than the cost of running $c$).

However, it is possible that some other candidacies outside of the "strict consensus interval" would still face no opposition, for the "citizen $y$" with the ideal policy point $\pi_y$ that would lead $y$ to have an incentive and a chance to beat such candidacies may not exist. Let us visualize this with the aid of Figure 1.

A citizen with an ideal preference located in $\pi_m - \frac{1}{2}c$ would face no opposition, for only other citizens with preferences below $\pi_m + \frac{1}{2}c$ would defeat him, and these citizens would rather accept $\pi_m - \frac{1}{2}c$ as the policy outcome than to bear the costs of running and implementing their favorite policy. This is no longer true for citizen $l$. He is at a distance $d$ to the left of $\pi_m - \frac{1}{2}c$. Any citizen $h$ whose ideal outcome lies in the circle of radius

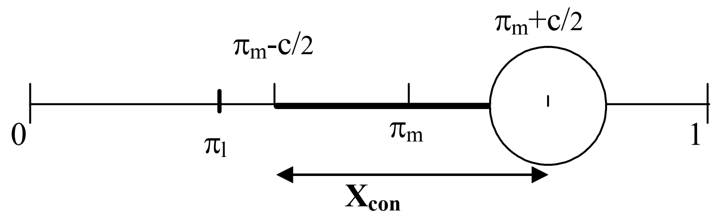


Figure 1: Potential consensus candidates

$d$ around $\pi_m + \frac{1}{2}c$ can beat $l$ because $\pi_h$ is closer than $\pi_l$ to $\pi_m$, and the candidate who is closer to the median always wins a two-way race. Any citizen $h$ in this circle also has the will to run and defeat $l$, for the distance between $\pi_l$ and $\pi_h$ is higher than the cost of running for office. Now, what if there is no such citizen $h$ whose preference lies in the circle of radius $d$? Then $l$ can run unopposed. The bigger the empty circle around one of the extremes of $X_{con}$, the further away from the median in the other direction that a candidate can run alone in equilibrium.

In particular, if the distance from the high extreme of $X_{con}$ to the nearest citizen's ideal point is $d_b$, then a citizen $k$ with preferences to the left of the lower limit of $X_{con}$ by a distance less than $d_b$ can also run unopposed. Similarly if there is no citizen within a distance $d_a$ of the lower extreme of $X_{con}$, then a citizen within $d_a$ of the higher limit of $X_{con}$ may also run alone in equilibrium. Let $X^*_{con}$ denote this "extended consensus interval" which now includes every citizen who can run unopposed if he chooses to present his candidacy (we will call these citizens "potential consensus candidates").

Formally, let $d_b = \min\limits_{i \in N} |b - \pi_i|$, $d_a = \min\limits_{i \in N} |a - \pi_i|$ and $X^*_{con} = (\underline{x} - d_b, \overline{x} + d_a)$, where $a$ and $b$ are a function of the location of the median $\pi_m$ and the entry cost $c$. Throughout this section, $a = \pi_m - \frac{1}{2}c$ and $b = \pi_m + \frac{1}{2}c$, thus they coincide with the limits of $X_{con}$, but this will not always be the case when the number of citizens is even.

From Proposition 3, condition $(ii)$ we can then obtain $X^*_{con} = \left[\pi_m - \frac{1}{2}c - d_b, \pi_m + \frac{1}{2}c + d_a\right]$.

We are now ready to rewrite condition *(ii)* in Proposition 3 as:

$(ii)$ $\pi_i \in X^*_{con}$

This form of expression will prove useful later on. The idea behind condition $(ii)$ in Proposition 3 is that whoever may beat the single candidate has no incentive to do so. If citizen $i$ is to be a single candidate in equilibrium, she must be a "potential consensus candidate", that is, her ideal point must lie within the "extended consensus interval" $X^*_{con}$.

We have seen that conditions for the existence of a single candidate equilibrium have changed, due to the appearance of $q$ in condition *(ii)* in Proposition 3. Only "potential consensus candidates", those with a favorite policy contained in $X^*_{con}$, have a chance of

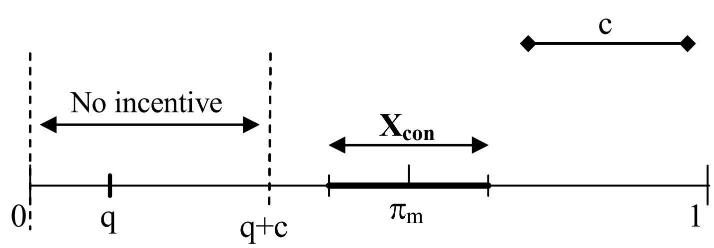


Figure 2: Single candidate equilibria with an extreme status quo

being the single candidate in an equilibrium. But this is a necessary condition, not a sufficient condition.

The value of the status quo policy affects which of these potential consensus candidates can be the single candidate in equilibrium.

**Proposition 4** *Let $q \in \left[0, \pi_m - \frac{3}{2}c - d_b\right]$, then there is a single candidate equilibrium in which citizen $i$ is the unopposed candidate if and only if $\pi_i \in X^*_{con}$.*

**Proof.** If $i$ is a potential consensus candidate, then by definition, it meets the requirements in condition *(ii)* of Proposition 3. And given that $q \in \left[0, \pi_m - \frac{3}{2}c - d_b\right]$, for any $\pi_i \in X^*_{con}$, condition *(i)* is also verified, since $\pi_i \in X^*_{con}$ implies $\pi_i \geq \pi_m - \frac{1}{2}c - d_b$, and then $\pi_i - q \geq \left(\pi_m - \frac{1}{2}c - d_b\right) - \left(\pi_m - \frac{3}{2}c - d_b\right) \geq c$. If $\pi_i \notin X^*_{con}$, then $i$ cannot be a single candidate in equilibrium. ■

**Corollary 1** *Let $q \in \left[\pi_m + \frac{3}{2}c + d_a, 1\right]$, then a single candidate equilibrium exists in which citizen $i$ is the unopposed candidate if and only if $\pi_i \in X^*_{con}$.*

This result is obtained by symmetry.

The idea in this Proposition (and Corollary) is that, when the status quo is too low (too high), each potential consensus candidate has an incentive to present himself and stand as a single candidate.

We can see how this happens in Figure 2. Citizens near the status quo have no incentive to present themselves as the single candidate. They are within a "no incentive area" which extends to a distance $c$ (the cost of entry) from the status quo. The potential consensus candidates are far enough from this zone (throughout Figures 2 to 4 we will assume that there are no "gaps" or empty neighborhoods around $X_{con}$, thus the "consensus interval" is $X_{con} = X^*_{con}$). Their favorite policies are shown in a thicker black line, which extends around the median policy.

However, if $q$ takes bigger values, the "no incentive area" will be shifting to the right (Figure 3). The citizens whose favorite policy is in the lower part of $X^*_{con}$ (thick dotted line in Figure 3) will no longer have an incentive to present themselves as lone candidates.

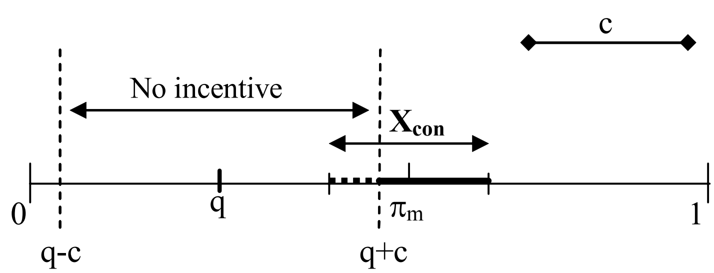


Figure 3: Single candidate equilibria with a not so extreme status quo

They would rather accept the new status quo policy, which is not too far from their ideal. Among the potential consensus candidates only those whose ideal policy is sufficiently high to be far apart from an increasing status quo can be single candidates in equilibrium. These possible equilibrium outcomes are marked, once again, with a thicker black line in Figure 3.

**Proposition 5** *Let* $q \in \left[\pi_m - \frac{3}{2}c - d_b, \pi_m - \frac{1}{2}c + d_a\right]$*, then a single candidate equilibrium exists in which i is the unopposed candidate if and only if* $\pi_i \in \left[q + c, \pi_m + \frac{1}{2}c + d_a\right]$ *.*

**Proof.** Let $\pi_i \in \left[q + c, \pi_m + \frac{1}{2}c + d_a\right]$ this means that $\pi_i \geq q + c$ or equivalently $\pi_i - q \geq c$, so that condition *(i)* in Proposition 3 is met.

Since $\pi_m - \frac{3}{2}c - d_b \leq q \leq \pi_m - \frac{1}{2}c + d_a$ then $\pi_m - \frac{1}{2}c - d_b \leq q + c \leq \pi_m + \frac{1}{2}c + d_a$ which implies that $\left[q + c, \pi_m + \frac{1}{2}c + d_a\right] \in \left[\pi_m - \frac{1}{2}c - d_b, \pi_m + \frac{1}{2}c + d_a\right]$ , therefore $\pi_i \in X^*_{con}$, and condition *(ii)* in Proposition 3 is also met.

We have yet to prove that there are no other possible single candidate equilibria. This is almost immediate:

Let $\pi_j \notin X^*_{con}$ then $\pi_j$ cannot be single candidate equilibrium, because it does not meet condition *(ii)* in Proposition 3.

Let $\pi_j \in X^*_{con}$ but $\pi_j \notin \left[q + c, \pi_m + \frac{1}{2}c + d_a\right]$, then $\pi_j < q + c$ or equivalently $\pi_j - q < c$, so we can see that condition *(i)* is not met. ■

There is a strong implication in this Proposition. Let $q = \pi_m - \frac{1}{2}c + d_a$, then there is only one possible single candidate equilibrium. The single candidate's favorite policy is $\pi_i = \pi_m + \frac{1}{2}c + d_a$. This is just a very specific case, but if the status quo takes exactly this value, we have reduced the multiple possible single candidate equilibria in Besley and Coate´s model to just one.

Once again, we obtain the following Corollary by symmetry:

**Corollary 2** *Let* $q \in \left[\pi_m + \frac{1}{2}c - d_b, \pi_m + \frac{3}{2}c + d_a\right]$*, then a single candidate equilibrium exists in which i is the unopposed candidate if and only if* $\pi_i \in \left[\pi_m - \frac{1}{2}c - d_b, q - c\right]$ *.*

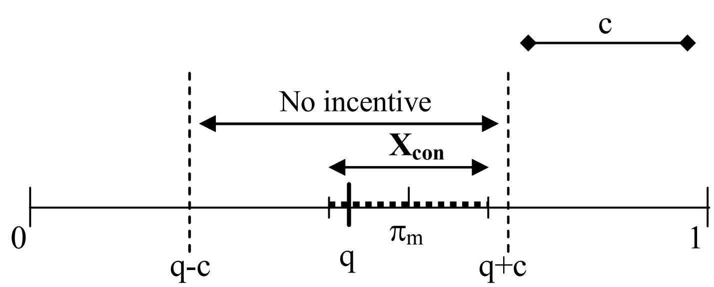


Figure 4: No single candidate equilibrium with a moderate status quo

The next result is one of our main contributions. It shows that existence of single candidate equilibria in Besley and Coate´s model is dependant on the value of the status quo policy. If such a policy takes a moderate value, around the median of the favorite policies of the citizens in the society, then there will be no political equilibrium in which only one candidate runs for office. To reiterate, a moderate status quo rules out political equilibrium in which there are no competed elections.

Comparing Figure 2 and Figure 3, we can imagine what will happen if the status quo keeps shifting to the right: The dotted thick line (those potential consensus candidates who wish to not run for office unopposed) gets bigger and bigger, and the thick black continuous line gets shorter and shorter until it disappears. Therefore, there is no single candidate equilibrium. (Figure 4)

The intuition is simple. Those who could stand alone as unopposed candidates wish not to do so, and those who have an incentive to present themselves to defeat the default policy are too far apart from the median and as a consequence, if any of them presents his candidacy, then another citizen closer to the median will also run to defeat him.

For example, let us imagine that the median citizen´s ideal policy is the default policy. In this case, neither the median citizen nor any of the potential consensus candidates would like to assume the cost of presenting himself as a candidate. Only extremists would be willing to do so. But if any one of them presents his candidacy, either the median voter or another one of the potential consensus candidates will now present himself to prevent this extremist from choosing policy. This is not an equilibrium, since the extremist now obtains only costs presenting himself and losing, so he would drop from the race. If the extremist drops his candidacy, then so will the moderate, for once the menace of an extreme outcome has disappeared, there is no point in assuming the cost of running for office.

**Theorem 1** *If* $q \in \left(\pi_m - \frac{1}{2}c + d_a, \pi_m + \frac{1}{2}c - d_b\right)$, *then there is no equilibrium with a single candidate.*

**Proof.** Condition to run unopposed: $\pi_i \in X^*_{con}$.

Condition to be willing to run: $\pi_i \in [0, q - c] \cup [q + c, 1]$. But:

$q \in \left(\pi_m - \frac{1}{2}c + d_a, \pi_m + \frac{1}{2}c - d_b\right)$ implies that $X^*_{con} \bigcap [[0, q - c] \cup [q + c, 1]] = \emptyset$ ■

We have now completed our study of the implications of the different possible values of the status quo policy with regards to the existence of uncontested elections when the number of citizens is odd. We have seen that there are two relevant intervals:

First, there is an interval around the median of the favorite policies, which contains the potentially unopposed candidates, the source of single candidate equilibria. This interval is fixed. And there is a second relevant interval, around the status quo: Citizens whose favorite policies are in this interval will not be interested in running for office unopposed.

The interval around the status quo can be imagined as a dark cloud: If it is far from the median, it does not affect the equilibria; as it gets closer to the median, it "hides" part of the first relevant interval, or the totality of it. As in an eclipse, partial or total eclipse, hopes of a single candidate equilibrium fade and disappear from view when the two intervals, one around the median, another around the status quo, are one on top of each other. We can visualize this effect by comparing Figure 2 (two separated intervals), Figure 3 ("partial eclipse") and Figure 4 ("total eclipse"). If the "dark cloud" was blown by the wind to even higher values, single candidate equilibria will resuscitate once more, first the lower values near the median, the whole interval $X^*_{con}$ afterwards.

At this point we find it worthwhile to briefly digress in order to note that our results to note are not robust to the consideration of a benefit from holding office $\gamma > 0$. If the benefit from holding office is positive but less than the cost of entry, results are qualitatively the same, but the interval of "potential consensus candidates" shrinks exactly as if the cost of entry was lowered by the amount of benefit from holding office. For one candidate equilibria, we can consider $c' = c - \gamma$ to be the effective cost of entry, and our results hold true by simply replacing $c$ by $c'$. However, if $\gamma > c$, then condition *i)* in Proposition 3 is trivially satisfied and never binds, $X^*_{con} = \pi_m$, our results do not apply, and the median is the single candidate in equilibrium, regardless of the status quo value.

In the absence of high benefits from holding office capable of offsetting the cost of entry, the inexistence of single candidate equilibria when the status quo takes a moderate value provides a stronger motivation to pay attention to multi-candidate equilibria. We now proceed to study if the status quo plays any role in determining such equilibria.

Proposition 2 in Section 3 characterized equilibria with two candidates. We now argue that it holds as stated for any value of status quo policy:

Condition *2.i)* says that both candidates would tie in an election, since their ideal policy points are located at exactly the same distance from the median. This electoral result will occur regardless of the value of the status quo.

Condition *2.ii)* states that, given a one half chance of winning, both citizens prefer to maintain their candidacy regardless of the value of the status quo policy. The intuition is that, in a two candidates equilibrium, if one of the two decided to drop his candidacy, he would consider that his opponent will still run for office, and will, of course, win. This possibility is so undesirable for the citizen who is hesitating to run, that he will present his candidacy and pay the entry cost in order to have at least a fifty per cent chance of avoiding such an unattractive outcome. The status quo would only come into play if both of them drop their candidacy simultaneously, which may not occur since the citizens do not coordinate their actions. Therefore, both citizens prefer to maintain their candidacies and this kind of equilibrium is not affected by the value of the status quo policy.[1]

The same argument applies to any situation which involves the entry decisions of a bigger number of citizens: If $n$ citizens were running in equilibrium, each one of them stays in the race because he expects the electoral outcome to significantly worsen if he dropped his candidacy and the race was only between the remaining $n-1$ citizens. Three-candidate equilibria did not exist when the status quo was extreme, and will not exist if the status quo is moderate or anything in between, for the status quo does not enter into the calculations of the outcome of an hypothetical election between the other two opponents, calculations that each one of the candidates has to do in order to decide whether to run or not. If one of the three dropped out of the race, sincere voting will result to elect a winner between the remaining two, and no attention will be paid to the status quo. Besley and Coate present conditions under which at least one of the candidates always has an incentive to step out of the race. Under these conditions, the location of the status quo is irrelevant to the inexistence of equilibria with three candidates.

Multi-candidate equilibria with more than three candidates are more involved. Besley and Coate argue that there cannot be three tying winning candidates, but equilibria with one or two winners and several "spoilers" (candidates who are losing) are possible. In order for $n$ candidates to run for office in equilibrium, the following conditions must hold:

1. Voters for any of the spoilers must be indifferent between all winning candidates.

2. Common beliefs about the voting behavior that would result if any of the candidates dropped his candidacy are such that each one of the candidates, winners and spoilers alike, expect the political outcome to significantly shift away from his preferences if he is the only one to quit the race.

Condition 2 sets a very demanding requirement in beliefs, which makes the existence of these kind of equilibria less plausible. Informally, such beliefs imply that spoilers are not spoiling the chances of the candidates with preferences close to theirs, but on the contrary they are preventing some far-off unwanted candidate from winning. The assumption of Euclidean preferences lead naturally to the opposite beliefs (i.e. a far left candidate's decision to run is generally believed to handicap the chances of victory for a moderate leftist candidate, not that of a rightist candidate).

[1] The consideration of the benefits from holding office would require to replace $c$ by $c'' = c - \frac{1}{2}\gamma$, making condition *2.ii)* trivially satisfied for $\gamma \geq 2c$.

Nevertheless, these kind of equilibria might exist. If they do, the location of the status quo need not affect them, but it may well do so if the value of the status quo somehow influences the common beliefs about the voting behavior off the equilibrium path (beliefs about what would happen if one of the candidates quit the race). The status quo might affect beliefs by providing a cue (a reference point) to each citizen about which candidate the majority of voters with similar preferences would select as the "probable winner" that it is strategically convenient to support to victory if the equilibrium was broken by the withdrawal of a candidacy.

We conclude this section by summarizing its main results: As the status quo is less extreme, there are less citizens who may run as the single candidate, and if the status quo is sufficiently close to the median voter's favorite policy, uncontested elections will not occur (unless the benefits from holding office offset the cost of running for office). The status quo has no influence over the existing two-candidate equilibria, nor over the inexistence of three candidate equilibria, and the influence over equilibria with more than three candidates depends on the specification of beliefs.

# 5 Equilibria with Even Number of Citizens

In this section we extend the analysis of equilibria to encompass both an even and odd number of citizens. We do so for a general expression of the status quo value, so taking $q$ to be zero would allow the reader to compare the differences from Besley and Coate (1997) that are due exclusively to $N$ being even. As in their one-dimensional model, in this section we assume that citizens care only about the implemented policy. We will argue that the influence of the status quo on policy outcomes is qualitatively the same for either an even or an odd number of citizens.

We require a bit more notation to deal with the new definition of a median with an even number of citizens:

Let us order the citizens according to their ideal policy from first (furthest low or left) to last (furthest high or right). Then let $\pi_{lm}$ (for "lower median") denote the ideal point of the citizen in the $\frac{N}{2}$th position, and let $\pi_{hm}$ (for "higher median") denote the ideal point of the citizen in the $\left(\frac{N}{2}+1\right)$th position. Denote by $g$ (for "gap") the distance $g = \pi_{hm} - \pi_{lm}$. If this "gap" $g$ is zero, we can say $\pi_{lm} = \pi_{hm} = \pi_{m.}$

Let us also extend our definition and intuition of $d_a$, $d_b$, $X_{con}$ and $X^*_{con}$ (which we previously defined for the specific case in which there is an odd number of citizens) to the general case in which we may have any number of citizens:

$X_{con} = [\underline{x}, \overline{x}]$ is the "strict consensus interval" and its definition remains unchanged, though its extension will now also depend on the gap $g$ between medians, and not only in the entry cost $c$. Candidates with an ideal point within it would be able to run unopposed no matter what other citizens' ideal policies may be.

$d_a$ and $d_b$ are (respectively) the minimum distance from $a$ and $b$ to some citizen's ideal policy. And $X^*_{con} = [\underline{x} - d_b, \overline{x} + d_a]$ is the interval such that any citizen whose ideal policy lies in it is a potential consensus candidate, and would face no opposition if he chose to run for office.

What does change is the location of $a$ and $b$, which are functions of the location of the two medians ideal policies $\pi_{lm}$ and $\pi_{hm}$, and of the cost of entry $c$. $a$ and $b$ are such that a citizen $k$ with an ideal policy $\pi_k = a$ is the only citizen that would run against any hypothetical candidacy $\pi_y \in (\overline{x}, 1]$ and only a citizen $p$ with a preferred policy $\pi_p = b$ would run against any conceivable candidacy $\pi_y \in [0, \underline{x})$.

Summarizing what we have just said: lone candidates with favorite policies between $\underline{x}$ and $\overline{x}$ will never face opposition in equilibrium; candidates with preferences just below $\underline{x}$ would see how a candidate with an ideal point of $b$ runs to defeat them, but if there is no one around $b$, then candidates to the left of $\underline{x}$ can still run unopposed as long as their distance to the left of $b$ is no bigger than the radius of the empty interval around $b$. Similarly, candidates to the right $\underline{x}$ can run unopposed as long as there is an empty interval around $a$.

Now we are ready to generalize Proposition 3 in order to encompass both the $N$ even and the $N$ odd cases:

**Proposition 6** *For any number of citizens, a single candidate equilibrium in which $i$ is the unopposed candidate exists if and only if:*
*(i) $|\pi_i - q| \geq c$, and*
*(ii) $\pi_i \in X^*_{con}$.*

Proposition 6 holds regardless of the number of citizens essentially due to the way in which we have chosen to define $X^*_{con}$: The first condition guarantees that $i$ would like to run unopposed; and the second condition says that no one would like to oppose him if $i$ chose to run.

However, in order to have a complete characterization of the single candidate equilibria, we are required to specify the limits of $X^*_{con}$, or in other words we need to specify $\underline{x}$, $\overline{x}$, $a$ and $b$, which determine (respectively), the lower and upper limits of the "strict consensus interval", $d_a$, and $d_b$.

**Case 1.1** *If $N$ is odd, or $N$ is even and $g = 0$, then $\underline{x} = \pi_m - \frac{1}{2}c$; $\overline{x} = \pi_m + \frac{1}{2}c$; $a = \pi_m - \frac{1}{2}c$ and $b = \pi_m + \frac{1}{2}c$.*

Section 4 dealt with the odd case. For the other subcase, it is enough to note that if $N$ is even but the "gap" between medians is zero, then the situation is the same as if the two identical medians were only one and the number of citizens was odd: Whoever is closer to the two medians in a two way race will win, so the model works exactly the same as in the $N$ odd case.

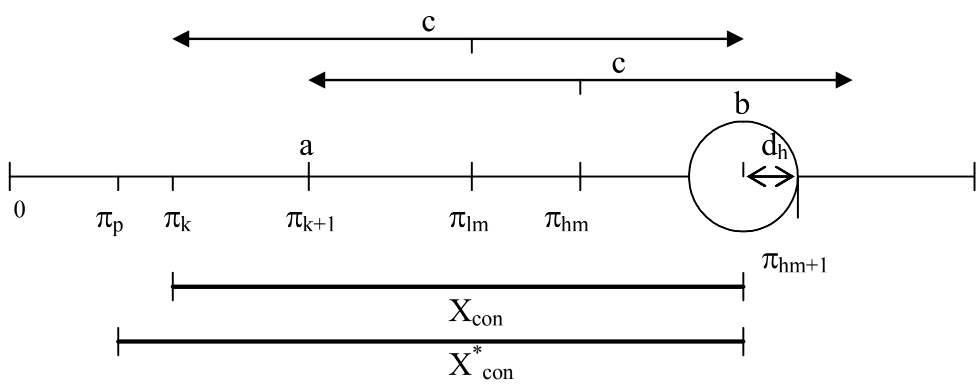


Figure 5: Extended consensus interval

**Case 1.2** *If $N$ is even, and $0 < g \leq \frac{1}{2}c$, then $\underline{x} = \pi_{lm} - \frac{1}{2}c$; $\overline{x} = \pi_{hm} + \frac{1}{2}c$; $b = \pi_{lm} + \frac{1}{2}c$; $a = \pi_{hm} - \frac{1}{2}c$.*

If a citizen $k$ whose favorite policy is $\pi_k = \pi_{lm} - \frac{1}{2}c$ presents his candidacy, only citizens with ideal preferences above $b = \pi_{lm} + \frac{1}{2}c$ would be willing to run against $k$, and they do not do so because the "lower median" $lm$ would not vote for them against $k$, and therefore they would not win, but at most tie if they gathered support from the "high median" $hm$. However, in order for a citizen $h$ to be willing to run for a tied race, then the distance from her ideal policy $\pi_h$ to $\pi_k$ ought to be twice the cost of running and such big distance implies that the high median would actually prefer $k$ to win, so $h$ would not even tie but lose if she run for office.

The inequality $\pi_h \geq \pi_k + 2c$ implies $\pi_h - \pi_{hm} \geq 2c - (\frac{1}{2}c + g) \geq \frac{1}{2}c + g = \pi_{hm} - \pi_{k.}$ Therefore, $\pi_{lm} - \frac{1}{2}c \in X_{con}$ and citizen $k$ is a "potential consensus candidate". Symmetrically, so would a citizen with an ideal preference of $\pi_{hm} + \frac{1}{2}c$, and of course so would anyone in between. But now, imagine a citizen $p$ with an ideal point anywhere below that of $k$. Then a citizen with an ideal point of $b$ would be willing to run against $p$ and would gather the vote of the low median, thus defeating $p$. Therefore, $p$ cannot be a single candidate in equilibrium... Unless there is no one around $b$ to defeat $p$, as it is the case in Figure 5, where no citizen has as ideal policy anywhere in between $\pi_{hm}$ and $\pi_{hm+1}$, so the interval of potential consensus extends all the way down to $\pi_p$.

**Case 1.3** *If $N$ is even, and $\frac{1}{2}c < g \leq 2c$, then $\underline{x} = \pi_{hm} - c$; $\overline{x} = \pi_{lm} + c$; $b = \pi_{hm} + c$; $a = \pi_{lm} - c$.*

A good way to visualize why these are the correct values is to start with $\pi_{lm}$, and then move away to more extreme positions. The low median could be a single candidate, for no one can beat him, and those who can tie (the high median and whoever is near the high median) are not willing to pay the cost of running for a tied race. As we marginally move to lower ideal values close to $\pi_{lm}$, the same argument holds (adding that only the low median could defeat them and is clearly unwilling to run to do so), until we reach

$\pi_k = \pi_{hm} - c$. A citizen with an ideal point of $b = \pi_{hm} + c$ will be willing to pay the cost of running just for a tied race. In fact, for any citizen with ideal points below that of $k$, the high median and all those to the right of the median will rather have $b$ than $\pi_k$, thus the race will be at least tied, and the result is that no citizen with an ideal point below $\pi_{hm} - c$ can run unopposed, once again unless there is no citizen with an ideal point of $b$ or around $b$.

**Case 1.4** *If $N$ is even, and $g > 2c$, then $X^*_{con} = \phi$, there is no potential consensus candidate, and there is no single candidate equilibrium.*

If the distance separating the low and high medians is sufficiently high, or alternatively, if the cost of running for office is sufficiently small and there is some distance between the ideal points of the two medians, then the medians would rather run, even against each other, and tie rather than letting an uncontested election happen.

With regards to condition *(i)* in Proposition 6, an analysis of the effect of different values of the status quo policy over single candidate equilibria would prove to be analogous to the one we did for the $N$ odd case:

If the status quo policy is extreme (either too low or too high), then any potential consensus candidate may stand as an unopposed candidate in equilibrium.

As the status quo takes more moderate values, some of the potential consensus candidates would find no incentive to run alone to replace it, so the set of possible uncontested candidates in equilibrium shrinks as the status quo policy comes approaches the median(s).

If the status quo policy comes too close to the midpoint between the medians (how close depends on the extension of the interval of the potential consensus candidates), then no single candidate is guaranteed to run alone in equilibrium (in some knife-edge results it may be the case that a particular citizen in one of the extremes of $X^*_{con}$ may succeed to run unopposed due to some other citizen who is indifferent between running or not, choosing not to do so).

The idea is the same as what we have already iterated in the previous section: Of all the potential consensus candidates, those who are too close to the status quo will not run alone in equilibrium.

This is true regardless of the number of citizens being odd or even, or the distance between the ideal points of any citizen.

Also, it follows that if all the potential consensus candidates' ideal policies are nearby the status quo, there will be no uncontested election.

Since consensus candidates are relatively moderate, the conclusion is that a moderate status quo will effectively rule out uncontested elections.

Turning to two-candidate equilibria, we recall Proposition 2 in Section 3 and note that it holds both for the odd case, and for the even case in which both medians share the same ideal policy preference. However, if the medians' ideal policies are not the same, Proposition 2 no longer applies. Instead, we obtain the following result:

**Proposition 7** *If $N$ is even and $g > 0$, a two-candidate equilibrium in which $i$ and $j$ run against each other exists if and only if:*
*2.i)* $\frac{1}{2}(\pi_i + \pi_j) \in (\pi_{lm}, \pi_{hm})$, *and*
*2.ii)* $|\pi_i - \pi_j| \geq 2c$.

The intuition does replicate that of Proposition 2:

The first condition says that the race between the two candidates must be tied. This occurs whenever the society splits into halves: The low median and everyone to his left back one candidate; the right median and everyone to her right back the other candidate. Society will split into halves whenever the midpoint between the ideal policies of the candidates lies in between the ideal points of the medians.

The second condition should be familiar: It guarantees that both candidates are willing to run against each other.

We notice that $N$ being even instead of odd does not alter the fact that the status quo plays no role in the determination of two-candidate equilibria. Similarly, regardless of the number of citizens being odd or even, pure strategy equilibria with three candidates will not exist (under the mild conditions enumerated in Section 3), and the status quo will not affect equilibria with more than three candidates so long as it does not affect the beliefs of the candidates about the voting behavior that would result if one of them dropped out of the race (we refer to the previous section for the intuitions in support of this argument).

However, an important difference has arisen between the results in this section and the preceding one:

When the number of citizens is odd, two-candidate equilibria are generically non-existent. That is, they do not exist for almost all of the possible distributions of ideal policies that the citizens may have, or equivalently, they exist only for a set of measure zero of distributions of citizen's ideal policies.

Condition *(i)* in Proposition 2 stated: $\frac{1}{2}(\pi_i + \pi_j) = \pi_m$. Given that the number of citizens is finite, whereas the ideal preference of the median is located in a continuous interval and thus it can take any of an infinite number of possible values, it follows that only for a limited few out of an infinite number of possibilities will condition *(i)* be satisfied.

Contrary to this negative result, if $N$ is even and the two medians' ideal points are distinct, two-candidate equilibria are no longer generically non-existent, for condition *(i)* in Proposition 7 offers a whole interval of possible values for the average of the ideal policies of $i$ and $j$. Let $\pi = (\pi_1, \pi_2, ..., \pi_N)$ be the vector composed of all the ideal points of the $N$ citizens. If $N$ is even, and $c < \frac{1}{2}$, there exists an open set $U \subset [0,1]^N$ such that if $\pi \in U$, then there exists a two candidate equilibrium.

If $N$ is odd, there does not exist such an open set, and two candidate equilibria are generically non-existent.

From Propositions 6 and 7 together, we infer that the further apart the ideal policies of the two medians are in relation to the cost of entry (in other words, the smaller $c$ in relation to $g$), the more difficult it is for a single candidate to run unopposed in equilibrium, and the less stringent the requirements are for two candidate competition to be the electoral equilibrium.

In fact, if the number of citizens is even, for almost all possible distributions of ideal points, there exists some positive value of the entry cost for which a two-candidate equilibrium exists. We state the result in the following corollary.

**Corollary 3** *If the number of citizens is even, there exists a set $U \subset [0,1]^N$ with Lebesgue measure zero such that for any vector $\pi = (\pi_1, \pi_2, ..., \pi_N)$ of citizens' policy preferences not in $U$, there exists a positive value $c_\pi$ such that if the cost of entry $c$ is less than $c_\pi$, then there exists a two-candidate equilibrium.*

From the set of all the possible distributions of ideal policies $\{\pi_1, \pi_2, ..., \pi_n\}$ that a group of $N$ citizens ($N$ even) forming a society may have, the proportion of such distributions of ideal policies in which the two medians have the same ideal policy is zero. If the two medians have a different ideal policy, then for a cost of entry equivalent to less than a half of the distance between their favorite policies, they would run against each other in equilibrium. Thus for the complement of the set in which the medians share the same ideal point, there is some cost of entry that guarantees the existence of a two-candidate equilibrium.

A two-candidate equilibrium is also almost sure to exist in a society with an even number of citizens if we introduce a sufficiently large benefit from holding office. Specifically, if the benefit from holding office $\gamma$ is bigger than $2c - g$, then no citizen can be a single candidate in equilibrium, and there exists an equilibrium in which the two medians run against each other, whenever they have different ideal policies. This result contrasts with the case in which the number of citizens is odd, when a big benefit from holding office leads to the existence of a single candidate equilibrium for any value of the status quo policy, but it is similar to Osborne and Slivinski's (1996) results for equilibria under plurality rule.

The findings in this section have shown that as the gap separating the ideal policies advocated by the medians increases relative to the cost of entry, the set of citizens who may win an uncontested election in equilibrium shrinks and eventually becomes empty, whereas the possibility of a two-candidate competition outcome becomes more and more likely until such an of equilibrium surely exists.

# 6 Conclusions and Extensions

This paper has achieved two goals: Besley and Coate's (1997) model of representative democracy in its unidimensional version has been extended to consider societies with either an even or odd number of citizens, and to allow for any possible value of the status quo policy.

The status quo value has proven to be relevant in the characterization of equilibrium outcomes unless the reward for holding office is big compared to the cost of running as candidate. In the case with an odd number of citizens and an extreme status quo presented by Besley and Coate (1997), equilibria with two candidates are generically non-existent and the natural prediction of the model is an uncontested election in which an unopposed candidate wins. We have shown that a moderate value of the status quo policy eliminates the incentive to run as single candidate and choose the policy outcome that moderate citizens have when the status quo is extreme. As a consequence, if the status quo is moderate and there are no other incentives to run, uncontested elections will not occur in equilibrium.

We find that two-candidate equilibria exist more generally in a society with an even number of citizens. The influence of the status quo policy is qualitatively the same regardless of the number of citizens, but that of introducing a benefit from holding office is not. These differences between polities with an even or an odd number of citizens might suggest that the Besley and Coate (1997) model is better suited to explain the behavior of voters and candidates in a small society or committee such as the one in our example, rather than to explain bigger elections in which the exact number of citizens should play no significant role.

Nevertheless, with the assumptions of an even number of citizens and a moderate status quo, the model predicts no uncontested elections, and a race with two candidates is the most natural equilibrium. We believe this represents a gain in realism to explain big elections of a single representative, such as the US presidential elections. Therefore, we argue in favor of discarding the assumptions of an odd number of citizens or an extreme status quo when using the Besley and Coate (1997) model to analyze elections with a large electorate.

Some other extensions to Besley and Coate (1997) that we think deserve further research are the following:

1. To consider an endogenous status quo. Status quo could be a function of past implemented policies in a dynamic model. This is in fact very likely to be the case in many policy decisions. If there are no new proposals, probably the implemented policy will look to the past.

2. To allow candidates to share the cost of running for office with voters who would be willing to help in affording such expenses. In this approach, a candidate would be the

leader of a political party, and all the citizens who help in financing the campaign would be party members.

3. To develop a version of the model with incomplete information about the distribution of the ideal policies of the citizens.

We let the study of these questions for further research.

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